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THE MATRIX OF NANOSCALE EMITTERS: THE CROSS EFFECTS AND THE TUNNELING CURRENT

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Cross effects arising in a matrix of nanoscale emitters are studied. When the tips are situated on the surface of the cathode sufficiently far from each other, one can treat them as independent. However, when the tips are closely set, the computation of the tunnelling current must take into account the cross effects and the field strength on their spikes should be decreased accordingly. In this paper we obtain the corresponding formulae.

Key words: scanning microscopy, cross effects, tunneling current.

At present time in different modern instruments, which principle of operation is based on the phenomenon of cold emission, an idea arises to use several or even many conducting tips instead of one, as well as necessity of consistent correct description of their mutual influence, that is cross effects. Changing the shape and geometric sizes of conducting probes, as well as the distance between them, one can achieve optimal relations when using them simultaneously in the scanning tunneling microscope and other analogous instruments with the purpose of the improvement of their work. Let us note that recent papers in the field of scanning tunneling microscopy are devoted to carbon nanotubes [1] and graphene [2–6], that is one of the most perspective directions of modern physics.

Obviously, if tips are situated on the surface of the cathode sufficiently far from each other (in other words, if the average distance l between them is much larger, then the distance d between electrodes, that is $l \gg d$), then we can neglect their mutual influence and consider them as independent. At the same time cross effects arise between closely set tips (that is for $l \ll d$). They lead to decrease of the absolute value E of strength $\mathbf{E}$ of the electric field close to a spike of every tip (emitter), therefore the corresponding tunneling current also noticeably decreases.

Emitters have nanoscales, being favorable to considerable increase of the quantity E close to them, and for correct determination of total cold emission current density, generally speaking, calling of mathematical apparatus of quantum mechanics is necessary. However, for not very little values of the distance l (in other words, for

$l^2 \gg S$, where S is an effective surface area of one emitter) cross effects have not quantum, but classic, electrostatic nature. In order to take them into account, it is necessary to solve a problem of distribution of the potential φ and field strength $\mathbf{E}$ between electrodes with highest possible accuracy. An attempt to do it, made in [7], was not crowned with success, because the first order approximation with respect to inhomogeneity of the field, obviously, is inapplicable close to the emitter, and exactly this spatial domain is of greatest interest. Neither other theoretical attempts, nor methods of computational modeling with the help of the program package "COMSOL Multiphysics" and others also brought desired results. Firstly, they did not achieve adequate accuracy and, secondly, they were confronted by different unlikely surmountable difficulties.

In this connection, in this paper, reasoning from [8], we propose and substantiate a sufficiently simple semiempirical formula, determining the quantity E on a spike of a tip in presence of neighboring tips depending on their average surface density on the plain surface of the cathode. This formula is in good agreement with experimental data and allows determining tunneling current density.

Following [8], by the example of only one solitary tip let us demonstrate appreciable increase of the quantity E close to its spike in comparison with the quantity U/d, corresponding to the case of the homogeneous electric field in the spatial domain, confined between two plain parallel electrodes, to which the voltage U is applied.

First of all, let us make use of a model of a tip in the form of a two-sheeted hyperboloid, which canonical equation reads

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{d^2} = -1\,, \tag{1}$$

where a, b, d are some positive constants. Assuming $b = a$, from (1) we obtain the canonical equation of the hyperboloid of revolution:

$$\frac{x^2 + y^2}{a^2} - \frac{z^2}{d^2} = -1\,. \tag{2}$$

Introducing the polar coordinate $\rho = \sqrt{x^2 + y^2}$ on the plane xy, from (2) we obtain

$$\frac{\rho^2}{a^2} - \frac{z^2}{d^2} = -1, \quad \frac{z^2}{d^2} - \frac{\rho^2}{a^2} = 1\,. \tag{3}$$

This is the canonical equation of the hyperbola in coordinates (z, ρ). Its vertex has coordinates $z = d$, $\rho = 0$, and the focus has coordinates $z = \sqrt{a^2 + d^2} \equiv f$, $\rho = 0$. Following [8], let us construct a family of confocal hyperbolas:

$$\frac{z^2}{\lambda^2} - \frac{\rho^2}{f^2 - \lambda^2} = 1\,, \tag{4}$$

where λ is a parameter of the considered family. When $\lambda = d$ we obtain the equation (3) of the surface of the tip, when $\lambda = 0$ we obtain the equation $z = 0$ of the plain surface of the anode, and when $0 < \lambda < d$ we obtain the equation of the some intermediate surface.

Obviously, the potential φ of the electric field in the considered case depends only on the introduced parameter λ. Let us write down Laplace's equation $\triangle\varphi = 0$, which it satisfies, in cylindrical coordinates, taking into account the cylindrical symmetry of the posed problem:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right) + \frac{\partial^2\varphi}{\partial z^2} = 0\,. \tag{5}$$

Switching from cylindrical coordinates (z, ρ) directly to the parameter λ, it can be shown (see [8]) that on the spike of the tip ($z = d$, $\rho = 0$; $\lambda = d$) strength of the electric field is directed along the axes z and is maximum with respect to the absolute value:

$$E = \frac{2Uf}{(f^2 - d^2)\ln[(f + d)/(f - d)]} = \frac{U}{d} \cdot \frac{2fd}{(f^2 - d^2)\ln[(f + d)/(f - d)]}\,. \tag{6}$$

where U is the voltage between electrodes, f is the distance between the plane modeling the anode and the focus of the hyperboloid of revolution modeling the cathode.

Thus, the quantity E on the spike of the tip, in accordance with (6), exceeds the quantity U/d in $\dfrac{2fd}{(f^2 - d^2)\ln[(f + d)/(f - d)]}$ times. For little difference between quantities f and d (in other words, for big curvature and, accordingly, a small radius of curvature of the spike) this number is sufficiently big. However, it appreciably decreases, if the considered tip is not solitary and there are other, neighboring tips close to it. Therefore, the corresponding current of cold emission also decreases.

In order to take into account cross effects, arising between closely set tips, we shall consider one of the most widespread basic cases, when their position on the cathode is approximately uniform. Then their same number on average falls on every element of its surface.

Let us introduce an average surface density of tips σ, equal to the ratio of their total number to the total surface area of the cathode, and assume that a critical value σ_c exists and for $\sigma < \sigma_c$, that is for sufficiently large distances between neighboring tips, the absolute value E of strength of the electric field on the spike of each of them is determined by the formula (6) and for arbitrary σ – by the modified formula

$$E = \frac{U}{d} + \left\{\frac{2Uf}{(f^2 - d^2)\ln[(f + d)/(f - d)]} - \frac{U}{d}\right\}\theta(\sigma), \quad \theta(\sigma) = \begin{cases} 1, \ \sigma \leqslant \sigma_c \\ \sigma_c/\sigma, \ \sigma \geqslant \sigma_c \end{cases}. \tag{7}$$

When $\sigma \to 0$, that is for very large distances between neighboring tips or, in other words, for very large surface area of the cathode, falling on one tip, from (7) we obtain $\theta(\sigma) = 1$ and the formula (6). Thus, in this limiting case tips are independent and cross effects are imperceptible.

In the opposite case, when $\sigma \to +\infty$, that is for very small distances between neighboring tips or, in other words, for their very big number, falling on the unit of the surface area of the cathode, from (7) we obtain $\theta(\sigma) \to 0$ and $E \to U/d$. Thus, in this limiting case tips are not independent and cross effects are so perceptible that the electric field is practically homogeneous and the absolute value E of its strength is determined by the standard ratio U/d.

Therefore, the proposed formula (7) demonstrates correct asymptotic behavior for very rare and very thick positions of tips on the surface of the cathode. Let us note that it is in good agreement with experimental data and allows determining tunneling current density, and the approach itself, leading to it, can find application in the field of scanning tunneling microscopy.

Obtained in this paper results allow to draw the following conclusion: we have proposed and substantiated the semiempirical formula (7) for the absolute value of strength of the electric field on the spike of one tip in presence of neighboring tips, taking into account cross effects between them and demonstrating correct asymptotic behavior.

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МАТРИЦЯ НАНОРОЗМІРНИХ ЕМІТЕРІВ: ПЕРЕХРЕСНІ ЕФЕКТИ І ТУНЕЛЬНИЙ СТРУМ

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Досліджено перехресні ефекти, що виникають у матриці нанорозмірних емітерів. Якщо голки розташовані на поверхні катоду достатньо далеко одна від одної, ми можемо вважати їх незалежними. Проте якщо голки є розташовані близько, для розрахунків тунельного струму потрібно брати до уваги перехресні ефекти та відповідно зменшити напруженість поля на їхніх вістрях. У цій статті отримано відповідні формули.

Ключові слова: сканувальна мікроскопія, перехресні ефекти, тунельний струм.

МАТРИЦА НАНОРАЗМЕРНЫХ ЭМИТТЕРОВ: ПЕРЕКРЕСТНЫЕ ЭФФЕКТЫ И ТУННЕЛЬНЫЙ ТОК

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Исследованы перекрестные эффекты, возникающие в матрице наноразмерных эмиттеров. Если иглы расположены на поверхности катода достаточно далеко друг от друга, мы можем считать их независимыми. Однако если иглы близко расположены, для расчета туннельного тока следует принимать во внимание перекрестные эффекты и соответствующим образом уменьшить напряженность поля на их остриях. В данной статье получены соответствующие формулы.

Ключевые слова: сканирующая микроскопия, перекрестные эффекты, туннельный ток.

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