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# NEW MECHANISM OF HADRON INELASTIC SCATTERING CROSS-SECTION BEHAVIOR IN $\varphi^3$ FIELD THEORY

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The method for taking into account the interference contributions to hadron inelastic scattering cross-section is developed within the framework of the simplest multiperipheral model. This method is based on the self-acting scalar  $\varphi^3$  field theory and the Laplace method using. It allows calculating the inelastic scattering cross-section with accounting all the interference contributions for the different number of secondary particles and obtaining absolutely new mechanism of hadron inelastic scattering cross-section behavior.

**Key words:** inelastic scattering cross-section, Laplace method, virtuality, Reggeon theory

## 1 Introduction

The inelastic scattering is one of the most common particles generation process in the particle and astroparticle physics. However, it is also one of the less studied process. Today existed the Reggeon theory [2] of hadron inelastic scattering cross-section includes 6 adjustable parameters and as the result repeats the experimental results Fig. 1. In fact telling us nothing about mechanisms such cross-section behavior. The formal difficulties, which appear in calculating of inelastic scattering cross-section with  $n$  secondary particles, are caused by the fact that there is a problem to take into account all the restrictions, which appear with regard to momentum-energy conservation, on the range of integration (1)

$$\sigma_n = \frac{1}{4n! \sqrt{(P_1 P_2)^2 - (M_1 M_2)^2}} \int \frac{d\vec{P}_3}{2P_{30}(2\pi)^3} \frac{d\vec{P}_4}{2P_{40}(2\pi)^3} \prod_{k=1}^n \frac{d\vec{p}_k}{2p_{0k}(2\pi)^3} \times \\ \times |T(n, p_1, p_2, \dots, p_n, P_1, P_2, P_3, P_4)|^2 \delta^{(4)} \left( P_3 + P_4 + \sum_{k=1}^n p_k - P_1 - P_2 \right), \quad (1)$$

where  $T(n, p_1, p_2, \dots, p_n, P_1, P_2, P_3, P_4)$  is scattering amplitude;  $M_1$  and  $M_2$  are the masses of colliding particles with four-momentums  $P_1$  and  $P_2$ ;  $\delta^{(4)}$  is a four-dimensional delta-function describing the conservation laws of energy and three momentum components in this process. Here it is also assumed that particles with four-momentums  $P_3$  and  $P_4$  are

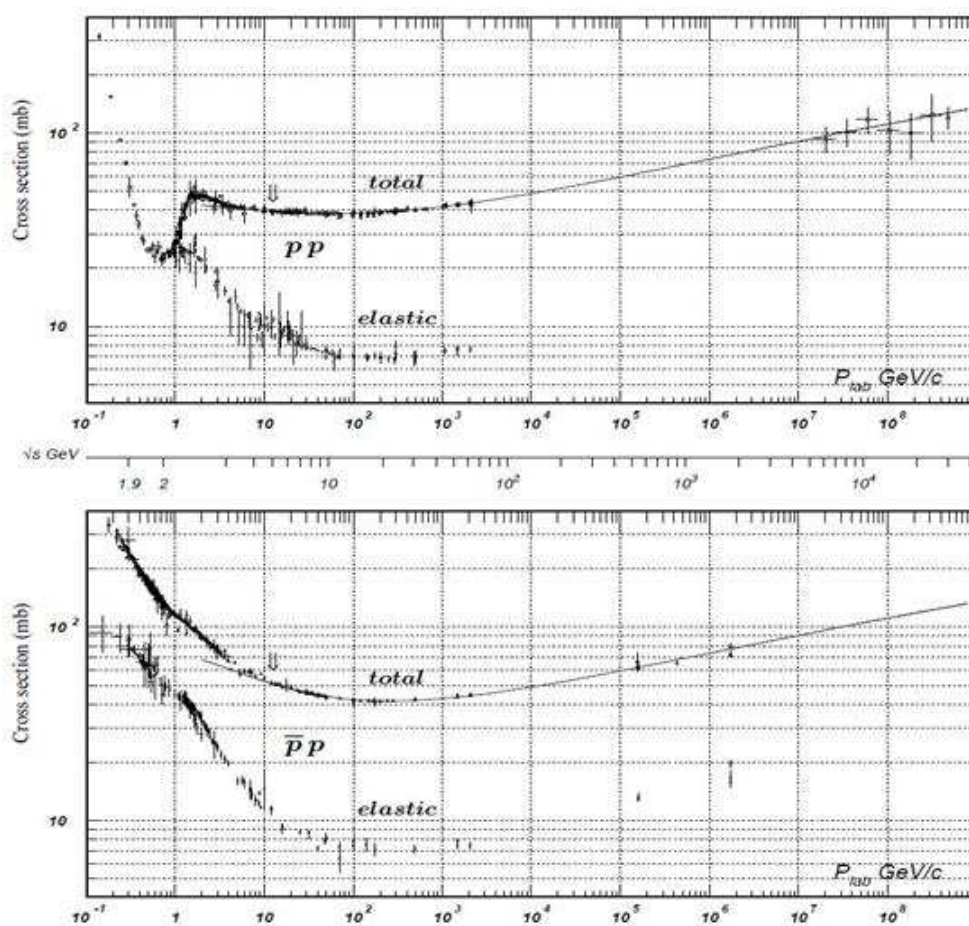


Рис. 1: The experimental ratio of total scattering cross-section and inelastic scattering cross-section to the energy [3].

the same sorts as  $P_1$  and  $P_2$ , respectively, and  $n$  secondary particles with four-momentums  $p_1, p_2, \dots, p_n$  are identical.

The traditional theory [2] avoids these difficulties by assuming that the main contribution in the Eq.(1) is made by the area where each next particle's velocity is less significant than the contribution of previous particle. As the result accounting of momentum-energy conservation becomes less difficult and it allows repeating the experimental results Fig. 1. It's also telling us nothing about mechanisms of cross-section behavior.

Introduced theory is based on the applying the Laplace method [4] in the multiperipheral model [3]. It helps to overcome these difficulties and simulate qualitatively the figure of pp inelastic cross-section with only one adjustable parameter, coupling constant. However, the main advantage of this theory that it allows, even in such simple model like  $\varphi^3$ , to analyze contributions of various Feynman diagrams.

## 2 The advantages of applying the Laplace method

The Laplace method or the method of maximization consists in finding the constrained maximum point of scattering amplitude squared modulus in (1) under four conditions imposed by  $\delta$ -function of (1). Then, expressing the scattering amplitude squared modulus as  $|T|^2 = \exp\left(\ln\left(|T|^2\right)\right)$ , it is possible to expand the exponent of the exponential function in Taylor series about a point of the constrained maximum, coming to nothing more than quadratic items [1].

The process of colliding two primary particles  $P_1$  and  $P_2$  is considered in central mass system. In such a frame of reference the initial and finite states have some symmetry, which is possible to use for solving the constrained maximization problem. In particular, the consideration of symmetries makes it possible to reduce the search of the constrained maximum of scattering amplitude to the search of the maximum of its restriction on a certain subset of physical process. This restriction is the function of substantially smaller number of independent variables than the initial amplitude [1].

When the scattering amplitude is expressed in terms of only independent variables, it is possible to search the ordinary and not constrained extremum. It is reached at zero values of transversal to collision axis components of the momentums of particle in finite state [1]. So, the restriction is a function of only longitudinal components of momentum  $p_{1\parallel}, p_{2\parallel}, \dots, p_{n\parallel}$ .

This and the other statements, like analysis of symmetry of the multiperipheral diagrams [1], [6], allow to calculate the inelastic cross-section value for some type of Feynman diagrams.

## 3 The contributions from different type of Feynman diagrams

The inelastic scattering process can be represented as combination of different Feynman diagrams. Today, through difficulties of calculation inelastic cross-section, just the "comb" type diagrams are examined. However, the Laplace method applying allows to calculate contributions from various much more complicated diagrams, like the "tree"

type diagrams. So, when the combination of “comb” and “tree” diagrams are considered, there is a difficulty to take into account the great number of interference contributions. The interference contributions appear as the result of the different way to chose the number of secondary particles on each “tree” and the number of “trees”. Moreover, the scattering amplitude of perturbation theory is the sum of the different Feynman diagrams contributions. In fact, the scattering amplitude of (1) represents as [1]

$$|A|^2 = \left( \sum_{k_l} A_{k_l} \right)^* \left( \sum_{k_r} A_{k_r} \right) \quad (2)$$

Therefore, the calculation of the interference contributions (2) of great number of secondary particles is the very labour intensive process, even with computer centre. As the result, the inelastic scattering cross-section of only 8 secondary particles has been calculated. However, even with this number of secondary particles, the important results have been obtained.

## Results and conclusions

The inelastic scattering cross-section was calculated with accounting all the interference contributions for the different number of secondary particles.

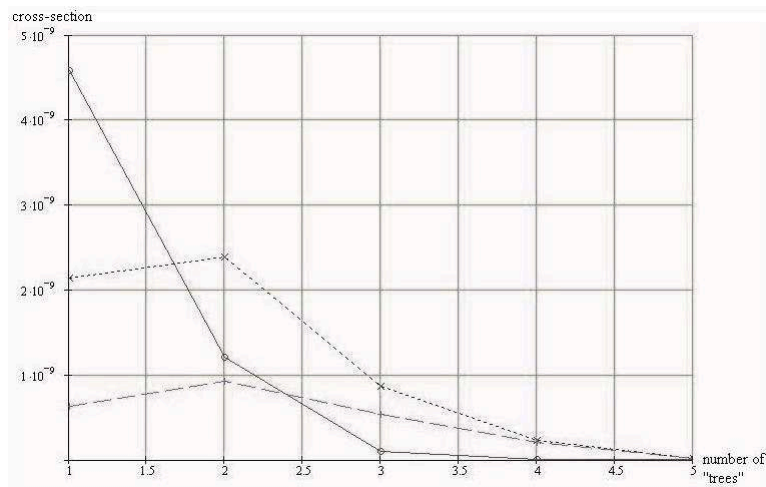


Рис. 2: The ratio of inelastic scattering cross-section with 5 secondary particles to the number of “trees”, for the value of energy: 5 GeV - solid line, 15 GeV - dotted line, 30 GeV - dashed line.

As we could see from Fig. 2, where the ratio of cross-section to the different number of “trees”, with the increasing of energy the contribution from diagrams with more “trees” becomes prevail. It means, that for the low area of energy the process of inelastic scattering is going with the mechanisms where low number of “trees” generated. It also gives the opportunity to confine the number of “trees” for the interference contributions calculation while the value of energy is low.

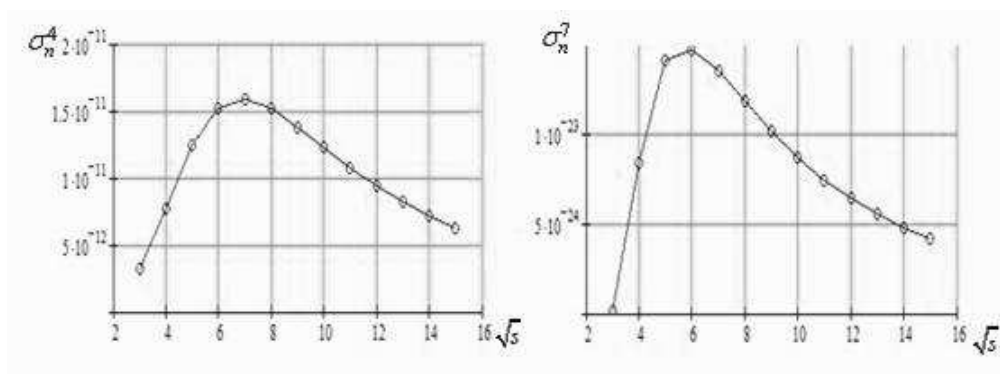


Рис. 3: The ratio of the value of energy to the inelastic scattering cross-section, where the number of secondary particles is: 4 -  $\sigma_n^4$ , 7 -  $\sigma_n^7$ .

The Fig. 3 demonstrates the ratio of the value of energy to the inelastic scattering cross-section. It is clear that if this figure of inelastic cross-section is superimposed on the figure of elastic cross-section Fig. 1, we would not obtain the exact experimental figure of total cross-section. However, the part of the low energy area, less than 10 GeV, it repeats qualitative. The reasons of this not exact repeating the experimental results for the energy area of more than 10 GeV, are that we were unable to calculate inelastic scattering cross-section with more than 8 secondary particles and that the  $\varphi_3$  model is too primitive. However, the obtained results will be applied to consider this method within the QCD model.

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**НОВИЙ МЕХАНІЗМ ПОВЕДІНКИ ПЕРЕРІЗУ  
НЕПРУЖНОГО РОЗСІЯННЯ  
АДРОНІВ У МОДЕЛІ  $\varphi^3$**

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В межах мультипериферичної моделі розвинуто метод розрахунку перерізів непружного розсіяння, що ґрунтується на використанні методу Лапласа, дозволяє враховувати інтерференційні внески при довільній кількості вторинних частинок і призводить до нових особливостей поведінки перерізів.

**Ключові слова:** переріз непружного розсіяння, метод Лапласа, віртуальність, теорія Редже

**НОВЫЙ МЕХАНИЗМ ПОВЕДЕНИЯ СЕЧЕНИЯ  
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АДРОНОВ В МОДЕЛИ  $\varphi^3$**

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В рамках мультипериферической модели развит метод расчета сечений неупругого рассеяния, основанный на использовании метода Лапласа, позволяющий учитывать интерференционные вклады при произвольном количестве вторичных частиц и приводящий к новым особенностям поведения сечений.

**Ключевые слова:** сечение неупругого рассеяния, метод Лапласа, виртуальность, теория Редже