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BEHAVIOR OF NONLINEAR REACTIVE SYSTEM UNDER EXTERNAL HARMONIC PERTURBATION NEARBY A CRITICAL STATE

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We consider behaviour of critically strained reactive system under external harmonic perturbation. While a reactor works nearby such a state, the role of external fluctuations increases. Because of non-linearity of the system response on external perturbations, the reactor crosses over from a steady operative state into an unsteady one, which is explosive. Then a question arises of how the value of the perturbation provoking the conversion from the steady state to the unsteady one, depends on its frequency. It is shown that in the immediate vicinity of the critical state, the basic equation of the reactor warming-up can be brought to the Riccati equation with a harmonic source. It is found with the parametric calculation method that, under condition of constant amplitude, the transition time increases with increasing frequency. A linear dependence of the external transition energy on the perturbation frequency is revealed.

Key words: reactor, external harmonic perturbation, nonlinear system, critical state, Riccati equation.

The productivity of the system, which can be, for example, a homogeneous reactor or just a particle of combustible, increases on approaching the critical state. However nearby the critical state the risk of a stability loss of the reactor operating condition grows under the perturbation effect that provokes the conversion to the unsteady and difficult controlled state. By this reason, from the scientific point of view the response of the reactor as a nonlinear system on external harmonic perturbation is the most interesting [1]. In the works [2,3] the similar task was examined in a wide temperature range.

The task of a present work is to find out a connection between the perturbation parameters, which are amplitude and frequency, when the system converts from the steady state to the unsteady one.

Taking into account the heat supply due to reactions and heat sink from the system to the environment the equation of the homogeneous reactor warming-up can be expressed as:

$$\frac{dT}{dt} = \frac{Q\rho cV}{c_p m} z e^{-\frac{E}{RT}} - \frac{\alpha S}{c_p m}(T - T_\infty) + \frac{\alpha S}{c_p m} T_A \cos\omega t, \qquad (1)$$

Here the first and second summands in a right part of the equation (1) define physically the heat supply and the heat sink of the system accordingly, the third summand is a source of the external harmonic perturbation. In these expressions Q is a reaction thermal effect; z is a particle generation rate in a reaction; ρ is a density of agent, c is its concentration; E is an activation energy, i.e. that energy which the mole of substance must possess to be responded; R is a universal gas constant; T is a reactor temperature; α is a heat-transfer coefficient; T_∞ is an environment temperature; V is a volume where a reaction takes place; S is a surface area limiting the volume; ω is the external perturbation frequency; T_A is an amplitude, i.e. a peak temperature of the external perturbation; c_p is a specific heat capacity at constant pressure; m is mass of the reactive system.

Supposing that the reactive volume has a spherical form for which, as is generally known, the Nusselt number approximates by 2. Then the equation (1) has the next type

$$\frac{dT}{dt} = \frac{Q}{c_p} zce^{-\frac{E}{RT}} - \frac{3\lambda}{c_p\rho r^2}(T - T_\infty) + \frac{3\lambda}{c_p\rho r^2} T_A \cos\omega t. \qquad (2)$$

Such denotations are here accepted: λ is an environment heat conductivity coefficient, r is a size of the reactor core.

We will make the equation (2) dimensionless. For this purpose we will do the known change of variables by Frank-Kamenetskii [4]:

$$\theta = \frac{E}{RT_\infty^2}(T - T_\infty), \beta = \frac{RT_\infty}{E}. \qquad (3)$$

Considering expressions (3) the equation (2) can be brought to the dimensionless temperature form:

$$\frac{d\theta}{dt} = \frac{1}{\tau_x} e^{\frac{\theta}{1+\beta\theta}} - \frac{\theta}{\tau_r} + \frac{T_A}{\tau_r}\frac{E}{RT_\infty^2}\cos\omega t, \qquad (4)$$

where $\tau_r = \dfrac{c_p\rho r^2}{3\lambda}$ is relaxation time; $\tau_x = \dfrac{c_p RT_\infty^2}{QzcE} e^{\frac{E}{RT_\infty}}$ is time of the chemical reaction.

For the wide row of substances a dimensionless parameter β is negligible quantity about 0.05.

Taking into account the infinitesimal parameter β and insertion of dimensionless time τ the equation (4) can be presented like that:

$$\frac{d\theta}{d\tau} = \kappa e^{\theta} - \theta + \theta_A \cos\omega'\tau. \tag{5}$$

Such denotations are here accepted: $\kappa = \frac{\tau_r}{\tau_x}$ is a Semenov's parameter, $\theta_A = T_A \frac{E}{RT_\infty^2}$ is a dimensionless amplitude of the external harmonic perturbation, $\tau = t/\tau_r$ is dimensionless perturbation time, ω' is dimensionless frequency

$$\omega' = \omega\tau_r. \tag{6}$$

The dimensionless frequency by its physical meaning is a ratio of two characteristic process times which are the relaxation time and period of the perturbation effect.

As is generally known [4], critical conditions are implied by not only equality of functions of heat supply and heat sink in tangency point but also their derivatives:

$$\{\, \kappa^* e^{\theta^*} = \theta^*, \kappa^* e^{\theta^*} = 1. \tag{7}$$

From (7) we find the critical parameters θ^*, κ^* as follows:

$$\{\, \theta^* = 1, \kappa^* = e^{-\theta^*} = e^{-1}. \tag{8}$$

Then the equation corresponds to the system that is reactive nearby a critical state:

$$\frac{d\theta}{d\tau} = e^{\theta-1} - \theta + \theta_A \cos\omega'\tau. \tag{9}$$

Because of a littleness of $\theta - 1$ we will expand the exponent in a Taylor series up to second order. Thus nearby criticism the equation (9) is brought to the Riccati equation with a source of external harmonic perturbation:

$$\frac{d\theta}{d\tau} = \frac{1}{2}(\theta - 1)^2 + \theta_A \cos\omega'\tau. \tag{10}$$

An analytical solution of the equation (10) does not exist because the right part contains the source written in a harmonic form. Therefore in the work the numerical parametric calculation of the equation solution with a variation of temperature amplitude θ_A and frequency ω' of the perturbation has been carried out (fig. 1).

At each set of parameters induction time τ_i was found, under which duration of the nonlinear critically strained system transition is implied from the stable initial state to the unsteady one that is explosive.

The results of dependence of the induction time τ_i on amplitude θ_A and frequency ω' of the external perturbation are presented in the fig. 2–3 accordingly.

In the first of them the drop-down dependence of the induction time on the perturbation amplitude is visible at the different fixed frequencies. This result looks obvious because for the nonlinear system of the exothermic reactive environment its response on the phase of warming-up is stronger than the response on the phase of the system cooling down.

On the contrary, in a fig. 3 there is an increase of the induction time with growth of frequency at the fixed external perturbation amplitude. It is consequent that with growth of frequency the nonlinear system response on the external perturbation decreases by virtue of its thermal inertia (6).

Thus the induction time is a function of amplitude and frequency $\tau_i = \tau_i(\theta_A, \omega')$ of the perturbation. At the fixed transition time the dependence of amplitude on frequency is found out. A form of this dependence is brought in the fig. 4 for different transition times. It is shown that curves 1–3 approximate with high accuracy ($\sim 1\,\%$) by a linear function.

In the fig. 5 we can see the cross-section of 3D plot by planes at constant induction time. The general property of heavy lines is the linear dependence between perturbation energy and frequency.

So, the energy of the external perturbation, which is enough for the system conversion from the initial steady state to the unsteady explosive one, is proportional to the perturbation frequency. This result calls to the well known quantum-mechanical relation in which also, as in our case, energy is in proportion to frequency in the first degree.

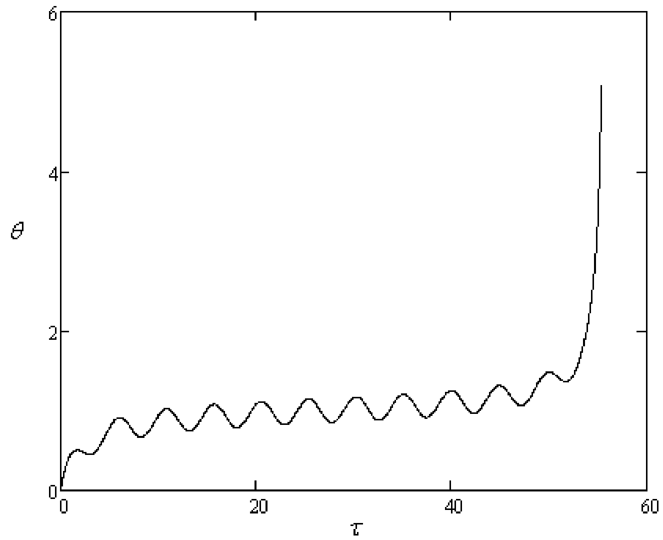

Fig. 1. Temperature-time curve of the strained system under the external perturbation with $\theta_A = 0,2$, $\omega' = 1{,}3$

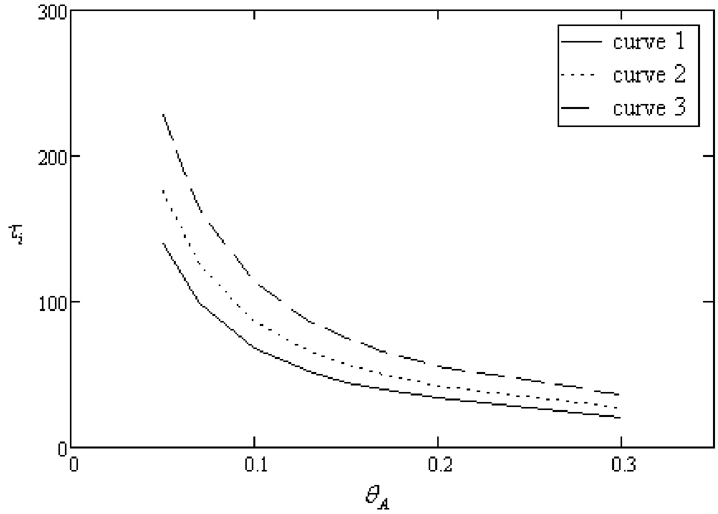

Fig. 2. Dependence of induction time on external perturbation temperature at fixed frequency. Values of frequencies 0,8, 1, 1,3 correspond to the curves 1, 2, 3

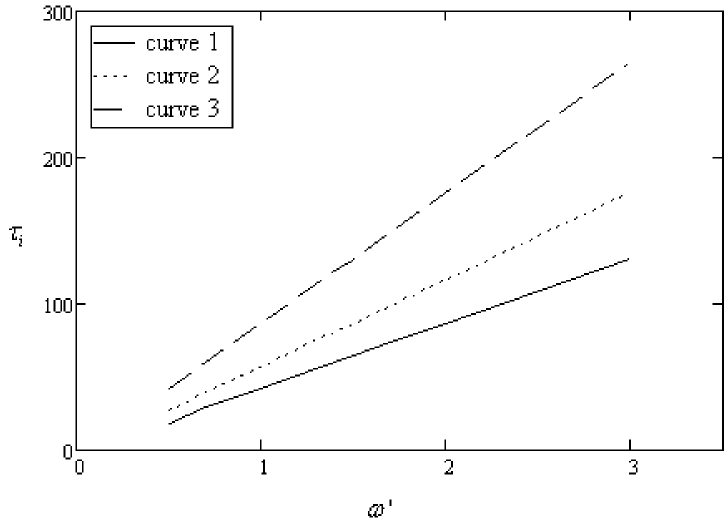

Fig. 3. Dependence of induction time on external perturbation frequency at fixed amplitude. Values of amplitudes 0,1, 0,15, 0,2 correspond to the curves 1, 2, 3

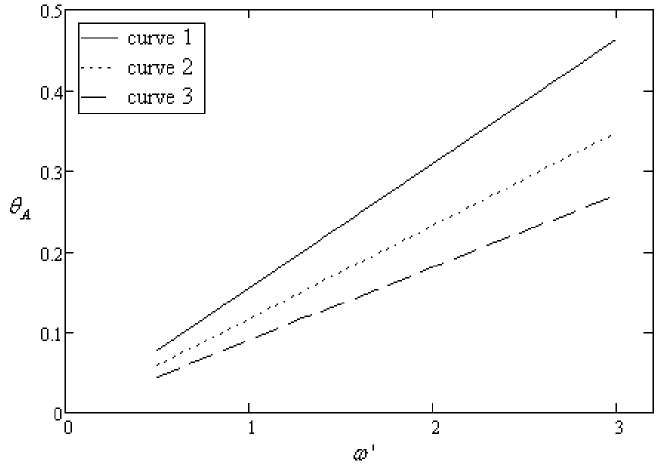

Fig. 4. Dependence of amplitude on frequency of external perturbation of the strained system, for which a transition from initial steady to the unsteady explosive one happened in same induction time. Induction time 55, 74, 96 correspond to the curves 1, 2, 3

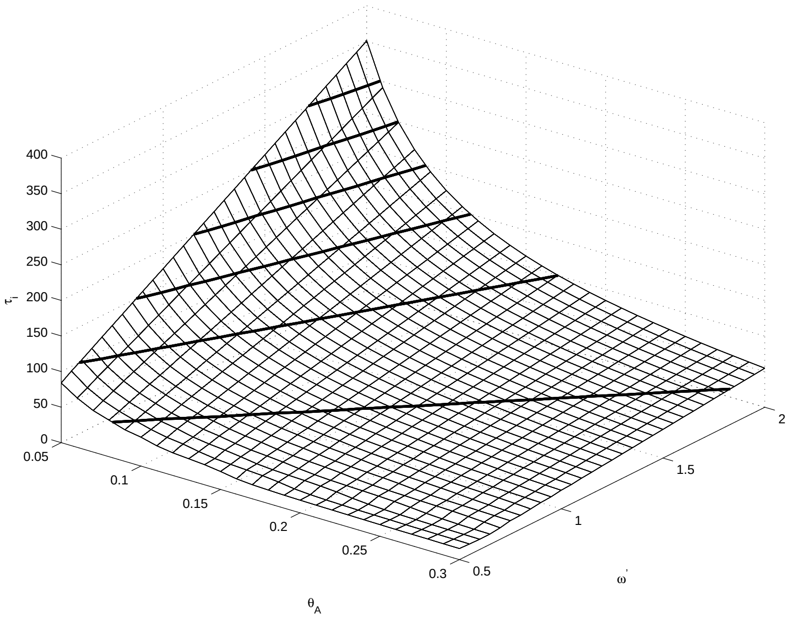

Fig. 5. The set of solutions $\tau_i = \tau_i(\theta_A, \omega')$ of the equation (10) forms 3D plot. Heavy lines correspond to the curves of dependences of amplitude on frequency of external perturbation in cross-sectional planes at constant induction time

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ПОВЕДІНКА НЕЛІНІЙНОЇ РЕАГУЮЧОЇ СИСТЕМИ ПІД ЗОВНІШНІМ ГАРМОНІЙНИМ ВПЛИВОМ ПОБЛИЗУ КРИТИКИ

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Розглянуто поведінку критично загостреної реагуючої системи під впливом зовнішнього гармонійного збурення. Під час роботи реактора поблизу критичного стану зростає роль зовнішніх флуктуацій. Внаслідок нелінійності відгуку такої системи на зовнішній вплив реактор переходить зі стійкого робочого стану до нестійкого — вибухового. Вивчається питання зв'язку величини збурення з частотою його впливу під час переходу реактора до нестійкого стану. Показано, що безпосередньо поблизу критики основне рівняння розігрівання реактора можна привести до рівняння Ріккаті з гармонійним джерелом. Методом параметричного розрахунку знайдено, що за незмінної амплітуди зовнішнього збурення час переходу зростає з підвищенням частоти. Визначено лінійну залежність зовнішньої енергії переходу від частоти впливу.

Ключові слова: реактор, зовнішнє гармонійне збурення, нелінійна система, критичний стан, рівняння Ріккаті.

ПОВЕДЕНИЕ НЕЛИНЕЙНОЙ РЕАГИРУЮЩЕЙ СИСТЕМЫ ПОД ВЛИЯНИЕМ ВНЕШНЕГО ГАРМОНИЧЕСКОГО ВОЗДЕЙСТВИЯ ВБЛИЗИ КРИТИКИ

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Рассмотрено поведение критически обостренной реагирующей системы под воздействием внешнего гармонического возмущения. При работе реактора вблизи критического состояния возрастает роль внешних флуктуаций. Вследствие нелинейности отклика такой системы на внешнее воздействие реактор переходит из устойчивого рабочего состояния в неустойчивое — взрывное. Изучается вопрос связи величины возмущения с частотой его воздействия при переходе реактора в неустойчивое состояние. Показано, что непосредственно вблизи критики основное уравнение разогрева реактора может быть приведено к уравнению Риккати с гармоническим источником. Методом параметрического расчета найдено, что при неизменной амплитуде внешнего возмущения время перехода увеличивается с ростом частоты. Выявлена линейная зависимость внешней энергии перехода от частоты воздействия.

Ключевые слова: реактор, внешнее гармоническое возмущение, нелинейная система, критическое состояние, уравнение Риккати.

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