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Quantifying concurrence of two-qubit quantum states on a quantum computer

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The concurrence of two-qubit quantum states is considered. Relation between the concurrence and the parameters of an arbitrary two-qubit quantum state is presented and quantified with quantum computing. For this purpose the mean values of spin operators are calculated on a quantum device. Results obtained on a quantum computer are in a good agreement with theoretical ones.

Key words: entanglement, concurrence, two-qubit quantum states, quantum computing.

1. Introduction

Quantum entanglement is a phenomenon that occurs in a system of quantum particles. If quantum particles are in entangled state, quantum state of each particle in the system cannot be written independently on states of the other particles. Entanglement is possible over long distances [1] which is used in quantum communications and quantum cryptography [1–6]. Also, entanglement plays an important role in quantum metrology for reducing the statistical error of measurement [7–9]. Usage of the entanglement gives an opportunity to reach the quantum supremacy. For instance, in papers [10–13] is shown that the quantum entanglement is present in the Shor's algorithm.

Among entangled states a lot of attention has been paid to the studies of the quantum graph states, namely, the states that can be represented with graphs. In paper [14] a quantum graph state is prepared on IBM's quantum computer. It is shown that a quantum computer can be in a maximally entangled state. In papers [15,16] the geometric measure of entanglement of quantum graph states is studied analytically and using quantum computations.

In this work we prepare quantum protocols for measuring the concurrence of two-qubit quantum states. We also present analytical relation between the concurrence and parameters of the state. The concurrence of two-qubit quantum states is calculated on IBM's quantum computer.

The paper is organized in the following way. In Section 2 we present theoretical relation between the concurrence and the parameters of two-qubit quantum state in a general case. Then we consider relation of the parameters and the mean values of the spin operator, and correlators. In Section 3 we present quantum protocols for calculation of the mean values of the spin operator and correlators. We also demonstrate results of the concurrence obtained theoretically and using quantum computations on the quantum computer IBM Lima for three different states. Conclusions are done in the last Section.

2. Concurrence of two-qubit quantum states

In general case a two-qubit quantum state can be written as

$$|\Psi\rangle = a\,|00\rangle + be^{i\phi_1}\,|01\rangle + ce^{i\phi_2}\,|10\rangle + de^{i\phi_3}\,|11\rangle\,, \tag{1}$$

where a, b, c, d are amplitudes that satisfy equation

$$a^2+b^2+c^2+d^2=1, \tag{2}$$

which occurs due to normalization condition $\langle\Psi|\Psi\rangle=1$. Parameters ϕ_1, ϕ_2, ϕ_3 are phases.

The concurrence for the state (1) can be calculated as

$$C = 2|ade^{i\phi_3} - bce^{i(\phi_1+\phi_2)}|, \tag{3}$$

see, for instance, [17,18]. The expression can be rewritten as follows

$$C = 2\sqrt{a^2d^2+b^2c^2-2abcd[\cos\phi_3(\cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2)+ +\sin\phi_3(\sin\phi_1\cos\phi_2+\cos\phi_1\sin\phi_2)]} = 2\sqrt{a^2d^2+b^2c^2-2abcd\cos(\phi_3-\phi_1-\phi_2)}, \tag{4}$$

where $\cos(\phi_3-\phi_1-\phi_2)$, accodding to relations (5)-(9) below,

$$\cos(\phi_3-\phi_1-\phi_2) = \left[\frac{\langle\sigma_1^x\sigma_2^x\rangle - \langle\sigma_1^y\sigma_2^y\rangle}{1+\langle\sigma_1^z\rangle+\langle\sigma_2^z\rangle+\langle\sigma_1^z\sigma_2^z\rangle}\times \right.$$
$$\left.\times\frac{(\langle\sigma_2^x\rangle+\langle\sigma_1^z\sigma_2^x\rangle)(\langle\sigma_1^x\rangle+\langle\sigma_1^x\sigma_2^z\rangle)-(\langle\sigma_2^y\rangle+\langle\sigma_1^z\sigma_2^y\rangle)(\langle\sigma_1^y\rangle+\langle\sigma_1^y\sigma_2^z\rangle)}{\sqrt{((1+\langle\sigma_1^z\rangle)^2-(\langle\sigma_2^z\rangle+\langle\sigma_1^z\sigma_2^z\rangle)^2)((1-\langle\sigma_1^z\rangle)^2-(\langle\sigma_2^z\rangle-\langle\sigma_1^z\sigma_2^z\rangle)^2)}}\right]+$$
$$+\left[\frac{\langle\sigma_1^x\sigma_2^y\rangle - \langle\sigma_1^y\sigma_2^x\rangle}{1+\langle\sigma_1^z\rangle+\langle\sigma_2^z\rangle+\langle\sigma_1^z\sigma_2^z\rangle}\times \right.$$
$$\left.\times\frac{(\langle\sigma_2^y\rangle+\langle\sigma_1^z\sigma_2^y\rangle)(\langle\sigma_1^x\rangle+\langle\sigma_1^x\sigma_2^z\rangle)-(\langle\sigma_2^x\rangle+\langle\sigma_1^z\sigma_2^x\rangle)(\langle\sigma_1^y\rangle+\langle\sigma_1^y\sigma_2^z\rangle)}{\sqrt{((1+\langle\sigma_1^z\rangle)^2-(\langle\sigma_2^z\rangle+\langle\sigma_1^z\sigma_2^z\rangle)^2)((1-\langle\sigma_1^z\rangle)^2-(\langle\sigma_2^z\rangle-\langle\sigma_1^z\sigma_2^z\rangle)^2)}}\right].$$

The parameters of the quantum state can be found with evaluation of the mean values of spin operators. Namely the following relations are satisfied

$$a=\frac{1}{2}\sqrt{1+\langle\sigma_1^z\rangle+\langle\sigma_2^z\rangle+\langle\sigma_1^z\sigma_2^z\rangle},\quad b=\frac{1}{2}\sqrt{1+\langle\sigma_1^z\rangle-\langle\sigma_2^z\rangle-\langle\sigma_1^z\sigma_2^z\rangle}, \tag{5}$$

$$c = \frac{1}{2}\sqrt{1 - \langle\sigma_1^z\rangle + \langle\sigma_2^z\rangle - \langle\sigma_1^z\sigma_2^z\rangle}, \quad d = \frac{1}{2}\sqrt{1 - \langle\sigma_1^z\rangle - \langle\sigma_2^z\rangle + \langle\sigma_1^z\sigma_2^z\rangle}, \tag{6}$$

$$\cos\phi_3 = \frac{\langle\sigma_1^x\sigma_2^x\rangle - \langle\sigma_1^y\sigma_2^y\rangle}{4ad}, \quad \sin\phi_3 = \frac{\langle\sigma_1^x\sigma_2^y\rangle + \langle\sigma_1^y\sigma_2^x\rangle}{4ad}, \tag{7}$$

$$\cos\phi_2 = \frac{\langle\sigma_1^x\rangle + \langle\sigma_1^x\sigma_2^z\rangle}{4ac}, \quad \sin\phi_2 = \frac{\langle\sigma_1^y\rangle + \langle\sigma_1^y\sigma_2^z\rangle}{4ac}, \tag{8}$$

$$\cos\phi_1 = \frac{\langle\sigma_2^x\rangle + \langle\sigma_1^z\sigma_2^x\rangle}{4ab}, \quad \sin\phi_1 = \frac{\langle\sigma_2^y\rangle + \langle\sigma_1^z\sigma_2^y\rangle}{4ab}, \tag{9}$$

$$\cos(\phi_3 - \phi_1) = \frac{\langle\sigma_1^x\rangle - \langle\sigma_1^x\sigma_2^z\rangle}{4bd}, \quad \sin(\phi_3 - \phi_1) = \frac{\langle\sigma_1^y\rangle - \langle\sigma_1^y\sigma_2^z\rangle}{4bd}, \tag{10}$$

$$\cos(\phi_2 - \phi_1) = \frac{\langle\sigma_1^x\sigma_2^x\rangle + \langle\sigma_1^y\sigma_2^y\rangle}{4bc}, \quad \sin(\phi_2 - \phi_1) = \frac{\langle\sigma_1^y\sigma_2^x\rangle - \langle\sigma_1^x\sigma_2^y\rangle}{4bc}, \tag{11}$$

$$\cos(\phi_3 - \phi_2) = \frac{\langle\sigma_2^x\rangle - \langle\sigma_1^z\sigma_2^x\rangle}{4cd}, \quad \sin(\phi_3 - \phi_2) = \frac{\langle\sigma_2^y\rangle - \langle\sigma_1^z\sigma_2^y\rangle}{4cd}. \tag{12}$$

In the next section we construct quantum protocols for detection of the concurrence and present results of quantum computations on IBM's quantum device.

3. Evaluation of the concurrence on IBM's quantum computer

Mean value of the operator σ_i^z in an arbitrary state $|\Psi\rangle$ can be measured on a quantum computer using its relation with the probabilities

$$\langle\Psi|\,\sigma_i^z\,|\Psi\rangle = |\langle 0|\Psi\rangle|^2 - |\langle 1|\Psi\rangle|^2 = P_{|0\rangle} - P_{|1\rangle}, \tag{13}$$

where $P_{|0\rangle}$, $P_{|1\rangle}$ are the probabilities of reduction to state $|0\rangle$ and state $|1\rangle$, respectively, after the measurement in the standard basis performed on the i-th qubit.

To calculate the mean value of the operator σ_i^x, or σ_i^y in an arbitrary state we need to modify the initial state of a particular qubit before the measurement. Operators σ^x, σ^y can be represented as

$$\sigma^x = e^{-i\frac{\pi}{4}\sigma^y}\sigma^z e^{i\frac{\pi}{4}\sigma^y} = RY^+\left(-\frac{\pi}{2}\right)\sigma^z RY\left(-\frac{\pi}{2}\right), \tag{14}$$

$$\sigma^y = e^{i\frac{\pi}{4}\sigma^x}\sigma^z e^{-i\frac{\pi}{4}\sigma^x} = RX^+\left(\frac{\pi}{2}\right)\sigma^z RX\left(\frac{\pi}{2}\right). \tag{15}$$

So, the mean values $\langle\Psi|\,\sigma^x\,|\Psi\rangle$, $\langle\Psi|\,\sigma^y\,|\Psi\rangle$ can be written in the following form

$$\langle\Psi|\,\sigma^x\,|\Psi\rangle = \langle\Psi|\,RY^+\left(-\frac{\pi}{2}\right)\sigma^z RY\left(-\frac{\pi}{2}\right)|\Psi\rangle = \langle\Psi_i^y|\,\sigma^z\,|\Psi_i^y\rangle = |\langle 0|\Psi_i^y\rangle|^2 - |\langle 1|\Psi_i^y\rangle|^2, \tag{16}$$

$$\langle\Psi|\,\sigma^y\,|\Psi\rangle = \langle\Psi|\,RX^+\left(\frac{\pi}{2}\right)\sigma^z RX\left(\frac{\pi}{2}\right)|\Psi\rangle = \langle\Psi_i^x|\,\sigma^z\,|\Psi_i^x\rangle = |\langle 0|\Psi_i^x\rangle|^2 - |\langle 1|\Psi_i^x\rangle|^2, \tag{17}$$

where $|\Psi_i^y\rangle = RY(-\pi/2)\,|\Psi\rangle$, and $|\Psi_i^x\rangle = RX(\pi/2)\,|\Psi\rangle$.

In the case of mean value $\langle\Psi|\,\sigma_i^z\sigma_j^z\,|\Psi\rangle$, $(i \neq j)$ there is the following relation to the probabilities obtained on a quantum computer after measurements in the standard basis

$$\begin{aligned}\langle\Psi|\,\sigma_i^z\sigma_j^z\,|\Psi\rangle &= |\langle 00|\Psi\rangle|^2 + |\langle 11|\Psi\rangle|^2 - |\langle 01|\Psi\rangle|^2 - |\langle 10|\Psi\rangle|^2 = \\ &= P_{|00\rangle} + P_{|11\rangle} - P_{|01\rangle} - P_{|10\rangle},\end{aligned} \tag{18}$$

where $P_{|00\rangle}$, $P_{|11\rangle}$, $P_{|01\rangle}$, $P_{|10\rangle}$ are probabilities of the reduction of state $|\Psi\rangle$ to the basis states $|00\rangle$, $|11\rangle$, $|01\rangle$, $|10\rangle$, respectively.

Similarly, taking into account identities (14), (15) we write

$$\begin{aligned}\langle\Psi|\,\sigma_i^\alpha\sigma_j^\beta\,|\Psi\rangle &= \langle\Psi_{ij}^{\alpha\beta}|\,\sigma_i^z\sigma_j^z\,|\Psi_{ij}^{\alpha\beta}\rangle\,, \\ \langle\Psi|\,\sigma_i^\alpha\sigma_j^z\,|\Psi\rangle &= \langle\Psi_i^\alpha|\,\sigma_i^z\sigma_j^z\,|\Psi_i^\alpha\rangle\,,\end{aligned} \tag{19}$$

where $|\Psi_{ij}^{\alpha\beta}\rangle = R_i^\beta R_j^\alpha\,|\Psi\rangle$, $|\Psi_i^\alpha\rangle = R_i^\beta\,|\Psi\rangle$. The indices read $\alpha, \beta = (x, y)$ and $R_i^x = RX_i(\pi/2)$, $R_i^y = RY_i(-\pi/2)$.

Quantum protocols for calculation of the mean values of operators σ_0^z, σ_0^x, σ_0^y and the correlator $\sigma_0^x\sigma_1^y$ in a general two-qubit quantum state $|\psi\rangle = U\,|00\rangle$ considering relations (16)-(18) are presented in Fig. 1.

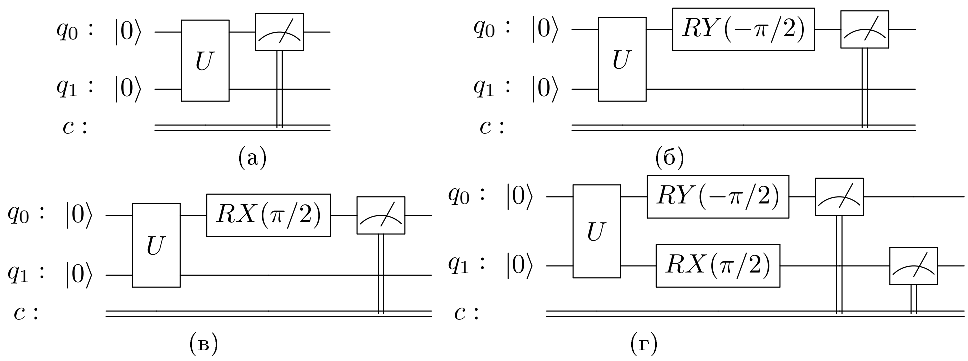

Fig. 1: Quantum protocols for calculation the meanvalues of operators σ_0^z (a), σ_0^x (b), σ_0^y (c) and $\sigma_0^x\sigma_1^y$ (d) in a general two-qubit quantum state.

On a quantum device we examine the dependence of the concurrence on the parameters of quantum states in particular cases. Namely, we consider quantum state of the form

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + e^{i\alpha_1}\,|01\rangle + e^{i\alpha_2}\,|10\rangle + e^{i\alpha_3}\,|11\rangle\right), \tag{20}$$

and study the dependence of the entanglement on the phase angels. The state (20) can be obtained using the identity $|\psi\rangle = P_0(\alpha_2)P_1(\alpha_1)CP_{01}(\alpha_3 - \alpha_2 - \alpha_1)H_0H_1\,|00\rangle$. The analytical result for the entanglement reads

$$C_{|\psi\rangle} = \frac{1}{\sqrt{2}}\sqrt{1 - \cos(\alpha_3 - \alpha_1 - \alpha_2)}. \tag{21}$$

To prepare the state $|\psi\rangle$ (20) we consider the following quantum protocol:

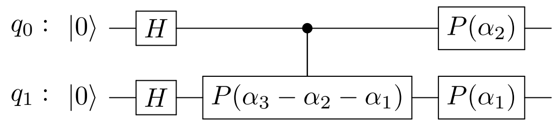

Fig. 2: Quantum protocol to prepare state $|\psi\rangle$ (20) on a quantum computer.

For the state $|\psi\rangle$ (20) we set a pair of variables from $\alpha_1, \alpha_2, \alpha_3$ equal to 0, the nonzero parameter is changed with a step $\pi/12$ starting from 0 and finishing on 2π. Results of quantum calculation of the concurrence using (4) and theoretical result are presented in Fig. 3.

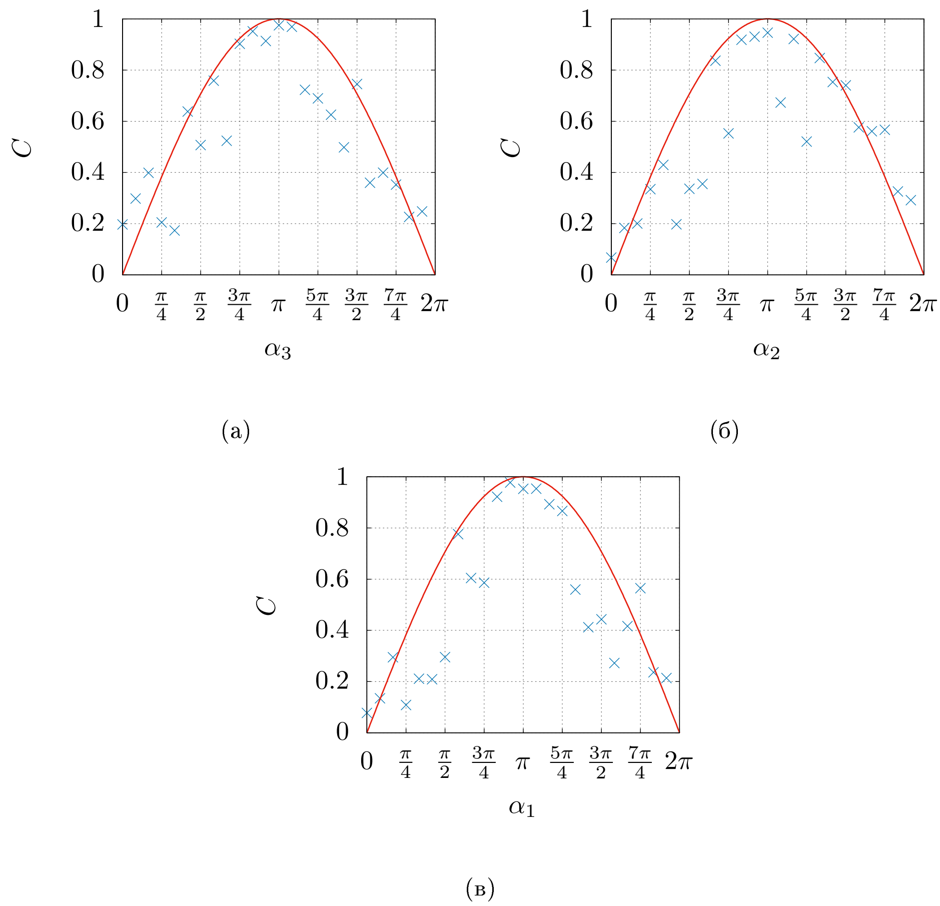

Fig. 3: Results of quantum calculations of concurrence for the state $|\psi(0, 0, \alpha_3)\rangle$ (a), $|\psi(0, \alpha_2, 0)\rangle$ (b), $|\psi(\alpha_1, 0, 0)\rangle$ (c) on a quantum computer IBM Lima (marked with crosses) and theoretical result (continuous line).

We also consider the Bell states and their generalizations. Namely, we examine the following two-qubit states

$$|\chi\rangle = \sigma_0^x CNOT_{01} \sigma_0^x RY(\theta)_0 |00\rangle = \cos(\theta/2)|01\rangle + \sin(\theta/2)|10\rangle, \quad (22)$$

$$|\phi\rangle = CNOT_{01} RY(\theta)_0 |00\rangle = \cos(\theta/2)|00\rangle + \sin(\theta/2)|11\rangle, \quad (23)$$

which can be prepared on a quantum computer using quantum protocols:

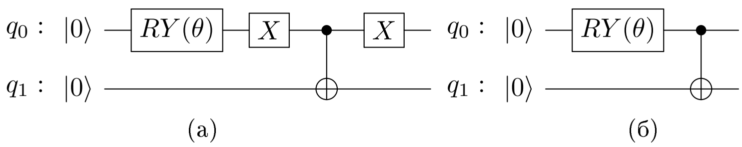

Fig. 4: Quantum protocols for preparation of the states (22) (a) and (23) (b).

In the case of states (22), (23) the analytical result for the entanglement reads

$$C_{|\chi\rangle,|\phi\rangle} = |\sin\theta|. \quad (24)$$

For these states we calculate concurrence with quantum programming taking into account its relation with correlators, using quantum protocols presented in Fig. 1 (protocols a, b, c, d to calculate $\langle\sigma_0^z\rangle$, $\langle\sigma_0^x\rangle$, $\langle\sigma_0^y\rangle$, $\langle\sigma_0^x\sigma_1^y\rangle$ respectively). For other correlators, protocols look in a similar way, considering equation (19). For calculations parameter θ is changed with step $\pi/12$ from 0 to 2π. Results of quantum computations and theoretical ones for the states (22), (23) are presented on Fig. 5.

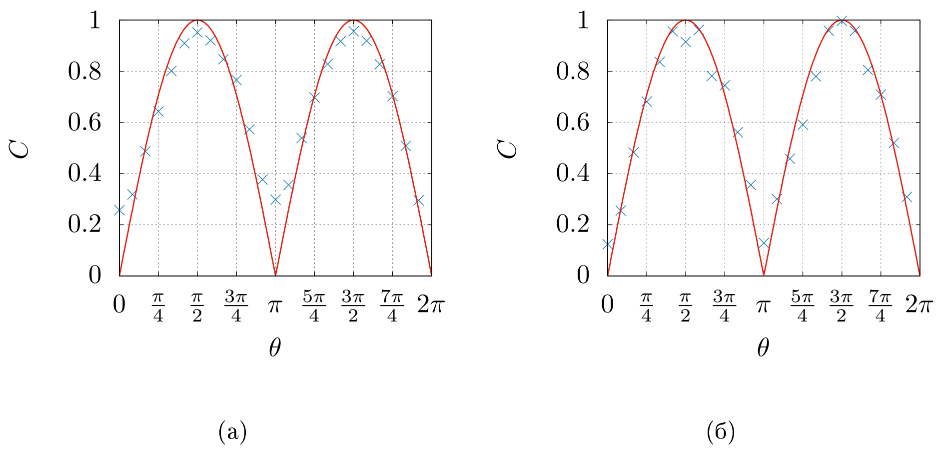

Fig. 5: Results of quantum calculations of concurrence of the state $|\chi(\theta)\rangle$ (22) (a), and of the state $|\phi(\theta)\rangle$ (23) (b) for different values of parameter θ calculated on quantum computer IBM Lima (marked with crosses) and theoretical results (continuous line).

At the end of this section, it is worth noting that there is another way of calculating of the concurrence of quantum states with quantum programming which is based on its relation with mean spin [18]. This is worth to be considered as separate studies.

Conclusions

Concurrence of 2-qubit quantum states has been studied theoretically and using quantum computations on IBM's quantum computer IBM Lima. Relation between the concurrence and parameters of an arbitrary quantum state has been presented. We have considered two-qubit states given by (20), (22), (23). Results of quantum calculations for concurrence of states (22), (23) are in good agreement with the theoretical ones (see Fig. 5). For the set of states with different phases (20) concurrence calculated with quantum programming are not in such a good agreement with analytical results (see Fig. 3). This is because quantum protocols for detection of the concurrence of state (20) are more gate expensive. According to (5)-(6) measuring amplitudes requires only finding mean values of the operator σ^z, so we do not need to additionally use rotation gates as when we are measuring phases (see equations (7)-(9)), thus there are more gate errors when measuring phases.

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Обчислення узгодженості двокубітних квантових станів на квантовому комп'ютері

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Ми розглянули задачу знаходження заплутаності стану двох кубітів за допомогою квантового програмування на квантовому комп'ютері компанії IBM. А саме, вивчається залежність узгодженості від параметрів квантового стану за допомогою обчислень на квантових комп'ютерах компанії IBM. Дослідження заплутаності квантових станів є важливим та актуальним. Це обґрунтовується тим, що квантова заплутаність є ключовою для створення квантового каналу, що використовується у квантових комунікаціях (квантовій телепортації, квантовій криптографії). Також заплутаність квантових станів є ключовим ресурсом для досягнення квантової переваги (виконання на квантових комп'ютерах задач за менший час та з меншою затратою ресурсів у порівнянні з класичними), створення алгоритмів корекції похибок квантових обчислень. У статті побудовано квантові протоколи для дослідження залежності узгодженості двокубітних квантових станів від їх параметрів. Розглядаються такі стани $(|00\rangle + \exp(i\alpha_1)\,|01\rangle + \exp(i\alpha_2)\,|10\rangle + \exp(i\alpha_3)\,|11\rangle)/2$, $\cos(\theta/2)\,|01\rangle + \sin(\theta/2)\,|10\rangle$, $\cos(\theta/2)\,|00\rangle + \sin(\theta/2)\,|11\rangle$. Узгодженість розраховується на квантовому комп'ютері на основі визначення середніх значень спінових операторів та кореляторів. Обчислюється її залежність від фазових параметрів, та амплітуд. Квантові протоколи реалізовані на п'ятикубітному квантовому комп'ютері IBM Lima. Результати квантових обчислень залежності узгодженості від амплітуди добре узгоджуються з теоретичними розрахунками. Не таку добру відповідність узгодженості, обчисленої за допомогою квантового програмування з теоретичними розрахунками отримано при дослідженні залежності узгодженості від фазових параметрів. Це пов'язано з більшою складністю квантових протоколів для таких досліджень.

Ключові слова: заплутаність, узгодженість, двокубітні квантові стани, квантові обчислення.