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THE SELF–CONSISTENT THEORY OF DELOCALIZATION OF A QUANTUM PARTICLE IN A RANDOM FIELD

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Based on the analysis of the configuration space of an one–particle system, the presence of the quantum interference is shown to result in the effective interaction of an exchange–type. The quenched disorder is represented by analogy with the thermodynamic fluctuations for which the halfwidth of scattering of the particle levels plays the role of temperature. With regard to fermions within the framework of Anderson locator approach, the gap in the single excitation spectrum caused by the condensation of pairs of the coupled sites is shown to determine the density of the extended states. Its dependence on the level scattering width and the chemical potential shift from the band center is found. The law of collective mode dispersion is established. For the charged fermions, the collective mode is of an ordinary diffusion type, but in the case of the neutral ones (quantum crystal), the zero–sound mode of oscillations of the extended fermion density appears within the long–wave limit. The dependencies of the zero–sound velocity and typical values of its frequency and wave number on the temperature and parameter of the quantum dilatation are examined. The dependencies of the extended state condensate density and effective interaction parameter on the level scattering width W are determined for Bose case. The collective excitations are demonstrated to reduce to the first sound which is transformed, as the value W decreases, into a pure dissipative mode, and the second sound the velocity of which critically depends on W.

Key words: delocalization, level scattering, single and collective excitation.

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I. INTRODUCTION

Beginning with Anderson paper [1], in the quantum theory the problem of describing the stochastic system experiencing the ergodicity breaking has arose. The straightest way in accounting the non–ergodicity effects consists in the study of Kubo relaxation function [2,3]. Its static pole provides the non–ergodicity parameter defining the compressibility in the localized state; in the extended one, the relaxation function has an ordinary diffusion pole. It appeared however that the self–consistency equations obtained in Ref. [3] do not bring to a critical behaviour of the diffusion coefficient in the vicinity of the threshold of mobility [46], and, hence, the method [2,3] does not make a representation of Anderson transition like in the case with the phase transition scheme.

The greatest promise in this direction is held by the application of the nonlinear σ–model (see Ref. [4] and references therein). In so doing, however, the problem of choice of the long–range order parameter turns to be quite important. The cause of this importance lies in the fact that the ergodicity breaking presents a more intricate phenomenon than the symmetry breaking investigated within the framework of the standard theory of phase transitions [5]. So, it was found [6] that, for the system of noninteracting fermions, the density of particles at Fermi level (compressibility) behaves in a non–critical manner and, hence, cannot be used as an order parameter. A scheme [7] was proposed where its role was played by a certain functional. Recently, however, Kirkpatrick and Belitz [8] proceeding from the assumption that the localization process is caused by the competition of effects of the quenched disorder and electron interaction, have shown on the basis of the renormalization group approach that appearance of the interaction gives rise to the fixed point that is stable at the dimensionality $d > 4$ (but not $d \geq 2$ as in Ref. [7]). The density of states therewith exhibits the critical behaviour and Anderson transition is represented by standard Landau scheme.

The present paper is concerned with the derivation of the mean–field theory for noninteracting particles — both fermions and bosons. In contradistinction to Refs. [4, 6, 8] we do not proceed from the field approach but from the quantum statistical theory [9]. The main idea of our approach consists in the fact that formally the quenched disorder can be considered as thermodynamic fluctuations for which the energy level scattering width W plays the role of temperature. Another fact of a fundamental nature is the consideration of the effective interaction. It results from the presence of the quantum interference which stipulates dividing the configuration space of the system into the subspaces of single and collective excitations [10]. With regard for this fact, the one–particle problem reduces to the site analog of the BCS model in the superconductivity theory. Accordingly, the ergodicity breaking under Anderson transition is described by analogy with the phase transformation of order $2+\delta$ where $\delta \to 0$ is the addition caused by the sites resonating and corresponding to the logarithmic singularity [11].

Methodically, when investigating the localization problem, one should highlight the mutually complementary approaches by Anderson [1] and Edwards [12]. In the context of the site representation by Anderson, the localized state is recognized as initial, and the transition involves formation of the extended state band. The application of Edwards wave approach assumes, on the contrary, that the extended state is initial, and rearrangement of the system is localization related. Since the time of representation of the indicated models, a great deal of papers have been published (see review [4] and references therein). Here one may set off the scaling theories [6,13,14], application of the replica method [15,16], the supersymmetrical approach [17,18], diagram methods [19] — [23], the above–mentioned mode–coupling method [2,3]. Most of these papers contained the consideration of the collective excitation behaviour as represented by Edwards. In addition to the evident methodical advantages, it can be probably further explained by the fact that determination of the basic experimentally–measurable values (of the conductivity and polarization type) is achieved within the framework of such an approach in the most natural way.

In the context of the locator representation based on Anderson approach [1], the system perturbation resulting in delocalization is associated with overlapping the wave functions. As already noted, such an approach makes possible the representation of the critical behaviour of one–particle characteristics of the extended state density type [11]. Needless to say, the locator approach adequately reflects the system collective behaviour as well. So, the self–consistent consideration of Green two–particle function $\varphi(\mathbf{K},\Omega)$ showed [22] that, within the limits of low values of the wave vector $\mathbf{K}$ and frequency Ω of the collective excitations, it has a characteristic feature of $\varphi \propto (\Omega + iD\mathbf{K}^2)^{-1}$ the pole of which is determined by the diffusion coefficient D. Notice that the self–consistency scheme [11] allowed for finding the system behaviour throughout the entire level scattering region W while the diagram method [22] represents only the regimes of the weak ($W \ll W_c$) and strong ($W \sim W_c$) coupling where W_c is a critical scattering of levels.

In the context of Edwards propagator approach [12], the role of perturbation is played by a random potential. The diagram investigations [19] — [23] of the collective mode made it possible not only to represent the diffusion pole of the function $\varphi(\mathbf{K},\Omega)$ but also to prove that the diffusion coefficient zeroing is not ensured by the ladder sequence the terms of which cancel each other, but by the fan diagrams obtained from the ladder ones through turning the hole propagator [47]. The approach [19] — [23] however describes only the limit of the weak coupling ($W \ll W_c$) for the systems of different dimensionality. Consideration of the entire range of the values W for the three–dimensional system is obtained within the limits of the equivalent mode–coupling method [2,3]. It is based on Zwanzig–Mori technique where the correlator $\varphi(\Omega)$ of the fermion density is expressed in the form of the continued fraction of the second order. Its kernel $M(\Omega) = -\Omega + \mathrm{i}/D(\Omega)$ is the correlator of the force acting on the particle, and describes the memory effects. The mode–coupling approximation consists in the fact that the memory function $M(\Omega)$ is expressed via the initial correlator $\varphi(\Omega)$ in the linear form $M(\Omega) = \lambda\varphi(\Omega)$ where the effective coupling constant $\lambda \propto W^2$ is determined by the width W of the energy level scattering. In Refs. [2, 3] the very absence of interaction ensures the linear nature of coupling [48] $M \propto \varphi$. The self–consistent investigation has shown that, in the static limit $\Omega \to 0$, the frequency dependencies $D(\Omega)$, $g(\Omega)$ of the diffusion coefficient and compressibility take the following form: $D(\Omega) = \text{const} \equiv D$, $g(\Omega) = i(\rho/m)D/\Omega$ — in the extended state; $g(\Omega) = \text{const} \equiv g$, $D(\Omega) = -i(m/\rho)g\Omega$ — in the localized state (here, m and ρ are the mass and volume concentration of particles respectively).

Though, in combination, Anderson and Edwards approaches mutually complement the delocalization picture, direct join, needless to say, cannot afford satisfaction. Along with it, recently a paper [24] has appeared where, on the basis of a self–consistent consideration of Fermi and Bose excitations of a superconductor, it is illustrated that the standard case is described by the BCS theory, and the HTSC case is described on the basis of the concept of Bose–Einstein condensation of strongly coupled Cooper pairs. The vertex function determining the fermion self–energy therewith plays the role of Green function of bosons corresponding to Cooper pairs. As a result, the self–consistent consideration of Green's Fermi and Bose functions permitted describing, in a self–consistent manner, the single and collective excitations of a superconductor.

The present article is devoted to realization of the program [24] on the basis of the locator representation of Anderson model. The qualitative study of the wave function type in Section II reveals that, in accordance with the results [16,17], system symmetry group turns to be noncompact in which connection its configuration space should be divided into two orthogonal subspaces. This permits reducing Anderson's Hamiltonian to the BCS type. Section III deals with the derivation, in a standard manner, of Dyson equation for Green one–particle function the self–energy function of which is expressed via the vertex function of the effective interaction of particles. For the latter an equation of Bethe–Salpeter type is found that concludes the self–consistent description of the system. In Section IV, the obtained equations are used to describe single excitations, and in Section V — collective excitations. The scheme under discussion allows for representing not only the peculiarities of behaviour of extended states of a particle but a microscopic picture of transformation of the fermion diffusion mode into the sound mode associated with the zero–sound oscillations of the extended quantum particle density. Such a mode is realized in quantum crystals [25]. In Bose case, the collective excitations are reduced to the first sound which is transformed into a pure dissipative mode under delocalization, and the second sound which velocity critically changes close to the threshold of delocalization.

II. STATEMENT AND QUALITATIVE ANALYSIS OF THE PROBLEM

Let us consider N_0 noninteracting quantum particles having in the sites $\mathbf{r}_l$, $l = 1, \ldots, N_0$ of the regular lattice the energy levels ε_i distributed according to the law $P_i \equiv P(\varepsilon_i)$ where the function $P(\varepsilon)$ is bell–shaped with the width W and value $\bar{\varepsilon} = N_0^{-1}\sum_i \varepsilon_i$ in the center of the cupola. Assume that the overlapping of the site wave functions $\varphi_i^l(\mathbf{r}) \equiv \varphi_{\varepsilon_i}(\mathbf{r}-\mathbf{r}_l)$ is characterized by the integral $J_{ij}^{lm} = \int \varphi_i^{l*}(\mathbf{r})\varphi_j^m(\mathbf{r})\,\mathrm{d}\mathbf{r}$, and the spectrum E_α of the system is assigned by the distribution function $w_\alpha = w(E_\alpha)$. In Lorentz quantum model, the function $P(\varepsilon)$ characterizes the distribution of energy levels of the nonoverlapping scatterers, and the system spectrum defined by the relationship $w(E)$ describes the test particle which is in the field of the interfering scatterers.

When there is no overlapping ($J_{ij}^{lm} = 0$), the wave function $\Phi_E(\mathbf{r})$ of the state $E = E_\alpha$ takes the form:

$$\Phi_E(\mathbf{r}) = \sum_i a_E^i \varphi_i^l(\mathbf{r}). \tag{1}$$

The localized state is realized therewith in which the test particle initially placed in the site l stays therein for indefinitely long time. It means that only one coefficient $a_E^i = \delta_{E\varepsilon_i}$ out of all coefficients in (1) is not of a zero value. Consequently, both distributions $P(\varepsilon)$ and $w(\varepsilon)$ coincide.

The overlapping of the wave functions $\varphi_i^l(\mathbf{r})$ leads to the qualitative rearrangement of the system. To clear up a situation, consider two adjacent sites $\mathbf{r}_1$ and $\mathbf{r}_2$ with the ε_1 and ε_2 levels and the overlapping integral $J_{12}^{12} \equiv J$. Applying the variation principle for the energy $E = \int \Phi^*(\mathbf{r})H\Phi(\mathbf{r})\mathrm{d}\mathbf{r}$ where $H = H_1 + H_2$, $H_i\varphi_i = \varepsilon_i\varphi_i$, $i = 1,\, 2$, it is easy to show that the states of the test particle in these sites are defined by the wave functions

$$\begin{aligned}
&\Phi_0 = u\varphi_1 - v\varphi_2,\\
&\Phi_1 = u\varphi_2 + v\varphi_1,\\
&\left.\begin{matrix} u^2 \\ v^2 \end{matrix}\right\} = \frac{1}{2}\left(1 \pm \frac{\varepsilon_1 - \varepsilon_2}{\epsilon}\right),\\
&\epsilon \equiv \left[(\varepsilon_1-\varepsilon_2)^2 + J^2(\varepsilon_1+\varepsilon_2)^2\right]^{1/2},
\end{aligned} \tag{2}$$

to which the energies $E_{0,1} = (1/2)\left[(\varepsilon_1+\varepsilon_2) \mp \epsilon\right]$ correspond. Here, at normalization of the functions $\Phi_{0,1}$, the condition $J \ll 1$ is accounted for, and the designations $u \equiv a_{E_0}^1 = a_{E_1}^2$, $v \equiv -a_{E_0}^2 = a_{E_1}^1$; $\varphi_1 \equiv \varphi_1^l(\mathbf{r})$, $\varphi_2 \equiv \varphi_2^l(\mathbf{r})$, $l = 1,\, 2$ are introduced for brevity sake. At $|(\varepsilon_1-\varepsilon_2)\,/\,(\varepsilon_1+\varepsilon_2)| \gg J$, the site function overlapping can be neglected, and the test particle states at the pair of sites reduce to the states at the isolated scatterers. In the opposite case of $|(\varepsilon_1-\varepsilon_2)\,/\,(\varepsilon_1+\varepsilon_2)| \le J$, the site representation loses its significance, and we have to do only with the states at the pair of coupled sites (PCS). Regarding the isolated PCS, the wave function Φ_0 corresponds to the low–energy state E_0, and the function Φ_1 does to the high–energy state E_1. It is apparent that, under transition from the Lorentz quantum model to the equivalent two–particle problem with the absence of interaction, the functions Φ_0, Φ_1 within the limits of $\varepsilon_1 = \varepsilon_2$ are transformed into the antisymmetric and symmetric combinations of the single site functions $\varphi_i^l(\mathbf{r})$. This suggests that the basic state of fermions corresponds to the low–energy region characterized by the value E_0 while, with respect to bosons, it is realized in the high–energy region which corresponds to the value E_1. As discussed in Section IV, this fact essentially affects the delocalization picture.

This conclusion which will be strengthened further on the basis of the standard scheme of the quantum theory of many–particle systems seems to be quite unexpected at first glance. Indeed, since there is no interaction, the system behaviour (in particular, the situation with the mobility threshold), as might appear, should not be sensitive to the type of statistics. With such a line of reasoning, however, completely disregarded is the phenomenon of the quantum interference which, as is known, leads to the effective interaction of an exchange type. In the discussed two–site problem, this makes itself evident in the fact that expression for the spectrum of noninteracting particles is formally in agreement with the corresponding expression for the spectrum of the degenerate system the interaction potential of which has matrix elements $V_{11} = \varepsilon_1$, $V_{22} = \varepsilon_2$, $|V_{12}| = |V_{21}| = J(\varepsilon_1+\varepsilon_2)/2$ (see Ref. [26]). Thus, it turns out that consideration of the quantum interference of noninteracting particles gives rise to the effective interaction which is dependent on the spectrum of the system. It is the cause of the critical behaviour of the one–particle system including various kinds of behaviour of Fermi and Bose particles.

The problem presented makes it apparent that the presence of the PCSs radically changes the nature of the system. So, if in their absence to each site corresponds a unique value ε_i, the appearance of the PCS results in splitting the ε_i into two levels E_α, and a pair of components $\Phi_\alpha(E, \mathbf{r})$ corresponding to them separates out of the one–particle function (1), where the low–energy state $\alpha = 0$ relates to fermions and the high–energy state $\alpha = 1$ — to bosons. At the given values of the overlapping integral $I \equiv J\bar{\varepsilon}$ and the energy scattering W the specific number of PCSs apparently amounts to $\sim I/W$ with respect to the total number of sites $N_0 \to \infty$. However, the delocalization process reduces to the formation of an infinite cluster of the PCSs, hence, not all of the $(I/W)N_0 \to \infty$ contribute to this process. In accordance with Refs. [27, 28] at $I/W = \mathrm{const}$, the system represents a set of clusters various in size and shape an adequate description of which requires the application of the fractal representation. To elucidate the qualitative aspect of the problem, however, it will suffice to make use of the mean–field approximation within the frameworks of which the fractal set reduces to a collection of N_c optimal clusters each of which contains $\mathcal{N}_c$ of PCSs (according to the Anderson concept [1], the most probable Green function corresponds to such clusters). Physically speaking, transition from a complete ensemble of clusters to a set of the optimal ones (as the I/W increases, the lattices reduce to the maximal cluster) is based on that fact that the formation of small–sized clusters corresponds to the short–range order of the system while the delocalization process is associated with the appearance of the long–range order. Therefore, in spite of the fact that the total number of the PCSs $\sim (I/W)N_0$ is infinite, at small values of I/W, their overwhelming majority are contained in small clusters, and the number of PCSs $\mathcal{N} = \mathcal{N}_c N_c$ in the optimal clusters is negligible in comparison with the total number of sites $N_0 \to \infty$ ($\mathcal{N}/N_0 = 0$). As the I/W parameter increases, ever more PCSs are incorporated into optimal clusters, and with the formation of the infinite one ($\mathcal{N} \to \infty$) the ratio $\mathcal{N}/N_0$ takes the finite value. It means that the value $2\mathcal{N}/N_0 \equiv \eta^2$ defining the specific part of sites which found themselves in the optimal clusters can be considered as the delocalization parameter.

Consider now in which manner the presence of the PCSs clusters affects the wave function of the localized state (1). According to (2), the formation of the isolated PCS results in separating out the two–site term $\Phi_\alpha(E,\mathbf{r})$ from the sum (1). Unification of $\mathcal{N}$ PCSs into clusters is responsible for the increase in the number of terms (1) appearing in the cluster component $\Phi_\alpha(E,\mathbf{r})$ from 2 to $2\mathcal{N}$. For $\mathcal{N} \gg 1$ the site states $\varphi_i^l(\mathbf{r})$ rearranges into the quasi–Bloch states $\varphi_\mathbf{k} \sim \Theta(\mathbf{r})\exp(i\mathbf{kr})$ where the form function $\Theta(\mathbf{r}) = 1$ inside the clusters and $\Theta(\mathbf{r}) = 0$ outside the clusters. As a result, the wave function of the state E assumes the form:

$$\Phi_E(\mathbf{r}) = {\sum_i}' a_E^i \varphi_i^l(\mathbf{r}) + \sum_\mathbf{k} b_\alpha^\mathbf{k}(E)\varphi_\mathbf{k}(\mathbf{r}) \qquad (3)$$

$$\equiv \Phi'_E(\mathbf{r}) + \Phi_\alpha(E,\mathbf{r}), \quad \alpha = 0,\, 1,$$

where the wave vector $\mathbf{k}$ runs through $\mathcal{N}_c$ values corresponding to the number of PCSs in the given cluster, the prime by the i sum sign means that the summation is performed with respect to the uncoupled sites only. In correspondence with (4), the distribution function $w(E) = \int |\Phi_E(\mathbf{r})|^2 \mathrm{d}\mathbf{r}$ is written as follows

$$w(E) = AP(\varepsilon) + 2S(E), \qquad (4)$$

where the term $2S(E) = \sum_\alpha \int |\Phi_\alpha(E,\mathbf{r})|^2 \mathrm{d}\mathbf{r}$ describes the distribution of PCSs, and the dependence $P(\varepsilon)$ describes the distribution of isolated sites; coefficient A and form of dependence $S(E)$ are determined by the relation between the level scattering width W and characteristic value I of the overlapping integral. At $I/W \to 0$, when the specific number of PCSs is insignificant, we have $A \approx 1$, and the function $S(E)$ describing the effects of the short range order appears as a highly diffused bell characterized by the width $\sim W$ and height $\sim I/W$. Thus, within the limit $I/W \to 0$, the distribution $w(E) \approx P(\varepsilon)$ is primarily set by the superposition (1) of the site states; accordingly, the ground state energy is determined by the sum of the levels ε_i and amounts to $\bar\varepsilon N_0$. As the level scattering decreases, the number of PCSs increases and they form coupled clusters. This leads to an increase in the height and to the width narrowing of the bell–shaped dependence $S(E)$, though in the localized state when there is only short–range order, the height of the $S(E)$ is always finite. With formation of an infinite cluster of PCSs, in (4) the appearance is made by the δ–like term with the $\eta^2 = 2\mathcal{N}/N_0$ factor determining the specific number of the PCSs which fell into the Bose–Einstein condensate of extended states (see below eq. (16)). The ground state energy, therewith, decreases by the macroscopic value of $I\mathcal{N} = (IN_0/2)\eta^2$ which is defined by the long–range order parameter η.

The argument brought forward refers to the Fermi case where the delocalization process requires a preliminary formation of the PCSs the condensation of which is responsible for delocalization. The distinction of the Bose system lies in the fact that the single particles themselves can be condensed. Therefore, the delocalization parameter $\eta = (2\mathcal{N}/N_0)^{1/2}$ is proportional to the square root of the number of the condensate bosons but not the PCSs, as with fermions. Taking into account the present fact, the fermion system will be meant in what follows, as in the initial Anderson model [1]. For the boson system, by the value of $2\mathcal{N}$ should be meant the number of condensed bosons.

Pass on now to the development of the quantitative picture. According to Ref. [1], the description of the Anderson transition requires summation of the infinite series of the divergent terms resulting from the ergodicity breaking. In the framework of the Edwards approach, this difficulty is resolved by separating out the ladder sequence [19] — [23] or initial changing the pole structure of the relaxation function with the subsequent procedure of self–consistancy by the mode–coupling method [2,3]. In other words, allowance is initially made for the rearrangement of the system phase space. A similar situation exists in the theory of superconductivity where the rearrangement of the ground state results from the presence of the Cooper pairs [29]. In this connection, an idea suggests itself that, in the context of the Anderson locator approach as well, the phase space rearrangement associated with the presence of PCSs should be initially taken into account. In accordance with Ref. [22], within the framework of the site representation, this can be achieved in the same manner as in the Edwards approach — by summation of the ladder sequence. Below is shown that the mentioned rearrangement of the phase space is most naturally represented by replacement of the initial Anderson's Hamiltonian by a certain effective Hamiltonian defined over the functions conforming to the rearranged ground state.

The above outlined qualitative picture shows that the state E delocalization is manifested as the separation (4) of the corresponding wave function $\Phi_E(\mathbf{r})$ into the components $\Phi_\alpha(E,\mathbf{r})$ and $\Phi'_E(\mathbf{r})$ of the coupled and uncoupled sites. The former represents the superposition of quasi–Bloch states $\varphi_{\mathbf{k}}(\mathbf{r})$, the latter — site states $\varphi^l_i(\mathbf{r})$. At the given number $\mathcal{N}$ of PCSs contained in the optimal clusters, the fraction of the former is of the order of $2\mathcal{N}/N_0 = \eta^2$, and that of the latter — $1 - 2\mathcal{N}/N_0 = 1 - \eta^2$. The characteristic feature of the component $\Phi'_E(\mathbf{r}) = \sum'_i \alpha^i_E \varphi^l_i(\mathbf{r})$ lies in the presence of the free nonsummable index l at the site functions $\varphi^l_i(\mathbf{r})$. Physically, it means that the set of levels ε_i can be distributed, in an ambiguous fashion, by sites $\mathbf{r}_l$ in which connection the system of noncoupled sites display symmetry with respect to the group G' consisting of the $N'!$, $N' = N_0 - 2\mathcal{N} = N_0(1-\eta^2)$ permutations of the sites $\mathbf{r}_l$ by the levels ε_i. Collection of functions $\Phi'_E(\mathbf{r})$ complying with various arrangements of $\mathbf{r}_l$ by the ε_i levels is effected by the basis of the group G'. In the absence of overlapping, it reduces to the total group G of $N_0!$ permutations, and the configuration space of the system is determined by the set of $N_0!$ functions (1). Coupling $2\mathcal{N} = \eta^2 N_0$ sites into optimal clusters results in the reduction of the total group G to its subgroup G' the power of which is determined by the parameter η: at $\eta = 0$ in the macroscopic approximation (i.e. correct to the terms of $N_0^{-1} \to 0$), groups G' and G coincide, and at $\eta = 1$ the G' group reduces to an one element. The parameter η taking the value $\eta = 0$ in the localized state and $0 < \eta \le 1$ in the extended state conforms to the ordinary determination of the order parameter in the phase transition theory. The localized state marked by the total group G is of high symmetry and complies with the disordered phase ($\eta = 0$), and the extended state defined by its subgroup G' conforms to the ordered low symmetry phase ($\eta \neq 0$).

Thus, the overlapping of site wave functions results in partition of the complete configuration space $\{\Phi\}$ generated as a result of $N_0!$ permutations of sites l in (1), into subspaces $\{\Phi'\}$, $\{\Phi_\alpha\}$. The first subspace corresponds to uncoupled sites and, consequently, represents the single excitations. The pair of subspaces $\{\Phi_\alpha\}$, $\alpha = 0, 1$ formation of which results from the clusterization of PCSs corresponds to the collective excitations. As may be seen from the example of two coupled sites, when the energies ε_i completely coincide, the wave function of the state Φ_0 is antisymmetric, and that of the state Φ_1 is symmetric. Consequently, the ground state of fermions is realized in the first case, and that of bosons — in the second case. It is convenient to express the present fact entering the following projection operators:

$$|\Phi_0\rangle = P\,|\Phi\rangle\,, \quad |\Phi_1\rangle = Q\,|\Phi\rangle \qquad \text{for fermions;}$$
$$|\Phi_1\rangle = P\,|\Phi\rangle\,, \quad |\Phi_0\rangle = Q\,|\Phi\rangle \qquad \text{for bosons.} \tag{5}$$

Here, P separates out the subspace of real states, $Q = 1 - P$ does that for virtual states. The physical meaning of the performed partition of the configuration space lies in the fact that, in the case of fermions, the zone of real collective states corresponds to the low–energy subspace $\{\Phi_0\}$, and the zone of virtual states corresponds to the high–energy subspace $\{\Phi_1\}$; in the case of bosons, the reverse situation is observed. In what follows we observe that the given fact resulting from the various permutation symmetry of particles is responsible for the opposite signs of the exchange interaction of fermions and bosons.

The Anderson's site Hamiltonian

$$H = \sum_i \varepsilon_i \psi^{i+}_l \psi^i_m + \sum_{l\neq m} I^{lm}_{ij} \psi^{i+}_l \psi^j_m \equiv H_0 + U_0 \tag{6}$$

is symmetric with respect to the permutations [49] of the total group G. Thus, here again one is up against the ordinary situation inherent in the phase transition theory — the symmetry of the configuration space $\{\Phi'\} \otimes (\{\Phi_0\} \oplus \{\Phi_1\})$ proves to be below the Hamiltonian symmetry. To eliminate this degeneration, separate out of (6) the effective Hamiltonian defined on the appropriate basis. Taking the operator U_0 in (6) to be perturbation, we come to the following series [30]:

$$H_{\rm eff} = PH_0P + \sum_{n=0}^{\infty} PU_0 \left(\frac{QU_0}{\mathcal{E} - H_0}\right)^n P, \tag{7}$$

where $\mathcal{E}$ is the energy of the system in the ground state. Hence, taking into consideration only the overlapping of the z nearest neighbours, for the Hamiltonian $H' = H_{\rm eff} - \mu N_0$ written in view of the $E = \mu - \bar{\varepsilon}$ shift from the center of the band, in the second order by the parameter zI/W, we obtain

$$H' = \sum_l (\varepsilon_l - \mu) a^+_l a_l + \frac{V}{N_0} \sum_{lm} a^+_l b^+_{\bar{l}} b_{\bar{m}} a_m\,. \tag{8}$$

It is generally agreed here that the overlapping integral $I^{lm}_{ij} \equiv I$ does not depend on i, j, l, m; μ is the chemical potential of localized particles (for the extended particles it reduces to the Fermi energy E_F), $V \equiv -N_0(zI)^2/(\widetilde{\mathcal{E}} - \mathcal{E})$ the effective coupling constant, $\widetilde{\mathcal{E}}$ is the energy of the system in the virtual state. Operators a^+_l and a_l of creation and annihilation of particles in the ground state and operators b^+_l and b_l of the appropriate "antiparticles" in the virtual state are determined by the following equations:

$$a^+_l = \sum_i P_i \psi^{i+}_l, \qquad a_l = \sum_i \psi^i_l P_i;$$
$$b^+_l = \sum_i \psi^i_l Q_i, \qquad b_l = \sum_i Q_i \psi^{i+}_l, \tag{9}$$

where P_i and Q_i are the projection operators at the level i appearing in the complete operators P and Q in the form of products; the operators in the "site" $\bar{l}$ adjacent to the given l appears as $a_{\bar{l}} = z^{-1}\sum_m a_{l+m}$ where summation is performed by z of the nearest neighbours.

So, the spontaneous symmetry breaking with respect to the permutations of the levels ε_i by the sites $\mathbf{r}_l$ consisting in transition from group G in the localized state to its subgroup G' in the extended state, results in the problem of the effective system of interacting particles and antiparticles [50]. Notice that origination of efficient interaction in the systems where from the very beginning provision was made only for the wave function overlapping inherent not only in the Anderson model but in the spin glass [32] as well where effective interaction results from averaging the overlapping integral values. The distinctive property of our model lies in the fact that the nature of effective interaction does not depend on the sign of the overlapping integral and is defined only by the sign of the difference $\Delta\mathcal{E} \equiv \widetilde{\mathcal{E}} - \mathcal{E}$ of the energies of the system in the virtual and real states. According to (5), for fermions the zone of virtual collective states is located above the zone of real states ($\widetilde{\mathcal{E}} \equiv E_1$, $\mathcal{E} \equiv E_0$). Consequently, here $\Delta\mathcal{E} \equiv \widetilde{\mathcal{E}} - \mathcal{E} = E_1 - E_0 > 0$, and the effective interaction of fermions is of the attraction nature ($V < 0$). For bosons the roles of the indicated bands are interchanged ($\widetilde{\mathcal{E}} \equiv E_0$, $\mathcal{E} \equiv E_1$), and the situation is reversed ($V > 0$). It follows herefrom that consideration of the interference effects should most essentially influence the behaviour of fermions since their ground state, as in superconductors, proves to be unstable.

Leaving the statement of the formalism based on the effective Hamiltonian (8) for Section IV, notice that it coincides, in its form, with the model Hamiltonian BCS, with the distinction lying in the fact that the site representation figures instead of a momentum one, and the role of electrons with opposite momenta and spins is played by the particles and antiparticles at the adjacent sites (in addition, the sign of interaction is changed for bosons). Since Hamiltonian (8) hereinafter plays a leading part, it seems necessary to discuss the legitimacy of the infinite series approximation (7) used, when Hamiltonian is obtained, by the second order term over zI/W. Strictly speaking, the indicated series is convergent only in the limit of $zI/W \ll 1$ associated with the high localization region. Close to the threshold of mobility and particularly in the extended state where the value of zI/W can be arbitrarily large, the series (7) becomes asymptotic by nature. This means that at its approximation by the finite number of terms the best approximation is obtained if this number does not exceed a certain optimal value n_0 which grows as the zI/W parameter increases. At transition from (7) to (8), it is agreed that $n_0 = 1$, i.e. we restrict ourselves to the first nonvanishing term of the asymptotic series. It follows that Hamiltonian (8) can be a satisfactory approximation of the exact expression (7) only at not very large values of the expansion parameter zI/W. Below is shown (see (36)) that $zI/W_{c0} = 0.744$ at the critical point. Thus, even in the extended state ($W < W_{c0}$), the value zI/W near the critical point is of the order of one, and the finite series (8) is a good approximation of the exact expression (7). Physically, such approximation means that the two–particle effects of the exchange interaction are responsible for the critical behaviour of the system defined by Hamiltonian (8). In the next Section, this conclusion will be confirmed on the basis of the diagram techniques.

Notice that the represented situation with the asymptotic series is also characteristic to the microscopic theory of phase transitions based on both the quantum statistical [33] and renormalization group [34] approaches (the role of the expansion parameter therewith is played by the ratio between the characteristic value of the interaction energy and the temperature). Therefore, it is quite natural that within the framework of the localization scheme being stated which is constructed by analogy with the phase transition theory, we are facing all the problems of the latter. Among other things, since the expansion parameter zI/W is not always small, the results of the theory (type of the mobility threshold) are semiquantitative by nature. The main advantage of the system being developed is the possibility of self–consistent description reflecting the critical behaviour of the system.

Let us take up, finally, the question of the type of statistics of particles and antiparticles corresponding to the operators $a_l^{\pm}$, $b_l^{\pm}$ (for the sake of convenience here a_l and b_l are redenoted as a_l^- and b_l^-). According to the determination (9), they are obtained from the operators $\psi_l^{i\pm}$ of bare particles by way of projecting onto the subspaces $\{\Phi_0\}$ and $\{\Phi_1\}$. It means that the operators $a_l^{\pm}$ and $b_l^{\pm}$ anticommutate with one another in the Fermi case and commutate in the Bose case. For each of the subspaces we have got $[a_l^-, a_m^+]_\pm = \delta_{lm}$, $[b_l^-, b_m^+]_\pm = \delta_{lm}$ where the signs $\pm$ at the square brackets mean the anticommutator for fermions and the commutator for bosons; the commutation relations for the other combinations give zero. Introducing the $\varphi_{l0} \equiv a_l$ and $\varphi_{l1} \equiv b_l$, we come to the canonical relations:

$$\left[\varphi_{l\alpha}, \varphi^+_{m\beta}\right]_\pm = \delta_{lm}\delta_{\alpha\beta};$$

$$[\varphi_{l\alpha}, \varphi_{m\beta}]_\pm = \left[\varphi^+_{l\alpha}, \varphi^+_{m\beta}\right]_\pm = 0, \qquad (10)$$

where indices $\alpha, \beta = 0, 1$ indicate the subspaces $\{\Phi_{\alpha,\beta}\}$ of the real and virtual states.

III. DEVELOPMENT OF THE SELF–CONSISTENT SCHEME

Since it is convenient, when describing the single and collective excitations, to proceed from different (site or wave) representations, we shall first make recourse to the diagram method making no use of the explicit form of the appropriate Hamiltonian. This will permit to get the general view of equations for the required set of the Green functions.

As elucidated in Section II, a fundamentally important fact is the separation, resulted from the permutation symmetry breaking, of the pair of the subspaces $\{\Phi_\alpha\}$, $\alpha = 0, 1$ corresponding to the bands of real and virtual collective states. Therefore, the sites are to be characterized by not only their numbers l and m but by these bands indices α and β as well. The presence of the latter results in the division of the ensemble of the original particles into the excitations of the particle– and vacancy/hole–type the operators of which are obeyed to commutation relations (10). As indicated in Introduction, the approach being proposed is based on the concept of the system with the quenched disorder within the framework of a standard thermodynamic scheme. It is illustrated in Appendix that, in spite of the nonequilibrium character, such a system, when in the steady state, is described by the quasi–Gibbs distribution for which the role of temperature is played by the level scattering halfwidth $W/2$. Description of the present system is ensured by the use of Matsubara's Green function $G_{lm}^{\alpha\beta}(t) = -\left\langle \widehat{T}\varphi_{l\alpha}(t)\varphi_{m\beta}^{+}(0)\right\rangle$ where $\varphi_{l0} \equiv a_l$ and $\varphi_{l1} \equiv b_l$ are the operators of particles and holes determined by the relationships (9), t is the imaginary time, the angular brackets mean averaging over the quantum states, the other symbols are standard [9]. Making use of the commutation relations (10), it is easy to illustrate that the normal (diagonal) and anomalous (off–diagonal) components possess the following properties:

$$\mp G_{ml}^{11}(-t) = G_{lm}^{00}(t) \equiv G_{lm}(t),$$
$$G_{ml}^{10*}(t) = G_{lm}^{01}(t) \equiv F_{lm}(t), \tag{11}$$

where the upper sign refers to fermions, the lower one — to bosons. Thanks to this, it is sufficient to examine the behaviour of only two components — the normal $G_{lm}(t)$ (see fig. 1a) and the anomalous $F_{lm}(t)$ (see fig. 1b). Further, when making general computations, it is also convenient to use the matrix representation of the type $\widehat{G}_{lm}(t)$ where the cap signifies the exhaustive search for the α and β indices.

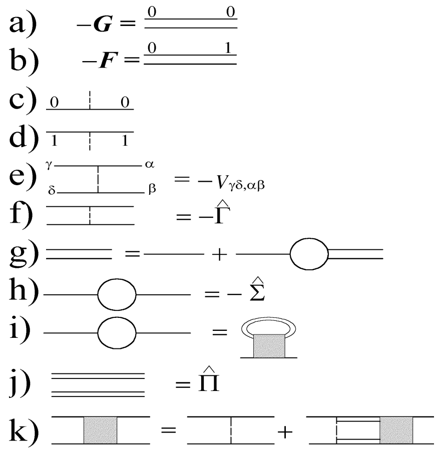

Fig. 1. Diagram representation of single and collective excitations.

In addition to the matrix nature of the Green function, the presence of the subspaces $\alpha = 0, 1$ results in two types of bare vertices depicted in fig. 1c and 1d where the solid line corresponds to the components of the Green locator function of a particle or vacancy, the dashed line corresponds to the overlapping integral. As in the theory of superconductivity [9], it is technically more convenient to switch from three–tail vertices to the appropriate four–tail vertices (see fig. 1e) of the effective interaction. Since the interaction between particles belonging to different bands is responsible for delocalization, we shall consider as non–zero only the components $V_{01,01} \equiv V_{00}$, $V_{10,10} \equiv V_{11}$, $V_{01,10} \equiv V_{01}$, $V_{10,01} \equiv V_{10}$ producing the $\widehat{V}$ matrix of the second rank. The $\widehat{\Gamma}$ matrix (see fig. 1f) of the vertex function is structured in the similar manner. However, while the $\widehat{V}$ matrix of the bare interaction is evidently diagonal, the complete vertex $\widehat{\Gamma}$ has, as will be seen below, all nonzero–components.

In the diagram representation the Dyson matrix equation takes an ordinary form [9] (see fig. 1g) where the double line corresponds to the exact Green function, and the single line corresponds to the bare one (the matrix of the latter is diagonal). The self–energy function (see fig. 1h) is expressed by the equation illustrated in fig. 1i. Thus, the problem reduces to the self–consistent determination of the vertex function $\widehat{\Gamma}$. Similar to Ref. [9], it can be shown that, at the expansion in terms of $\widehat{V}$, the main contribution is made by the terms containing the polarizer $\widehat{\Pi}$ (see fig. 1j) the matrix components $\Pi_{\alpha\beta}$ of which are determined in a similar way to the $V_{\alpha\beta}$ and $\Gamma_{\alpha\beta}$. Then the appropriate series reduces to the ladder sequence which can be represented in the form of the Bethe–Salpeter equation (fig. 1k). It closes the system of equations for the self–consistent description of a particle in a random field.

In the analytic representation, this system is written as follows:

$$\widehat{G}^{-1}(\omega_s) = i\omega_s\widehat{\delta} - (\varepsilon - \mu)\widehat{\tau}_3 - \widehat{\Sigma}(\omega_s), \tag{12a}$$

$$\Sigma_{\alpha\beta}(t) = G_{\beta\alpha}(-t)\Gamma_{\alpha\beta}(t), \tag{12b}$$

$$\widehat{\Gamma}^{-1}(\Omega_S) = \widehat{V}^{-1} \mp \widehat{\Pi}(\Omega_S), \tag{12c}$$

$$\Pi_{\alpha\beta}(t) = \left[G_{\alpha\beta}(t)\right]^2. \tag{12d}$$

Here, the frequencies ω_s and Ω_S of single and collective excitations are determined by the expressions $\omega_s = \pi(2s+1)W/2$ and $\Omega_S = \pi(2S+1)W/2$ in the case of the Fermi particles, and by the equations $\omega_s = \pi s W$ and $\Omega_S = \pi S W$ in the case of the Bose particles; $s, S = 0, \pm 1, \ldots$ are integers; ε is the bare energy of a particle; μ is a chemical potential; $\widehat{\delta}$ is a unit matrix; $\widehat{\tau}_3$ is a diagonal Pauli matrix with the elements $\tau_3^{00} = -\tau_3^{11} = 1$. The matrix structure of the Green function (12a) taking into account conditions (11) conforms to the Fermi case; in the Bose case, the matrices $\widehat{\delta}$ and $\widehat{\tau}_3$ are to be interchanged. With the derivation of the equation (12c) for the vertex function $\widehat{\Gamma}$ where the upper sign is in agreement with fermions, the lower one conforms to bosons, it is generally taken that the bare potential reduces to the constant $\widehat{V}$. A distinctive feature of the system obtained consists in the fact that the explicit expressions (12a), (12c) for the Green functions $\widehat{G}$, $\widehat{\Gamma}$ of the single and collective excitations are obtained in the frequency representation while the expressions (12b) and (12d) for the self–energy function $\widehat{\Sigma}$ and polarizer $\widehat{\Pi}$ require the application of the time representation. As far as the site and wave representations are concerned, their choice depends on the type of excitations meant.

IV. SINGLE EXCITATIONS

Further discussion essentially depends on the fact by what type of statistics the progenitor gas is characterized. At first, examine the case of fermions, and then — that of bosons.

a. Fermi case. If we are interested only in the behaviour of the single excitations [11], their description is obtained in the simplest way within the framework of the quasi–mean–value method [35]. Applying the standard procedure, it can be shown on the basis of the expression (8) that at $N_0 \to \infty$ the behaviour of the system is asymptotically defined by the approximating Hamiltonian which takes, in the self–consistent field approximation, the following form:

$$\mathcal{H} = \sum_l (\varepsilon_l - \mu) a_l^+ a_l - \frac{|V|}{2} \sum_l \left(\eta^* b_{\bar{l}} a_l + \eta a_l^+ b_{\bar{l}}^+\right) + \frac{|V|}{4} |\eta|^2 N_0. \tag{13}$$

Anomalous quasi–mean values

$$\eta = (2/N_0) \sum_l \langle b_{\bar{l}} a_l \rangle \equiv (2/N_0) \sum_{il} \left\langle Q_i \psi_{\bar{l}}^{i+} \psi_l^i P_i \right\rangle,$$
$$\eta^* = (2/N_0) \sum_l \left\langle a_l^+ b_{\bar{l}}^+ \right\rangle \equiv (2/N_0) \sum_{il} \left\langle P_i \psi_l^{i+} \psi_{\bar{l}}^i Q_i \right\rangle, \tag{14}$$

determining the amplitude of the particle transfer from the state Φ_0 at the site l to the state Φ_1 at the "neighbouring" site $\bar{l}$ (and vice versa for the η^*) represent the order parameter corresponding to the extended state. With an accuracy of the multiplier $(N_0/2)^{1/2}$, the values of η^* and η coincide with the condensate part of the PCS operators $\Psi_l^+ = a_l^+ b_{\bar{l}}^+$ and $\Psi_l = b_{\bar{l}} a_l$ determining the collective excitation mode at zero quasi–momentum:

$$\Psi_0 \equiv \langle \Psi(\mathbf{K}=0) \rangle = (2/N_0)^{1/2} \sum_l \langle b_{\bar{l}} a_l \rangle,$$
$$\Psi_0^+ \equiv \left\langle \Psi^+(\mathbf{K}=0) \right\rangle = (2/N_0)^{1/2} \sum_l \left\langle a_l^+ b_{\bar{l}}^+ \right\rangle. \tag{15}$$

It follows herefrom that Bose condensation of $\mathcal{N}$ PCSs implying $\Psi_0 = \mathcal{N}^{1/2}$ brings to the order $\eta \equiv (N_0/2)^{-1/2} \Psi_0 = (2\mathcal{N}/N_0)^{1/2}$. In so doing, the function of PCSs distribution by quasi–momentum

$$(N_0/2) S(\mathbf{K}) \equiv \left\langle \Psi^+(\mathbf{K}) \Psi(\mathbf{K}) \right\rangle \approx |\langle \Psi(\mathbf{K}) \rangle|^2 = |\Psi_0|^2 \, \delta_{\mathbf{K}0} \tag{16}$$

develops a δ–shaped peak of height $S(0) = 2\mathcal{N}/N_0 = \eta^2$. It is characteristically that this feature appears just in the two–particle Green function associated with the PCSs distribution but not in the single particle function. This is consistent with the results of the scaling theory [36] in accordance with which, close to the mobility threshold E_c, not the field variables φ_a^α ($\alpha = 0, 1$; $a = 1, \ldots, n$, $n \to 0$ is the dimensionality of the replica space) themselves exhibit a critical behaviour but the tensor $Q_{ab}^{\alpha\beta} = \varphi_a^\alpha \varphi_b^\beta$ caused by the spontaneous symmetry breaking (transition from the group $O(2n)$ to the noncompact group $O(n,n)$). Such a tensor plays a role of the order parameter in the field approach [4,8]. It is in agreement with the definition (14) of the order parameter in the form of the average of the pair of operators a_l, $b_{\bar{l}}$.

Diagonalization of Hamiltonian (13) is achieved through the transformation (compare with (2))

$$\alpha_{l+} = u_l a_l - v_l b_{\bar{l}}^+, \qquad \alpha_{l-} = u_l b_l + v_l a_{\bar{l}}^+, \tag{17}$$

where, by virtue of (10), $u_l^2 + v_l^2 = 1$. It results in giving:

$$\mathcal{H} = U + \frac{1}{2} \sum_l \epsilon_l \left(\alpha_{l+}^+ \alpha_{l+} + \alpha_{\bar{l}-}^+ \alpha_{\bar{l}-}\right) + \frac{1}{2} \sum_l (\varepsilon_l - \mu) \left(\alpha_{l+}^+ \alpha_{l+} - \alpha_{\bar{l}-}^+ \alpha_{\bar{l}-}\right), \tag{18}$$

where

$$U = \frac{N_0}{2} \left\{ \frac{|V|}{2} |\eta|^2 - \frac{1}{N_0} \sum_l \left[\epsilon_l - (\varepsilon_l - \mu)\right] \right\};$$
$$\epsilon_l = \sqrt{(\varepsilon_l - \mu)^2 + \Delta^2}, \tag{19}$$
$$\Delta = |V| \eta, \qquad \eta = \eta^*;$$
$$\left.\begin{matrix} u_l^2 \\ v_l^2 \end{matrix}\right\} = \frac{1}{2} \left(1 \pm \frac{\varepsilon_l - \mu}{\epsilon_l}\right).$$

The quantity U represents the energy of the ground state the wave function of which is:

$$|\Psi_0\rangle = \prod_l \left(u_l + v_l a_l^+ b_{\bar{l}}^+\right) |0\rangle , \tag{20}$$

where $|0\rangle$ is the wave function of the Fermi vacuum. At the noncoincidence of excitations defined by the operators α_{l+} and $\alpha_{\bar{l}-}$, they relate to the gapless law of dispersion $\epsilon_l^{(\pm)} = (1/2)\left[\epsilon_l \pm (\varepsilon_l - \mu)\right]$, and the phase transition is not possible. For the phase transition to be achieved within the framework of the developed scheme, it is necessary to postulate the coincidence of the behaviour of elementary excitations in the states Φ_0 and Φ_1 (physically, it means that creation of excitation at the site l in the state Φ_0 results in excitation appearance at the "neighbouring" site $\bar{l}$ in the state Φ_1). Then the energy ϵ_l of elementary excitations defined by the operator $\alpha_l \equiv \alpha_{l+} = \alpha_{\bar{l}-}$ is characterized by the presence of the gap Δ which is proportional to the order parameter η.

To further develop the theory, it is appropriate to assign a physical meaning to the averaging operation $\langle \ldots \rangle \equiv \int \ldots P(\varepsilon) \mathrm{d}\varepsilon$ that is achieved by assigning the probability $P(\varepsilon)$ to have the energy ε. For the reasons stated in Appendix, we shall apply the standard quasi–Gibbs distribution (A.1) characterized by the constant W. Then the statistic theory of the system under consideration with the quenched disorder can be achieved by analogy with the appropriate theory [37] of thermodynamic systems with temperature $W/2$. In this theory, the distribution $\nu_l = \langle \alpha_l^+ \alpha_l \rangle$ of the elementary excitation gas is defined by the Fermi function (A.2), and the corresponding distributions $p_l = \langle a_l^+ a_l \rangle$, $h_l = \langle b_l^+ b_l \rangle$ of interacting particles and antiparticles are associated with the ν_l by formulae

$$2p_l = 2h_{\bar{l}} = 1 + n_l , \tag{21}$$

$$n_l \equiv \frac{\mu - \varepsilon_l}{\epsilon_l}(1 - 2\nu_l) = \frac{\mu - \varepsilon_l}{\epsilon_l} \tanh \frac{\epsilon_l}{W} ,$$

following from (17). Making use of the identity

$$\sum_i \left\langle \psi_l^{i+} \psi_m^i \right\rangle = \left\langle a_l^+ a_m \right\rangle + \left\langle b_l b_m^+ \right\rangle + \left\langle b_l a_m \right\rangle + \left\langle a_l^+ b_m^+ \right\rangle , \tag{22}$$

resulting from the relation $P_i + Q_i = 1$, we get, as expected, $\sum_i \left\langle \psi_l^{i+} \psi_l^i \right\rangle = 1$ for the number of levels at the given site. Transition from the obtained site functions to the corresponding energy distributions is performed through multiplying by the level density $P(\varepsilon)$ specified by the equality (A.1).

The equation for the gap width is obtained by substitution of the inverted equalities (17) in the definitions (14):

$$\frac{|V|}{W} \int_{-\infty}^{\infty} \frac{\tanh\left(\sqrt{(x-e)^2 + d^2}/2\right) \mathrm{e}^{-|x|}}{\sqrt{(x-e)^2 + d^2}} \,\mathrm{d}x = 1. \tag{23}$$

Applied here is the transition $\sum_l \ldots \to N_0 \int \ldots P(\varepsilon) \,\mathrm{d}\varepsilon$, $d \equiv 2\Delta/W$, $e \equiv 2E/W$, $E = \mu - \bar{\varepsilon}$ — shift from the band center. As contrasted to the BCS model, the interaction parameter $V = -N_0 \Delta_0^2 / \Delta\mathcal{E}$, where $\Delta_0 \equiv zI$, is not a constant, its magnitude is determined by the difference $\Delta\mathcal{E}$ of the system energies in the states Φ_1 and Φ_0. For the isolated PCS, it is equal to ϵ_l, and with the presence of the extended phase, $\Delta\mathcal{E} = \sum_l \epsilon_l$. So, for V we get:

$$|V|^{-1} = \frac{W}{(2\Delta_0)^2} \int_{-\infty}^{\infty} \sqrt{(x-e)^2 + d^2}\, \mathrm{e}^{-|x|} \,\mathrm{d}x. \tag{24}$$

At transition to the extended state, the jump of the complete energy (analog of the thermodynamic potential) $\Delta F = F_d - F_l$ is determined by the formula

$$\Delta F = \frac{N_0}{4} \int_0^V \frac{\Delta^2}{V^2} \,\mathrm{d}V , \tag{25}$$

following from the equalities $(\mathrm{d}F/\mathrm{d}V)_{W,\mu} = \langle \partial\mathcal{H}/\partial V \rangle = \partial U/\partial V$ and (17) where $\eta = \Delta/V$. According to (25) during delocalization, as it usually is during transition to the low–symmetry phase, value F decreases (recall that $V < 0$).

The elementary excitation energy $\mathcal{E}_{\mathrm{ex}} = \sum_l \epsilon_l \nu_l$ is of the form:

$$\mathcal{E}_{\mathrm{ex}} = \frac{N_0}{4} W \int_{-\infty}^{\infty} \frac{\sqrt{(x-e)^2 + d^2}\, \mathrm{e}^{-|x|}}{1 + \exp\sqrt{(x-e)^2 + d^2}} \,\mathrm{d}x. \tag{26}$$

The obtained formulae contain, as a free parameter, in addition to the level scattering width W, the shift $E = \mu - \bar{\varepsilon}$ from the center of the band of localized states. It should be distinguished from the usual Fermi energy E_F . Really so, within the framework of the locator approach applied, the Lagrange multiplier μ incorporates the condition of conservation of the total number of localized fermions, and, thus, is subtracted from the energy ε_l. Appearance of the magnitude E_F in the framework of the propagator approach is associated with taking account of the condition of conservation of the total number of extended fermions. In this connection use is made of the wave representation, and the Lagrange multiplier $E_F = k_F^2/2m$ is subtracted from the space Fourier–form of the overlapping integral [2,3] $I(\mathbf{k}) = k_F^2/2m + k^2/2m$ where k_F is the Fermi wave number, m is a particle mass, Planck constant $\hbar = 1$.

To determine the relation between E and E_F, we shall measure all the energy dimensional magnitudes in the units of $\Delta_0 = k_D^2/2m$, k_D is the Debye wave number. Then at the given value of concentration $n = N/N_0$ of the localized particles (N is their total number), the Fermi energy of the d–dimensional degenerated system $E_F = (k_F/k_D)^2$ is determined by the evident equality

$$E_F = (1-n)^{2/d}. \tag{27}$$

On the other hand, the chemical potential μ in the equalities (18) and (19) is given by the condition [37]

$$\frac{\partial F}{\partial \mu} = -N_0 p, \tag{28}$$

$$F \equiv -\frac{W}{2}\ln\left\langle \exp\left(-\frac{2\mathcal{H}}{W}\right)\right\rangle = U + \frac{W}{2}\sum_l \ln(1-\nu_l),$$

where $p = N_0^{-1}\sum p_l$ is the concentration of particles specified by the operators (9). Taking into account the equalities (21), we obtain the relation $p = (1+n)/2$, and condition (28) reduces to the equality

$$n = N_0^{-1}\sum_l \frac{\mu-\varepsilon_l}{\epsilon_l}\tanh\frac{\epsilon_l}{W}\,, \tag{29}$$

$$n \equiv N_0^{-1}\sum_l n_l\,.$$

Expressions (27) and (29) offer the required relation $E_F(E)$, the shift $E = \mu - \bar{\varepsilon}$ itself is determined through the equality (29) by the assigned values n and W.

Previously, we applied the quasi–mean–value method bringing in shortest ways to the description of single excitations. Needless to say, the illustrated results can also be obtained by means of the Green method which allows for determining the Fermi function $\widehat{G}(\omega_s)$ from the system (12a)–(12d). Keeping in mind the fact that the Green procedure allows for describing not only each of the types of excitations but for taking into account their coupling, we shall demonstrate at first the way it reproduces the results of the quasi–mean–value method.

In the context of the locator approach, the functions $\widehat{G}$, ε and $\widehat{\Sigma}$ in the equation (12a) and functions $\widehat{\Gamma}$, $\widehat{V}$ and $\widehat{\Pi}$ in the equation (12c) should be corresponded the lattice index l, and splitting of the appropriate contribution (12b) and (12d) is achieved in transition to the wave representation. Due to the site and wave representation inversion of this kind, the applied locator approach radically differs from the Edwards propagator representation.

To define the explicit form of the Green function (12a), we shall consider that, as in the case of the theory of superconductivity, the self–energy function $\widehat{\Sigma}$ acquires, under delocalization, the off–diagonal components $\Sigma_{01} = \Sigma^*_{10} = \Delta$ corresponding to the gap Δ in the energy spectrum of the single excitations [9]. The matrix $\widehat{G}$ diagonalization then leads to the following expressions:

$$G_l(\omega_s) = -\frac{i\omega_s + (\varepsilon_l - \mu)}{\omega_s^2 + \epsilon_l^2}\,, \tag{30}$$

$$F_l(\omega_s) = -\frac{\Delta}{\omega_s^2+\epsilon_l^2}\,,$$

$$\epsilon_l = \left[(\varepsilon_l-\mu)^2 + \Delta^2\right]^{1/2},$$

$$\omega_s = \pi(2s+1)W/2, \quad s = 0, \pm 1, \dots$$

Substituting them into eq. (12b) for the off–diagonal components, and assuming that here the vertex $\widehat{\Gamma}$ reduces to the bare potential $\widehat{V}$, we arrive at the self–consistency equation which, as would be expected, coincides with eq. (23) for the gap Δ. Dependence of the chemical potential $E = \mu - \bar{\varepsilon}$ on the localized particle concentration $n = N/N_0$ follows from the relation $N = W\sum_s\sum_l G_l(\omega_s)$ which, with the application of the first one from the equalities (30), leads to eq. (29).

Polarizer (12d) has the following Fourier–form:

$$\Pi_l^{\alpha\beta}(\Omega_S) = \tag{31}$$

$$- W\sum_{s=-\infty}^{\infty} N_0^{-1}\sum_m G^{\alpha\beta}_{l+m}(\Omega_S - \omega_s)G^{\alpha\beta}_m(\omega_s).$$

Substituting the Green functions (30) here, upon summing over s, we shall arrive at the expressions:

$$\Pi^{00}(0) = \Pi^{11}(0) = -N_0^{-1}\sum_l \epsilon_l^{-1}\tanh(\epsilon_l/W) + A\Delta^2,$$

$$\Pi^{01}(0) = \Pi^{10}(0) = -A\Delta^2; \tag{32}$$

$$A \equiv \frac{1}{2N_0}\sum_l \epsilon_l^{-3}\left[\tanh(\epsilon_l/W) - (\epsilon_l/W)\cosh^{-2}(\epsilon_l/W)\right],$$

where, in view of the macroscopic equivalence of sites, there is no dependence on their number l. Making use of (32), it is easy to find the inverse vertex function (12c) responsible for the behaviour of the collective excitations of fermions (see Section Va).

The system of equations (23), (24) and (29) that offers the self–consistent description of the single excitations affords finding, by the assigned values of the level scattering W and concentration n, the value V of the effective interaction potential, gap width Δ and chemical potential shift E from the band center. Numerical solution to these equations results in the dependencies displayed in figs. 2 and 3. From these figures we notice that, as W increases, the value of $|V|$ first rises (for more details, see what is preceding (39)) and then, after a cusp, monotonically decays; an increase in the concentration n brings about a decrease in $|V|$. The gap width Δ monotonically decays as the level scattering W and concentration

n increase, and the shift E does as they decrease. Let us examine the form of these dependencies in detail.

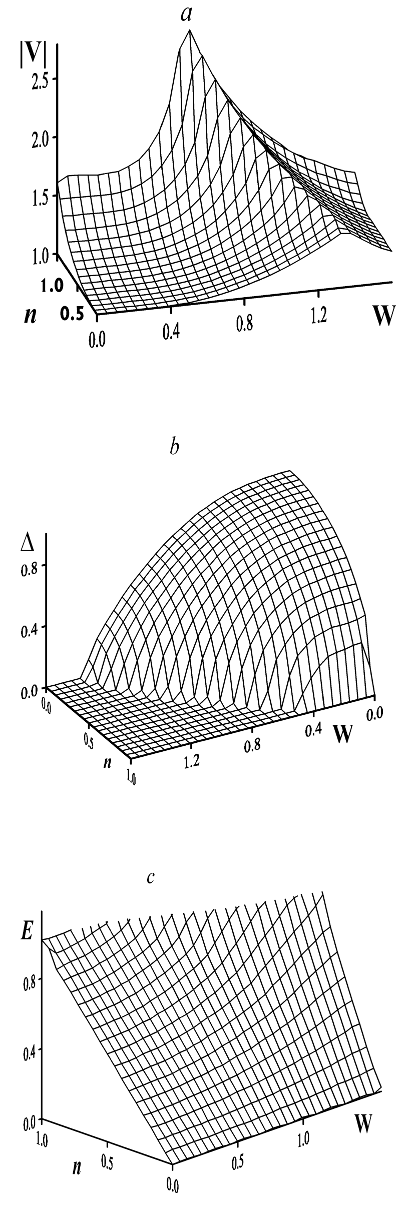

Fig. 2. a — Relationship between the effective interaction potential $|V|$ and the level scattering width W and concentration n; b — Relationship between the gap width Δ of single excitations and W, n; c — Relationship between the chemical potential shift E from the band center and W, n.

The ground system state defined by the wave function (20) is achieved at $W = 0$ where all the levels coincide ($P(\varepsilon) = \delta(\varepsilon - \bar{\varepsilon})$), and elementary excitations are absent ($\nu_l = 0$). In the center of the band ($E = 0$, $n = 0$), the order parameter and gap width take maximum values $\eta = 1$, $\Delta = 1$, the ground state energy and the total energy jump take minimum values $U = \Delta F = -N_0/4$, and the interaction parameter $|V| = 1$ (hereinafter, unless otherwise specified, the energy has dimensionality $\Delta_0 = zI$). Zones Φ_0 and Φ_1 are half full of particles and antiparticles ($p_l = h_{\bar{l}} = 1/2$). With the finite concentration $n \neq 0$, a shift from the band center is $E = n$, and $\Delta^2 = \eta^2 = 1-E^2$, $|V| = 1$, $\Delta F = -(N_0/4)(1-E^2)$,

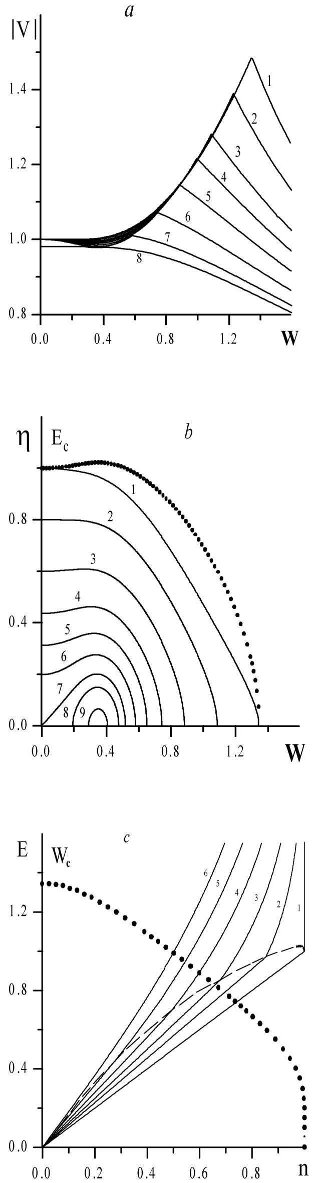

Fig. 3. a — Relationship between the effective interaction potential $|V|$ and the level scattering width W at various values of the chemical potential shift from the band center E: 0.0 (curve 1), 0.4 (2), 0.6 (3), 0.7 (4), 0.8 (5), 0.9 (6), 0.98 (7), 1.02 (8); b — Relationship between the extended state band width E_c (dotted line), order parameter η (solid lines) and W at various values of E: 0.0 (curve 1), 0.6 (2), 0.8 (3), 0.9 (4), 0.95 (5), 0.98 (6), 1.0 (7), 1.01 (8), 1.02 (9); c — Concentration relationships between the values E_c at which the extended state band expands (dashed line), the critical level scattering W_c (dotted line) and the values of E (solid lines) at various values of W: 0.0 (curve 1), 0.6 (2), 0.8 (3), 1.0 (4), 1.2 (5), 1.4 (6).

$p_l = h_{\bar{l}} = (1+E)/2$. Thus, as the value E increases, the order parameter, gap width and absolute value of the system total energy change under delocalization monotonically decay going into zero at the boundary value $E_c = 1$. The numbers of particles and antiparticles accordingly increase from 1/2 to 1.

There is no difficulty in understanding that the described ground state of a stochastic system corresponds to the ordinary spreading of the coincident energy levels $\varepsilon_l = \bar{\varepsilon}$ into the extended state band of width (in conventional units) $2E_c = 2zI$. The order parameter, therewith, determines the density of extended states in accordance with the following equality:

$$g(E) \equiv (2/\pi)\eta(E) = (2/\pi)\sqrt{1-E^2}\,. \tag{33}$$

Taking into account then that, in accordance with (30), the gap width determines the value of the anomalous Green function $F_l(\omega_s)$ the summation of which over frequencies ω_s and sites l gives the number of extended particles, it can be concluded that, in the general case $W \neq 0$, the dependence $\eta(W, E)$ defines the density of extended states [51] of an imperfect crystal with the energy E.

As the levels are scattered, originated are the elementary excitations the number of which $\nu_l \sim \exp(-2/W) \ll 1$. With a precision of the first non–zero terms over $W \ll 1$, we obtain:

$$\begin{aligned}
&\Delta^2 = \eta^2 = (1-E^2) - W^2/2, \qquad (34)\\
&|V| = 1,\\
&\Delta F = -(N_0/4)\left[(1-E^2) - W^2/2\right],\\
&\mathcal{E}_{\rm ex} = N_0 \exp(-2/W),\\
&E = n(1+W^2/2);\\
&E_c = 1 + W^2/4;\\
&2p_l = 2h_{\bar{l}}\\
&= \begin{cases} (1+E) - (1-E^2)(\varepsilon_l - \bar{\varepsilon}) \text{ at } |\varepsilon_l - \bar{\varepsilon}| \leq W/2; \\ 1+E \text{ at } |\varepsilon_l - \bar{\varepsilon}| \gg W. \end{cases}
\end{aligned}$$

The power character (as differentiated from the exponential one usually observed under phase transitions) of the dependence of the values of Δ, η, ΔF on the level scattering W results from the dependence of the interaction parameter V on the spectrum of the system which accounts for the site resonating [38]. According to (34), the last dependence displays only with the considerable level scattering ($W \sim 1$). For the subcritical shifts $E < E_{\rm cr}$, the level scattering results in decreasing the values of Δ, η, ΔF. Here the difference of the interaction parameter $|V|$ from unit resulting in the diversity in the boundary values of $E_{\rm cr}$ for the dependencies $\eta(W)$ ($E_{\rm cr} = (5^{1/2}-1)/2 = 0.618$) and $\Delta(W)$, $\Delta F(W)$ ($E_{\rm cr} = 2^{-1/2} = 0.707$).

The excitation energy $\mathcal{E}_{\rm ex}$ and the value of $C_{\rm ex} = {\rm d}\mathcal{E}_{\rm ex}/{\rm d}W$ which defines its rate of increase as the level scattering increases (an analog of the heat capacity in thermodynamics) are, according to (26), of the following form:

$$\begin{aligned}
&\mathcal{E}_{\rm ex} = N_0(1-E^2)^{-1}{\rm e}^{-2/W},\\
&C_{\rm ex} = 2N_0(1-E^2)^{-1}W^{-2}{\rm e}^{-2/W}; \qquad (35)\\
&W \ll 1, \quad E < 1.
\end{aligned}$$

Shift from the band center results in a power increase in the values of $\mathcal{E}_{\rm ex}$ and $C_{\rm ex}$ an exponential smallness of which results from the presence of a gap.

As viewed in figs. 2 and 3, the characteristic feature of the system behaviour consists in turning the values of Δ and η into zero, and cusp of the dependence $V(W)$ under the critical scattering W_c the maximum value of which (in conventional units)

$$W_{c0} = 2\sqrt{\ln(\pi/2)}\, zI = 1.344\, zI \tag{36}$$

is obtained at $n = 0$. The derived value of W_{c0}/zI is in satisfactory agreement with the value 3.3 obtained by Ref. [21] within the framework of the Edwards approach, and with the result 2.4 of the coherent potential approximation [39], and the result 1.8 in the diagram method [22]. The Götze approach [3] offers the value of W_{c0} dependent on the Fermi level E_F: with the value of $E_F = 1$ corresponding to the center of the band $E = 0$, we have $W_{c0}/zI = 1.2$. The shift from the band center caused by an increase in concentration n results in a monotonous decay of the critical value of W_c in compliance with the chart shown in fig. 3c with a dotted line. In the limiting cases with a logarithmic accuracy, we have [52]

$$\begin{aligned}
&W_c = W_{c0}(1 - an^2) \qquad \text{at} \quad n \ll 1,\\
&W_c = 2/\ln(1-n)^{-1} \qquad \text{at} \quad 1-n \ll 1; \qquad (37)\\
&a^{-1} \equiv 8(\ln 4 - 1)^2 \ln(\pi/2) = 0.539.
\end{aligned}$$

As regards the critical value of E_c, unlike the dependence of the critical field on temperature usually observed under phase transformations, in our case, the dependence $E_c(W)$ of the extended state band width on the level scattering (see the dashed curve in fig. 3b) is not monotonically dropping — at $W < W_m \simeq 0.35$, the scattering slightly expands the band (for $W^2 \ll 1$, see (34)), and at $0 < (W_{c0} - W)/W_{c0} \ll 1$, quickly narrows it:

$$\begin{aligned}
E_c &= 2^{-1/2}W_{c0}^2(1 - W/W_{c0})^{1/2} \qquad (38)\\
&= 1.277(1 - W/W_{c0})^{1/2}.
\end{aligned}$$

The nonmonotonic nature of the dependence $E_c(W)$ coincides with the one obtained in Ref. [39].

According to (24), an increase in the interaction parameter $|V|$ is caused by a decrease in the energy of elementary excitations as the level scattering increases (at the expense of dropping of $d = 2\Delta/W$), and its decrease is connected with the trivial integral increase in the right-hand side of eq. (24) occurring while the shift from the band center increases (at $W < W_c$, the first factor prevails, and at $W > W_c$, the second one predominates). In the center of the band ($E = 0$), the level scattering $W < W_c$ brings about a slow increase in the interaction parameter from the value of $|V| = 1$; under the finite shift $E < 1$, the decline $1 - |V| \propto W^n$ with a power of $n > 2$ takes place first (see (34)), and then an elevation occurs (see figs. 2a, 3a). At $1 < E < E_m,\ E_m \simeq 1.02$ in the domain $0 \le W < W_0$ where $W_0^2 \simeq 2(E^2-1)$, there is a constant value of $|V| = E^{-1}$; in the point $W = W_0$, a break occurs after which $|V|$ slightly drops at first and then rises. Close to the critical scattering ($0 < (W_{c0} - W)/W_{c0} \ll 1,\ (E/W_{c0})^2 \ll 1$), we have

$$|V| = \frac{2}{W_{c0}}\left[1 - \frac{W_{c0} - W}{W_{c0}} - 2\left(\frac{E}{W_{c0}}\right)^2\right]. \tag{39}$$

Above the critical point ($W > W_c$)

$$|V|^{-1} = E + (W/2)\exp(-2E/W). \tag{40}$$

A jump of the derivative $\mathrm{d}|V|/\mathrm{d}W$ stipulated by the spectrum rearrangement reaches at $W = W_c$ the maximum value of $4/W_{c0}^2 = 2.214$ at $E = 0$, and decreases to zero as E increases.

By virtue of the described dependence of the potential V on the system spectrum, observed is a different behaviour of the gap width Δ and the order parameter $\eta = \Delta/|V|$: as it usually is under phase transitions, the gap width monotonically decreases as the level scattering W increases (see fig. 2b), but the order parameter can change in a nonmonotonic manner as well (see fig. 3b). This nonmonotony is found beginning from the critical value of $E_{\rm cr} = 2^{-1/2}$ of the chemical potential shift from the band center. At $E_{\rm cr} < E < 1$, a small scattering W does not cause a decrease in the value of η but does its square–law increase which changes to linear at the very point of $E = 1$. Even greater change of the $\eta(W)$ dependence kind occurs over the interval $1 < E < E_m,\ E_m \approx 1.02$; the order parameter acquires the finite values not at zero–scattering but at $W > W_0$, $W_0 \simeq 2^{1/2}(E^2-1)^{1/2}$. Thus the region of existence of the extended state $W_0 < W < W_c$ is restricted here both from above and from below, the order parameter $\eta(W)$ changing with an infinite derivative at each of the boundaries (see fig. 3b). As indicated above, these features show up in a nonmonotonic change of the extended state band width $E_c(W)$ (see the dotted line in fig. 3b). Mathematically speaking, the nonmonotonic nature of the dependencies $\eta(W)$ and $E_c(W)$ is associated with the fact that fixed is not the concentration n but energy E when they are being determined. With allowance made for the relation (24), it means that the chemical potential of extended particles $E_F = (1-n)^{2/d}$ can change but corresponding value $\mu = E + \bar\varepsilon$ for the localized particles is fixed. The physical reason for such a choice is the fact that the extended state density $g = (2/\pi)\eta$ is determined in relation to the Fermi energy E_F.

Now, define the behaviour of the values Δ and η close to the critical point W_{c0} in the center of the band. Eliminating the parameter $|V|$ from eqs. (23) and (24), with a precision of the first non–zero terms over $\Delta/W_{c0} \ll 1$, we obtain ($E = 0$)

$$\begin{aligned}(W_{c0}/W)^2 &= \int_0^\infty \sqrt{x^2+d^2}\,\mathrm{e}^{-x}\,\mathrm{d}x \\ &= (\pi/2)d\left[H_1(d) - Y_1(d)\right],\end{aligned} \tag{41}$$

where $H_1(d)$ is the Struve function; $Y_1(d)$ is the Bessel function of the second kind [40]. At $d \ll 1$, we have $H_1(d) \approx (2/3\pi)d^2,\ Y_1(d) \approx -2/\pi d + (d/\pi)\ln(d/2)$, and it follows from (41):

$$\left(\frac{\Delta}{W_{c0}}\right)^2 \ln\left(\frac{\Delta}{W_{c0}}\right) = \frac{W - W_{c0}}{W_{c0}}. \tag{42}$$

Accordingly, for the order parameter we obtain:

$$\left(\frac{\eta}{0.903}\right)^2 \ln\left(\frac{\eta}{0.903}\right) = \frac{W - W_{c0}}{W_{c0}}. \tag{43}$$

Hence, in comparison with the ordinary phase transition of the second order where there is no logarithm in the formulae of the (42) and (43) type, a decay of the values of Δ and η close to W_c is slower: if, in the first case, the rate of decay $\mathrm{d}\Delta^2/\mathrm{d}W$ is constant, in the second case $\mathrm{d}\Delta^2/\mathrm{d}W = W_{c0}/\ln(\Delta/W_{c0})$ and at $W \to W_{c0}$ it slowly tends to zero. This difference does not affect the chart of the gap width $\Delta(W)$ itself but manifests itself with respect to its square $\Delta^2(W)$ which graph is tangent with the axis of abscissa is observed at $W = W_c$.

As pointed out above, the extended state density $g = (2/\pi)\eta$ should be determined with a fixed value of the energy E. To determine the compressibility $g(E)$ measured in the experiment, the value of this energy should by fixed by the condition (29). The form of the appropriate dependence $E(W,n)$ is illustrated in figs. 2b and 3b. As is seen therefrom, the dependence $E(n)$ in the extended region ($W < W_c$) is nearly linear and characterized by the asymptotic form depicted for $W \ll 1$ in (34). In the localized state, we get a relationship

$$\begin{aligned} n &= \alpha\ln(1+\alpha^{-1}) - \alpha^{-1}\ln(1+\alpha), \\ \alpha &\equiv \exp(2E/W), \end{aligned} \tag{44}$$

applicable at any value $W \ge W_c$. The transition from practically directly proportional dependencies $E(n)$, typical of the extended state, to the curves in the localized region which possess an appreciable nonlinearity is carried out as consistent with the dashed line of the dependence $E_0(n)$ which determines the value of the energy beginning from which, at the assigned concentration of n, the extended state is formed. According to fig. 3b, the curve $E_0(n)$ is convex–shaped with a monotonic energy increase over all the concentration values, exclusive of the region $n \approx 1$. In analytical form with a logarithmic accuracy, we have

$$\begin{aligned} &E_0 = bn \qquad \text{at} \quad n \ll 1, \\ &E_0 = 1 + |\ln(1-n)|^{-2} \qquad \text{at} \quad 1-n \ll 1; \\ &b \equiv \sqrt{\ln(\pi/2)}/(\ln 4 - 1) = 1.739. \end{aligned} \tag{45}$$

As is evident from the foregoing, the shift E from the center of the localized state band plays the role of the field conjugate with the order parameter. Its non–zero values are realized under the shift E which does not exceed the critical value of E_c (see the dotted curve in fig. 3b). The value proper of the chemical potential $\mu = E + \bar{\varepsilon}$, therewith, is determined by the concentration n of the localized particles and the level scattering W in accordance with the dependencies displayed in figs. 2b and 3b and relationships (34) and (44). The typical values of E_c and E_0 have the appearance of (38) and (45).

The change of the total energy $\Delta F = F_d - F_l$ caused by the delocalization reaches its minimum value (in conventional units) $-(N_0 z/4)I$ for the regular system ($W = 0$) in the band center $E = 0$. The value of ΔF increases, as the levels are scattered, remaining negative below the critical point W_c. What is mentioned last means the above–indicated instability in the localized state resulted from the presence of PCSs. In the vicinity of W_{c0}, we obtain the following expression

$$\begin{aligned} \Delta F &= \frac{N_0}{2}\left(\frac{W_{c0}}{2}\right)^3 \left(\frac{\Delta}{W_{c0}}\right)^4 \ln\left(\frac{\Delta}{W_{c0}}\right) \\ &= 0.228 N_0 \eta^4 \ln(1.107\eta), \end{aligned} \tag{46}$$

differing from an ordinary one by the presence of logarithm. This distinction is crucial in the sense that the difference of the "scattering capacities" $\Delta C = -W\,\partial^2 \Delta F/\partial W^2$, in the critical point, takes the value zero in compliance with the following equality

$$\begin{aligned} \Delta C &= -(N_0/2)\,(W_{c0}/2)^2 \,/\, \ln(\Delta/W_0) \\ &= -0.226 N_0/\ln(1.107\eta), \end{aligned} \tag{47}$$

and its derivative with respect to the value W tends to $-\infty$. By convention, this fact can be expressed by referring the transition to the extended state to the $2+\delta$ order where $\delta \to 0$ is a addition resulted from logarithm. Logarithms of this sort appear also in the expressions (37), (42)–(48). They owe their presence to the dependence of the effective interaction parameter V on the system spectrum, and their presence stipulated by the site resonating effect [38]. We shall point, in this connection, to the paper [41] where also appeared logarithms associated with the renormalization of the effective interaction. In our case, such a renormalization results from a change in the bare spectrum of the system discussed within the framework of the mean–value field theory. In Ref. [41], however, under investigation was considered the influence of the critical fluctuations. In the diagram representation, this is expressed by the fact that our self–consistent scheme (see Section III) is based on the ladder sequence summation, but in Ref. [41] under consideration was the parquet sequence. Therefore, the logarithmic corrections obtained here are not so intrinsic as in the theory [41] where their contribution proves to be nonperturbative. And that is the case, one can see from expressions (43) and (46) that the only distinction of the scheme being set forth from the Landau theory consists in replacing the value η^2 with the value $\eta^2 \ln \eta$ which can be considered, in the vicinity of W_c, as the smallness parameter. By contrast, the appropriate parameter $W^2 \ln(1-n)^{-1}$ in Ref. [41] can take arbitrary values in the region of $W \ll 1$, $1-n \ll 1$ (the value $(1-n)^{-1}$ in Ref. [41] is denoted as $\left(\Lambda^2/E\right)^{2K_4}$ where Λ is the cutoff parameter, $K_4 = (8\pi^2)^{-1}$).

It should be pointed out that similar to the transition to the spin glass, the found feature of the "scattering capacity" (47) can be perceived as a break of the dependence $C(W)$ at $W = W_c$. While, in the spin glass, the jump of the derivative $\mathrm{d}C/\mathrm{d}T$ in the cusp point is finite [32], in our case, however, $\Delta(\mathrm{d}C/\mathrm{d}W) = -\infty$, and, this break can be perceived, in the experiment, as a jump of the value C itself but not one of its derivative.

The analytical expression for the total energy of elementary excitations $\mathcal{E}_{\rm ex}$ can be obtained only in the limiting cases. At $W^2 \ll 1$, we have (34), and at $W \geq W_{c0}$, $E = 0$

$$\mathcal{E}_{\rm ex} = \frac{\pi^2 - 9}{24} N_0 W = 0.036 N_0 W\,. \tag{48}$$

Accordingly, for the "capacity" $C_{\rm ex} = \mathrm{d}\mathcal{E}_{\rm ex}/\mathrm{d}W$ we get $C_{\rm ex} = 0.036 N_0$.

b. Bose case. In conformity with (21), for fermions the distribution functions p_l and h_l coincide, and, macroscopically, Fermi particles and antiparticles corresponding to the operators a_l and b_l behave in like manner. The operators α_{l+} and $\alpha_{\bar{l}-}$ of the elementary Fermi excitations, however, represent, according to (17), the antisymmetric and symmetric combinations of the initial operators a_l and b_l. Hence, in a microscopic sense the latter cannot be identified. The situation for bosons is quite different since in this case the particles and antiparticles are identical not only in a macroscopic sense but in a microscopic one as well. It means that $b_l \equiv a_l$ should be assumed low everywhere. The pair of equalities (17) then will be reduced to a single transform

$$\beta_l = u_l a_l + v_l a_l^+\,, \tag{49}$$

where, in view of Bose commutation rules (10), $u_l^2 - v_l^2 = 1$. Along with the condition of positiveness of the effective interaction potential V, the given fact plays a leading part on further consideration of the system being defined by Hamiltonian (8).

A different sense is acquired by the delocalization parameter as well the microscopic nature of which is defined by formation of the Bose–condensate of collective excitations. While, in the Fermi case, the condensation process resulted from formation of PCSs, the Bose particles can condensate even without preliminary pairing. It means that now the anomalous quasi–mean values $a \equiv \langle a_l \rangle = \langle a_l^+ \rangle$ are non–zero not for pairs of operators but for each of them. Consequently, for the delocalization parameter instead of (14), we have

$$\eta \equiv N_0^{-1/2} a = N_0^{-1/2} \langle a_l \rangle = N_0^{-1/2} \langle a_l^+ \rangle . \tag{50}$$

Physically, it means that with delocalization the boson operators a_l^+ and a_l acquire the condensate component that is independent of the site number l. It is evident that the total number of bosons obeys the conservation condition

$$a^2 + \sum_l a_l^+ a_l = N_0. \tag{51}$$

In accordance with the standard procedure for investigation of the degenerate Bose system [42] in order to obtain the approximating Hamiltonian it is necessary to carry out the formal expansion by powers of the number of the condensate particles a^2 in the expression (8) where $a_l \equiv b_l$, and because of the nonconservation condition of the total number of the localized particles (see (51)), the Lagrange multiplier $\mu = 0$. As a result, we get the approximating Hamiltonian in following form:

$$\begin{aligned} \mathcal{H} = N_0 V \eta^4 + \sum_l \left(\varepsilon_l + 2V\eta^2\right) a_l^+ a_l \\ + V\eta^2 \sum_l \left(a_{\bar{l}} a_l + a_l^+ a_{\bar{l}}^+\right), \end{aligned} \tag{52}$$

where the chemical potential addition $\mu = -2V\eta^2$ is completely conditioned by ordering. Substitution of the inverted relation (49) into (52) gives

$$\begin{aligned} &\mathcal{H} = U + \sum_l \epsilon_l \beta_l^+ \beta_l \, ; \\ &U = N_0 \left\{ V\eta^4 - \frac{1}{2N_0} \sum_l \left[\left(\varepsilon_l + 2V\eta^2\right) - \epsilon_l \right] \right\}, \\ &\epsilon_l = \left(\varepsilon_l^2 + 4V\eta^2 \varepsilon_l\right)^{1/2}, \\ &\left.\begin{matrix} u_l^2 \\ v_l^2 \end{matrix}\right\} = \frac{1}{2}\left(\frac{\varepsilon_l + 2V\eta^2}{\epsilon_l} \pm 1\right). \end{aligned} \tag{53}$$

The obtained dispersion law $\epsilon(\varepsilon)$ differs from the corresponding Fermi relation (19) primarily in its gapless nature. Their resemblance consists in the coincidence of the elementary excitation energy ϵ_l with the bare value of ε_l in the limit $\varepsilon_l \to \infty$. In the opposite limit $\varepsilon_l \ll V\eta^2$, we have the square–root singularity $\epsilon_l \simeq 2\eta V^{1/2}\sqrt{\varepsilon_l}$ the coefficient of which is determined by the values of the order parameter η and effective interaction V. This feature accounts for the rearrangement of the Bose spectrum as a result of delocalization.

Inverting the equality (49), for the normal mean value of $p_l = \langle a_l^+ a_l \rangle$ with the use of (A.2), we obtain

$$2p_l = \frac{\varepsilon_l + 2V\eta^2}{\epsilon_l} \mathrm{cotanh} \frac{\epsilon_l}{W} - 1. \tag{54}$$

Substituting this equality into eq. (51) averaged over the level scattering, we find the equation for the order parameter (compare with (23))

$$\begin{aligned} &2\eta^2 + \int_b^\infty \vartheta(x,d) \mathrm{e}^{-x} \mathrm{d}x = 3, \\ &\vartheta(x,d) \equiv \frac{(x+2d)\mathrm{cotanh}\left(\sqrt{x^2+4dx}/2\right)}{\sqrt{x^2+4dx}}, \\ &d \equiv 2V\eta^2/W. \end{aligned} \tag{55}$$

By virtue of the Bose character of distribution (54), the integrand $\vartheta(x,d)$ possesses an unintegrable singularity in the lower limit $b \to 0$. This singularity is also present in the localized state when $d \equiv 0$ and $\vartheta(x,d) = \mathrm{cotanh}(x/2)$. Therefore, it is necessary to introduce the cutoff parameter b which is assigned by the normalization rule in the disordered state:

$$\int_b^\infty \mathrm{cotanh}\,(x/2)\, \mathrm{e}^{-x} \mathrm{d}x = 3. \tag{56a}$$

Elementary integration gives the equation $2\ln\left(1-\mathrm{e}^{-b}\right) + \mathrm{e}^{-b} + 3 = 0$ the root of which is $b = 0.157$. Performing the term–by–term subtraction of eq. (55) and (56a), we obtain

$$\eta^2 = \frac{1}{2}\int_b^\infty \left(\mathrm{cotanh}\frac{x}{2} - \vartheta(x,d)\right) \mathrm{e}^{-x} \mathrm{d}x. \tag{57a}$$

This equation is applicable in the region associated with small values of the parameter d. In the opposite limit $d \gg 1$ at $(xd)^{1/2} \ll 1$ we have the asymptotic form $\vartheta(x,d) \simeq x^{-1}$ substitution of which into (55) produces a nonphysical result $\eta = 0.844$ at $W = 0$. To avoid such an occurrence, write the condition (55) in a completely ordered state where $\eta = 1$ and $d = \infty$:

$$\int_b^\infty x^{-1} \mathrm{e}^{-x} \mathrm{d}x = 1. \tag{56b}$$

Performing the term–by–term subtraction of eq. (55) and (56b), we obtain the following expression

$$\eta^2 = 1 - \frac{1}{2} \int_0^\infty \left(\vartheta(x, d) - x^{-1}\right) \mathrm{e}^{-x} \mathrm{d}x, \tag{57b}$$

substituting (57a) over the region $d \gg 1$. Since the integrand loses divergence here, the cutoff at the lower limit becomes unnecessary ($b = 0$). In the intermediate region $d \sim 1$, the solutions of eqs. (57a,b) are sewed together.

For the effective interaction parameter $V = N_0 \Delta_0^2 / \sum_l \epsilon_l$ we have instead of (24) (here, as in (57b), divergence is absent and $b = 0$)

$$V^{-1} = \frac{W}{2\Delta_0^2} \int_0^\infty \sqrt{x^2 + 4dx}\, \mathrm{e}^{-x} \mathrm{d}x. \tag{58}$$

The set of eqs. (57a,b) and (58) affords by the assigned level scattering W finding the values of parameters η and V. As the levels are scattered, the order parameter η monotonically decreases from the value 1 at $W = 0$ to zero at $W \geq W_c$, $W_c = 5.617$. At first, the effective interaction parameter V monotonically decreases from infinite values, then, at $W_m < W_c$, a minimum value of $V_m < V_c$ is reached. Thereafter ($W_m < W < W_c$), the value of V increases to the value of $V_c = 0.356$ in the critical point W_c in which a cusp is observed, as in the case with fermions. In the localized state ($W > W_c$), the dependence $V(W)$ monotonically decays. So, the major distinction from the Fermi case consists in the infinite increase of the effective interaction potential at a small scattering of levels of bosons. This is associated with the gapless character of their dispersion law $\epsilon(\varepsilon)$ (see (53)).

In the limiting case $W \ll W_c$, the major contribution to the values of integrals in eqs. (57b) and (58) is made by the small x, so that, in the former, cotangent can be approximated by the use of their inverse argument. On the other hand, since here we have the parameter $d \gg 1$, it is possible to perform expansion in the degrees of the relation x/d. As a result, we have

$$\eta \simeq 1 - AW^{4/3}, \qquad A \equiv (\pi/2)^{1/3} 2^{-5} = 0.036; \tag{59}$$

$$V \simeq BW^{-1/3}, \qquad B \equiv (2/\pi)^{1/3} = 0.860. \tag{60}$$

Close to the localization point ($0 < W_c - W \ll W_c$), parameter d takes small values, and expansion in (57a) and (58) can be performed over it. The appropriate limiting expressions are of the following forms:

$$\eta \simeq C\sqrt{1 - W/W_c}\,, \qquad C \equiv (I/2)^{1/2} = 2.808; \tag{61}$$

$$V \simeq DW, \qquad D \equiv I^{-1} = 0.063, \tag{62}$$

$$I \equiv \int_b^\infty \sinh^{-2}(x/2) \mathrm{e}^{-x} \mathrm{d}x = 15.773.$$

The critical scattering of levels

$$W_c = (2I)^{1/2} = 5.617 \tag{63}$$

is far above the corresponding value (36) for fermions. That is also stipulated by the gapless character of the boson dispersion law. The exponent in the diminution law (59) at $W \ll W_c$ is reduced from 2 (see (34)) to 4/3 for the same reason. In the critical region, however, the logarithmic singularity (43) inherent in fermions is missing for bosons, and the dependence $\eta(W)$ takes the usual square–root form (61).

The stated behaviour of the order parameter (50) results, by virtue of the condition (51), from the change in the concentration of the noncondensate bosons $p = N_0^{-1} \sum_l p_l$. Making use of (54), (59)–(63), we obtain the following:

$$p \simeq EW^{4/3}, \tag{64}$$

$$E \equiv (\pi/2)^{1/3} 2^{-4} = 0.073 \quad \text{at } W \ll W_c;$$

$$p \simeq 1 - F(1 - W/W_c), \tag{65}$$

$$F \equiv I/2 = 7.886 \quad \text{at } 0 \leq W_c - W \ll W_c.$$

Thus, at an increase in the level scattering below the critical value (63), we obtain the linear decrease of the localized boson concentration from the limiting value 1, then the slowing down to the power law with exponent 4/3 occurs. With a complete coincidence of levels ($W = 0$) all bosons, as it must, are extended ($p = 0$).

The system total energy jump is determined by analogy with the Fermi case, and it results in (compare with (25))

$$\Delta F = -N_0 \int \eta^2(V) \left[1 - \eta^2(V)\right] \mathrm{d}V. \tag{66}$$

In the limiting cases, it follows therefrom

$$\Delta F \simeq -(N_0/48)W \qquad \text{at } W \ll W_c; \tag{67}$$

$$\Delta F \simeq GN_0(1 - W/W_c)^2, \tag{68}$$

$$G \equiv I^{1/2}/2^{3/2} = 1.404 \quad \text{at } 0 \leq W_c - W \ll W_c.$$

And so, as in the case of the effective potential $V(W)$, at the level scattering decrease the energy $\Delta F(W)$ exhibits nonmonotonic behaviour: at first, the rise of the value ΔF in the region $0 < W_c - W \ll W_c$ occurs according to (68), then it drops to negative values, and, in the limit $W \to 0$, linearly increases to 0 (see (67)). A conclusion follows herefrom that, in the Bose case, delocalization represents a phase transition of the first order. Attention is drawn to the linear behaviour of the dependence $\Delta F(W)$ at $W \ll W_c$ and the square–law in the vicinity of the critical point W_c. According to eq. (53) the internal energy U behaves in a like manner: in the region $W \ll W_c$ we have $U \simeq -(N_0/8)W$, and at $W_c - W \ll W_c$ obtained is the dependence (68) where $G = 21.147$.

Excitation energy $\mathcal{E}_{\rm ex} = \sum_l \epsilon_l \nu_l$ monotonically rises as the scattering W increases. In the limit of $W \ll W_c$, we have

$$\mathcal{E}_{\rm ex} \simeq H N_0 W^{7/3}, \qquad H \equiv (\pi/2)^{1/3} 2^{-2} = 0.290. \quad (69)$$

At $W = W_c$, we obtain $\mathcal{E}_{\rm ex} = 1.811 N_0$.

Let us show now in what way the results obtained can be reproduced within the framework of the Green formalism. We shall proceed from the equality (12a) in which we should interchange the matrices $\widehat{\delta}$ and $\widehat{\tau}_3$ and assume that the self–energy function reduces to the anomalous component $\widehat{\Sigma} = 2V\eta^2\widehat{\tau}_1$ where matrix $\widehat{\tau}_1$ has non–zero elements $\tau_1^{01} = \tau_1^{10} = 1$. In conformity with this, the chemical potential also takes the anomalous form [42] $\mu = -2V\eta^2$, and, instead of (30), we get

$$\begin{aligned} G_l(\omega_s) &= -\frac{i\omega_s + (\varepsilon_l + 2V\eta^2)}{\omega_s^2 + \epsilon_l^2}\,, \qquad (70)\\ F_l(\omega_s) &= \frac{2V\eta^2}{\omega_s^2 + \epsilon_l^2}\,;\\ \epsilon_l &= \left(\varepsilon_l^2 + 4V\eta^2\varepsilon_l\right)^{1/2},\\ \omega_s &= 2\pi W s, \quad s = 0, \pm 1, \dots \end{aligned}$$

Substituting the first one of these equalities into the determination of the number of noncondensate bosons $N' = -(W/2)\sum_s \sum_l G_l(\omega_s)$ and taking into account that summation over the even frequencies $\omega_s = 2\pi(W/2)s$, $s = 0, \pm 1, \dots$ produces the function cotanh [43], we obtain $N' = \sum_l p_l$ where distribution p_l appears as eq. (54). Then the application of N' as the second term in the condition (51) averaged over the distribution (A.1) brings, as one would expect, to eq. (55) for the order parameter. The boson polarizer (31) comprises the matrix elements

$$\begin{aligned} \Pi^{00}(0) &= \Pi^{11}(0)\\ &= -N_0^{-1}\sum_l \epsilon_l^{-1}{\rm cotanh}(\epsilon_l/W) - 4AV^2\eta^4,\\ \Pi^{01}(0) &= \Pi^{10}(0) = -4AV^2\eta^4; \qquad (71) \end{aligned}$$

$$\begin{aligned} A \equiv \frac{1}{2N_0}\sum_l \epsilon_l^{-3}&\left[{\rm cotanh}(\epsilon_l/W)\right.\\ &\left.+ (\epsilon_l/W){\rm sinh}^{-2}(\epsilon_l/W)\right]. \end{aligned}$$

They will be used in obtaining the inverse vertex function (12c) responsible for the behaviour of the collective Bose excitations (see Section Vb).

V. COLLECTIVE EXCITATIONS

a. Fermi case. As is seen from Sections II and IV, the fermion delocalization process results from the fact that, as the levels ε_l of neighbouring sites are drawn closer to one another, they form coupled pairs corresponding to the collective excitations of the Bose type. If the number of such pairs $\mathcal{N}$ makes up the finite part $\mathcal{N}/N_0 = \eta^2/2$ with respect to the total number of sites $N_0 \to \infty$, the delocalization process assumes the macroscopic character and is determined by the long–range order parameter $\eta = |\Delta|/V$. In the representation of single excitations, the delocalization is reflected through the appearance of the off–diagonal components $\propto |\Delta|$ in the matrix of the self–energy function $\widehat{\Sigma}$.

Let us show now how the delocalization manifests itself in the representation of the collective excitations. Application of the quasi–mean value method, therewith, turns to be insufficient, and a recourse should be made to the self–consistent scheme developed in Section III and taking account of the behaviour of the Green functions of both single and collective excitations. The latter is related to the vertex function $\widehat{\Gamma}(\mathbf{K}, \Omega)$ acquiring a condensate component under delocalization (in this Section, the gap Δ is supposed complex for generality)

$$\begin{aligned} \widehat{\Gamma}_0(\mathbf{K}, \Omega_S) &= (N_0/W)\widehat{\Gamma}_0 \delta_{\mathbf{K}0}\delta_{S0}\,, \qquad (72)\\ \widehat{\Gamma}_0 &= -\begin{pmatrix} |\Delta|^2 & \Delta^2 \\ (\Delta^*)^2 & |\Delta|^2 \end{pmatrix}, \end{aligned}$$

which, unlike the fluctuation function $\widehat{\Gamma}'(\mathbf{K}, \Omega)$, takes, in the static limit $\Omega_S = 0$, the finite value determined by the gap width Δ. Substituting (72) into the Fourier time transform of eq. (12b) and passing on to the Matsubara frequency ω_s, after inversion of the matrix (12a), we obtain, as expected, the expressions (30) for the components of the Green function $\widehat{G}_l(\omega_s)$ and energy spectrum ϵ_l. Their application in the determination of the polarizer (31) brings to the expressions (32) the substitution of which into (12c) produces

$$\widehat{\Gamma}_0^{-1} = \begin{pmatrix} B - A|\Delta|^2 & A\Delta^2 \\ A(\Delta^*)^2 & B - A|\Delta|^2 \end{pmatrix}, \qquad (73)$$

$$B \equiv V^{-1} + N_0^{-1} \sum_l \epsilon_l^{-1} \tanh(\epsilon_l/W),$$

where account is taken of the diagonal structure of the interaction matrix $\widehat{V}$, the parameter A is determined by the last equality (32). Inverting the matrix $\widehat{\Gamma}_0^{-1}$, we obtain, in the limit $B \to 0$, the postulated expression (72), with the constraint $B = -W/2N_0|\Delta|^2$. In the thermodynamic limit $N_0 \to \infty$, we thus arrive at the equality $B = 0$ which, with allowance made for the last relation (73) and determination (24) of the effective potential V, brings, as we might expect, to eq. (23) for the gap $|\Delta|$.

The simplest way to determine the hydrodynamic expression ($\mathbf{K}, \Omega \to 0$) for the fluctuation component $\widehat{\Gamma}'(\mathbf{K}, \Omega)$ of the vertex function is to apply the method developed in Refs. [20, 22]. Its main point consists in the fact that instead of the equation $\widehat{G}^{-1} = \widehat{G}_0^{-1} - \Delta_0\widehat{\delta}$, $\Delta_0 = zI$ for the Green function $\widehat{G}$, use is made of its analog $\left\langle \widehat{G} \right\rangle^{-1} = \widehat{\sigma}^{-1} - \Delta_0\widehat{\delta}$ for the function $\left\langle \widehat{G} \right\rangle$ averaged over the level scattering and locator $\widehat{\sigma} = \left\langle \widehat{G}_0 \right\rangle$. In addition, inserted is the effective interactor $\widehat{U} = \Delta_0\widehat{\delta} + \Delta_0^2 \left\langle \widehat{G} \right\rangle$ which, unlike the above–used potential V, contains the term in the first order over the overlap integral I. The Dyson equation replacing (12a) therewith assumes the form $\widehat{U}^{-1} = \Delta_0^{-1}\widehat{\delta} - \widehat{\sigma}$. The two–particle Green function [15] — [23]

$$\varphi(E; \mathbf{K}, \Omega) = -\frac{1}{2\pi i} \sum_{\mathbf{k}, \mathbf{k}'} \left\langle G^{\mathrm{R}}(\mathbf{k}_+, \mathbf{k}'_+; E + \Omega) G^{\mathrm{A}}(\mathbf{k}_-, \mathbf{k}'_-; E) \right\rangle \tag{74}$$

appears, in the ladder approximation, as (compare with (12c))

$$\varphi(E; \mathbf{K}, \Omega) = -\frac{1}{2\pi i} \left[\gamma^{-1} - \sum_{\mathbf{k}} U^{\mathrm{R}}(\mathbf{k}_+; E + \Omega) U^{\mathrm{A}}(\mathbf{k}_-; E) \right]^{-1}, \tag{75}$$

where $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{K}/2$; γ is an irreducible four–tail vertex, indices R and A of the retarded and advanced functions correspond to the selection of the components $\alpha = 0, 1$ in various subspaces of the system states. Hence, taking into account the Dyson equation and Ward identity, [22] we have

$$\sigma^{\mathrm{R}}_{\mathbf{k}_+}(E + \Omega) - \sigma^{\mathrm{A}}_{\mathbf{k}_-}(E) = \gamma \sum_{\mathbf{k}'} \left[U^{\mathrm{R}}(\mathbf{k}'_+; E + \Omega) - U^{\mathrm{A}}(\mathbf{k}'_-; E) \right] - \widetilde{\gamma}\,\Omega, \tag{76}$$

where the irreducible vertex $\widetilde{\gamma}$ has, contrary to the γ, two coinciding tails appropriate to the same sites. As a result we come to the conventional expression for the fluctuation component of the two–particle Green function [15] — [23]

$$\varphi'(\mathbf{K}, \Omega) = -\frac{\chi(\mathbf{K})}{\Omega + iD(\mathbf{K}, \Omega)\mathbf{K}^2}, \tag{77}$$

where $\chi(\mathbf{K})$ is the thermodynamic susceptibility equal, in the hydrodynamic limit $\mathbf{K} \to 0$, to the density of states $g(E)$ at the level E, $D(\mathbf{K}, \Omega)$ is the dispersing diffusion coefficient which at $\mathbf{K}$, $\Omega = 0$ is as follows [22]

$$D = \frac{1}{\pi g} \sum_{\mathbf{k}} \left(\frac{k}{m} \operatorname{Im} \langle G^{\mathrm{A}}_{\mathbf{k}} \rangle \right)^2, \tag{78}$$

where k is the $\mathbf{k}$ projection on $\mathbf{K}$, m is a particle mass. As a result of the above, for the fluctuation component of the vertex function $\widehat{\Gamma}'(\mathbf{K}, \Omega) = -2\pi i \widehat{\varphi}'(\mathbf{K}, \Omega)$ we obtain [20,22]

$$\left(\widehat{\Gamma}'(\mathbf{K}, \Omega) \right)^{-1} = \frac{1}{2\pi g} \begin{pmatrix} -i\Omega + D\mathbf{K}^2 & 0 \\ 0 & i\Omega + D\mathbf{K}^2 \end{pmatrix}, \tag{79}$$

where the matrix structure reflects the presence of two poles $\Omega = \mp iD\mathbf{K}^2$.

The sum of the expression (73) where $B = 0$ and eq. (79) produces, after the matrix inversion, the complete Green function of collective excitations:

$$\widehat{\varphi}(\mathbf{K},\Omega) = \frac{g}{\mathcal{D}(\mathbf{K},\Omega)}\begin{pmatrix} -\Omega - i\left((1/2)SK_0 - D\mathbf{K}^2\right) & -(i/2)(\Delta/|\Delta|)^2 SK_0 \\ -(i/2)(\Delta^*/|\Delta|)^2 SK_0 & \Omega - i\left((1/2)SK_0 - D\mathbf{K}^2\right)\end{pmatrix}, \tag{80a}$$

$$\mathcal{D}(\mathbf{K},\Omega) = \Omega^2 + \left((1/2)SK_0 - D\mathbf{K}^2\right)^2 - (1/4)S^2K_0^2, \tag{80b}$$

$$S^2 = 4\pi g A|\Delta|^2 D, \tag{80c}$$

$$K_0^2 = 4\pi g A|\Delta|^2/D. \tag{80d}$$

Condition $\mathcal{D}(\mathbf{K},\Omega) = 0$ brings to the dispersion law

$$\Omega = \pm DK\sqrt{K_0^2 - \mathbf{K}^2}\,. \tag{81}$$

As may be seen from fig. 4, its characteristic feature consists in the fact that the collective mode is of a reactive nature in the long–wave region $K < K_0$ limited by the wave number (80d), and a relaxation nature in the short–wave region $K > K_0$. At $K \gg K_0$, we obtain, as we might expect, the ordinary diffusion mode $\Omega = \mp iD\mathbf{K}^2$.

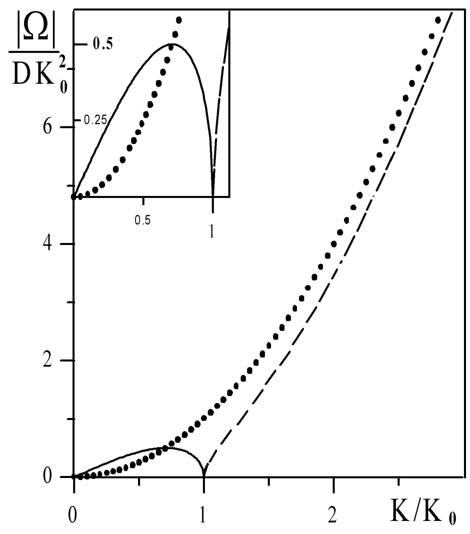

Fig. 4. Law of collective mode dispersion (the solid line corresponds to the real value of frequency Ω, the dashed — imaginary value, the dotted — diffusion mode).

In this way, the self–consistent consideration of single and collective excitations leads to the conclusion that apart from the diffusion regime realized in the mesoscopic region $K > K_0$, possible in the system is the propagation of oscillations characterized by the phase velocity S determined by the equality (80c). The dispersion law (81) acquiring the acoustic form in the limiting long–wave region $K \ll K_0$ is appropriate to the variations of density of extended fermions, that is to say, usual sound. It stands to reason that in the case fermions carry the charge e, as it is true with electrons, and density ρ the relaxation time $\tau = (m/4\pi\rho)^{1/2}e^{-1}$ conditioned by the Coulomb interaction is found to be much less than the inverse value Ω_0^{-1} of the characteristic sound frequency $\Omega_0 = SK_0$. Therefore manifestation of the present mode can be expected only for the uncharged fermions of the atoms He^3 type.

Such a situation is realized in quantum crystals [25] where the found sound mode results from the zero–oscillations. The level scattering halfwidth $W/2$ therewith reduces to the real temperature T (because of the smallness of the degeneracy temperature $T_F \propto \rho^{2/3}/m$ of the quantum crystals $T \ll 1$), concentration n of the localized atoms determines the value of the quantum dilatation parameter $\delta = 1 - n$, the fermion density $\rho = N_0/V$ gives the value of overlapping integral $I \propto \exp\left(-\mathrm{const}\cdot\rho^{-1/3}\right)$ determining the de Bour parameter. The curve of the temperature dependence $\Omega_0(T)$ of the typical sound frequency at the fixed values of δ is shown in fig. 5. By and large, it repeats the form of the appropriate dependence $\Delta(W)$ of the gap width (see fig. 2b). The exception is provided by the behaviour in the vicinity of the critical temperature of delocalization T_c where the singularity of $\Delta(W)$ is transformed into the linear dependence $\Omega_0 \propto T_c - T$. Since the diffusion coefficient $D(W)$ itself behaves in the critical region in a radical manner [3] it means the presence of singularities $S \propto (T_c - T)^{3/4}$ for the sound velocity (80c) and $K_0 \propto (T_c - T)^{1/4}$ for the boundary wave number (80d). In the quantum–crystal–essential limit of the weak coupling $T \ll T_c$, the formulae (80c), (80d), (32) and (34) lead to the result of $\Omega_0 = 4$. Then, taking into account that the diffusion coefficient diverges here according to the relation $D \propto T^{-2}$ [3], one can see that the sound velocity $S \propto T^{-1}$ increases indefinitely as well, and the boundary value of the wave vector $K_0 \propto T$, on the contrary, decreases. Shown in fig. 6 are the curves of the temperature dependencies of the values Ω_0, S and K_0 throughout the interval $0 \le T \le T_{c0}$ at $\delta = 1$ (the temperature dependence of the diffusion coefficient is taken from the paper

[3]). At $T \ll T_c$ the sound mode dispersion law appearing as a bell with a slanting long–wave side is realized on the narrow interval $K < K_0$ and over the wide frequency domain $\Omega < \Omega_0/2$ (velocity S therewith is very high). As the temperature rises the frequency domain $\Omega_0/2$ and velocity S monotonically decrease, and the bell width K_0 increases at the beginning and then quickly decreases. A decrease in the quantum dilatation parameter causes a decrease in the values Ω_0, S and K_0.

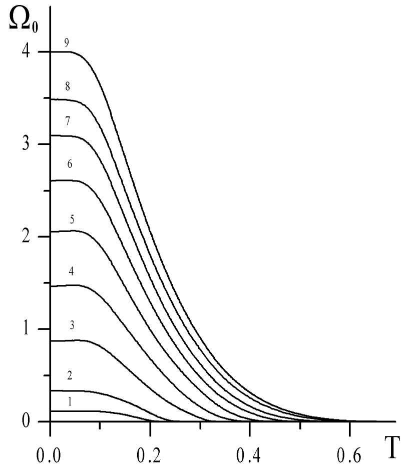

Fig. 5. Temperature dependence of the characteristic frequency of zero-oscillations at various values of the quantum dilatation parameter δ: 0.05 (curve 1), 0.1 (2), 0.2 (3), 0.3 (4), 0.4 (5), 0.5 (6), 0.6 (7), 0.7 (8), 1.0 (9).

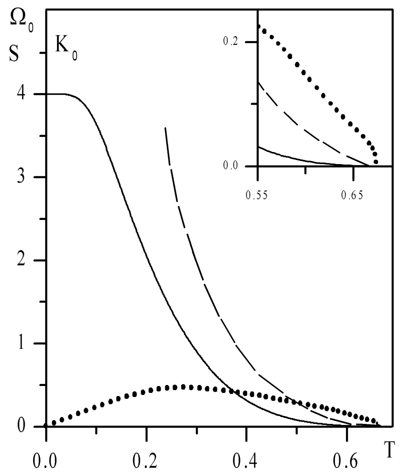

Fig. 6. Temperature dependence of the characteristic frequency Ω_0 of the sound mode (solid line), phase velocity S (dashed line) and boundary value of the wave number K_0 (dotted line) at quantum dilatation $\delta = 1$.

It follows herefrom that the most preferable (in terms of detection of the zero–sound mode) is the temperature region over the interval $0 < T < \min(T_F, T_m, T_c)$ the upper limit of which is given by the least value of degeneracy temperature T_F, melting point T_m or critical temperature T_c. Along with it, it is well to bear in mind that the Landau theory considering the interaction of fermions makes prediction about the zero–sound mode existence only at the high frequency Ω [9]. With consideration for the falling nature of the dependence $\Omega_0(T)$ in fig. 5, it means that the zero–sound appearance should be expected in the extremely low temperature region $T \ll \min(T_F, T_m, T_c)$ at the quantum dilatation $\delta = 1$.

b. Bose case. In Section IVb, we have considered approximation of the degenerate Bose–gas and we have shown that the effective interaction V conditioned by the quantum interference results in the delocalization of bosons in a random field. A real system of bosons with density ρ and mass m always possesses a repulsive interaction $V_0 = 2\pi\hbar^2\rho a/m$ the value of which is determined by the scattering amplitude a. Therefore, in the absence of the effective interaction ($V = 0$), the collective behaviour of bosons is characterized by the presence of the acoustic phonon branch $\Omega = \pm S_0 K$ with the sound velocity $S_0 = \left(4\pi\hbar^2\rho a/m^2\right)^{1/2}$ [42]. Evidently, the delocalization of bosons resulting, on the one hand, in softening the effective springs of the interatomic interaction and, on the other hand, in their transformation into pistons which simulate the viscosity forces should facilitate a decrease in the sound velocity S_0 and appearance of dissipation. Below is given a corresponding quantitative picture. Furthermore, it will be shown that the mode of the second sound appears the velocity of which is proportional to the delocalization parameter.

As is the case with fermions, we shall proceed from the assumption that, in the extended state, the vertex function acquires the form (compare with (72))

$$\widehat{\Gamma}_0(\mathbf{K}, \Omega_S) = (N_0/W)\widehat{\Gamma}_0 \delta_{\mathbf{K}0}\delta_{S0}, \tag{82}$$

$$\widehat{\Gamma}_0 = \begin{pmatrix} -\Delta^2 & \Delta^2 \\ \Delta^2 & -\Delta^2 \end{pmatrix}, \quad \Delta \equiv 2V\eta^2.$$

Performing further the same mathematical treatment as for fermions, and making use of the polarizer (71), instead of (73) we obtain

$$\widehat{\Gamma}_0^{-1} = \begin{pmatrix} B - A\Delta^2 & -A\Delta^2 \\ -A\Delta^2 & B - A\Delta^2 \end{pmatrix}, \tag{83}$$

$$B \equiv V^{-1} - N_0^{-1}\sum_l \epsilon_l^{-1}\mathrm{cotanh}\left(\epsilon_l/W\right),$$

where the parameter A is determined by the last equality (71). Inversion of the matrix (83) brings, as it might be expected, to the postulated expression (82) with the constraint $B = W/2N_0\Delta^2$. In the thermodynamic limit $N_0 \to \infty$, we obtain $B \to 0$ whence follows the equation

$$V^{-1} = N_0^{-1}\sum_l \epsilon_l^{-1}\mathrm{cotanh}(\epsilon_l/W), \tag{84}$$

which plays the part of the condition of self–consistency of type (23). Adding up the condensate component (83) to the inverted fluctuation vertex $\widehat{\Gamma}'(\Omega,\mathbf{K}) = -2\pi i\widehat{\varphi}'(\Omega,\mathbf{K})$ given by the Green function of phonons [42]

$$\varphi'(\Omega,\mathbf{K}) = \frac{\rho K}{2S_0}\left(\frac{1}{\Omega - S_0K + i0} - \frac{1}{\Omega + S_0K - i0}\right), \tag{85}$$

and inverting the matrix obtained, we arrive at the complete Green function of the collective Bose excitations (compare with (80))

$$\widehat{\varphi}(\Omega,\mathbf{K}) = \frac{\rho K}{2S_0}\mathcal{D}^{-1}(\Omega,\mathbf{K})\begin{pmatrix} \Omega + (S_0 + iS_1)K & iS_1K \\ iS_1K & \Omega - (S_0 - iS_1)K \end{pmatrix}; \tag{86a}$$

$$\mathcal{D}(\Omega,\mathbf{K}) = \Omega^2 - S_0^2K^2 + 2i\Omega S_1K, \tag{86b}$$

$$S_1 = \pi\rho A\Delta^2/S_0. \tag{86c}$$

According to equation $\mathcal{D}(\Omega,\mathbf{K}) = 0$, they possess the dispersion law

$$\Omega = -iS_1K \pm (S_0^2 - S_1^2)^{1/2}K, \tag{87}$$

which is of the acoustic form with the effective velocity $(S_0^2 - S_1^2)^{1/2}$ and relaxation time $(S_1K)^{-1}$. As may be seen from the determination (86c), in the localized state $S_1 = 0$, and the relationship (87) reduces to the usual law of phonon dispersion. The delocalization taking place as the level scattering $W < W_c$ decreases brings about a increase of the parameter S_1, which results in relaxation of the sound and reduction of its effective velocity. Beginning from a certain value W, fulfilled is the condition $S_1 = S_0$, and as the value W decreases further, the phonon mode (87) is taking a pure dissipative nature.

In order to separate out the second sound mode, one should take into account the space–time dispersion when determining the polarizer (12d). In the wave representation, it is written as

$$\Pi^{\alpha\beta}(\Omega,\mathbf{K}) = -i\sum_{\mathbf{k}}\int\frac{\mathrm{d}\omega}{2\pi}G^{\alpha\beta}(\mathbf{k}_+,\Omega-\omega)G^{\alpha\beta}(\mathbf{k}_-,\omega), \tag{88}$$

where $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{K}/2$. Substituting here the Green functions of free bosons (Planck constant $\hbar = 1$)

$$G^{\alpha\beta}(\omega,k) = \left[\omega\tau_3^{\alpha\beta} - (\mathbf{k}^2/2m)\delta^{\alpha\beta}\right]^{-1}, \tag{89}$$

we obtain the following expression:

$$\Pi^{\alpha\beta}(\Omega,\mathbf{K}) = -\sum_{\mathbf{k}}\left[\Omega\tau_3^{\alpha\beta} - (\mathbf{K}^2/4m + \mathbf{k}^2/m)\delta^{\alpha\beta}\right]^{-1}. \tag{90}$$

Its form shows that, in the hydrodynamic limit $\Omega,\mathbf{K}\to 0$, the parameter B in the inverse vertex (83) acquires the matrix structure and takes the value 0 in accordance with the following asymptotic form [24]

$$B^{\alpha\beta}(\Omega,\mathbf{K}) = \Omega\tau_3^{\alpha\beta} - (\mathbf{K}^2/4m)\delta^{\alpha\beta}. \tag{91}$$

As a result of this, the vertex function is written as following:

$$\widehat{\Gamma}(\Omega,\mathbf{K}) = \mathcal{D}^{-1}(\Omega,\mathbf{K})\begin{pmatrix} -\Omega + \mathbf{K}^2/4m + A\Delta^2 & A\Delta^2 \\ A\Delta^2 & \Omega + \mathbf{K}^2/4m + A\Delta^2 \end{pmatrix}, \tag{92a}$$

$$\mathcal{D}(\Omega,\mathbf{K}) = \Omega^2 - (\mathbf{K}^2/4m)^2 - 2A\Delta^2(\mathbf{K}^2/4m). \tag{92b}$$

The dispersion law

$$\Omega = \left[S_2^2\mathbf{K}^2 + (\mathbf{K}^2/4m)^2\right]^{1/2}, \tag{93a}$$

$$S_2^2 = A\Delta^2/2m \tag{93b}$$

derived from here takes the acoustic form in the long–wave limit $K^2 \ll 8mA\Delta^2$ and the parabolic form in the short–wave limit. The second sound velocity S_2 differs from zero only in the extended state where it increases as the level scattering decreases.

It is not difficult to understand that the represented picture of delocalization is realized in the quantum fluid He^4 where the level scattering halfwidth $W/2$ reduces to the usual temperature. The specific velocities (86c) and (93b) therewith behave in a critical manner in the vicinity of the critical point W_c, and diverge at $W \ll W_c$.

VI. CONCLUSION

The main prerequisites that allowed the theory presented to be developed include the consideration of the system ground state rearrangement resulted from the quantum interference and the assumption of the quasi–Gibbs character of the quenched disorder. The former provides for reduction of the one–particle problem Hamiltonian to the BCS form, and the latter reduces averaging to the standard procedure applied in the statistical physics.

The consideration given shows that for fermions the single excitation theory can be constructed by analogy with the microscopic theory of superconductivity [29] in which the role of the order parameter is played by the square root of the ratio between the number of the coupled sites and their total number, and the role of the conjugate field is played by the shift from the band center. A significant distinction from the theory [29] lies in the dependence of the effective interaction parameter on the spectrum of the system. This results in the anomalous behaviour of the order parameter in the vicinity of the critical level scattering that can be expressed referring the transition to the extended state to the $2+\delta$ order where $\delta \to 0$ is the addition reflecting the presence of the logarithmic factor. The total energy of the system therewith is a nonanalytic function of the order parameter, and the energy series expansion in terms of its powers is not possible.

In the Bose case the single excitations do not reduce to PCSs but to the particles themselves, and the delocalization parameter reduces to the density of the Bose–Einstein condensate. An important distinction of the developed approach from the standard theory [42] results, as in the case of fermions, from the dependence of the effective interaction on the system spectrum. In particular, at $W \to 0$ it assumes a diverging nature. Another feature consists in the fact that the delocalization process is realized by the mechanism of the first order phase transition.

Description of the collective excitations requires their self–consistent consideration along with single ones. The hydrodynamic behaviour of the system therewith is presented as the result of interference of the condensate and fluctuation components. For fermions, the former corresponds to the PCS ensemble, the latter describes the quantum diffusion process. Interference of these components results in appearance of the reactive mode corresponding to the oscillations of the density of extended fermions (zero–sound). In the Bose case delocalization brings about the first sound relaxation which results, as the level scattering decreases, in the pure dissipative behaviour of the quantum fluid. In addition, the mode of oscillations of the density of extended bosons (second sound) reveals itself.

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APPENDIX [53]

As is known, the choice of a specific kind of bell-shaped distribution $P(\varepsilon)$ does not influence the qualitative picture of localization. So, use was made of the Anderson step-like dependence, Gaussian distribution, Lorentzian (Lloyd model), etc [44]. For reasons of convenience only we, therefore, have the right to adopt the quasi–Gibbs distribution

$$P(\varepsilon) = \frac{1}{W}\exp\left(-\frac{2|\varepsilon-\bar{\varepsilon}|}{W}\right), \tag{A.1}$$

symmetrized about the mean energy $\bar{\varepsilon}$ and attaining a width W.

The formal convenience of such a choice is associated with the fact that it offers possibilities of application of the standard quantum statistics techniques for the thermodynamic system with a temperature $W/2$. So, for the distribution function $\nu_l = \langle \alpha_l^+ \alpha_l \rangle$ of elementary excitations with energy ϵ_l in the regular way we obtain

$$\nu_l = [\exp(2\epsilon_l/W) \pm 1]^{-1}, \tag{A.2}$$

where the upper sign corresponds to the Fermi statistics, the lower one — to the Bose excitations. At $W=0$, when the dependence (A.1) assumes the form $P(\varepsilon) = \delta(\varepsilon - \bar{\varepsilon})$ realized is the ground state in which the elementary excitations are not present ($\nu_l = 0$). At small values W, the dependence (A.2) takes the form of the Boltzmann distribution $\nu_l \simeq \exp(-2\epsilon_l/W)$. As the scattering W continues to grow, the type of statistics is beginning to take effect.

So, the number of Fermi elementary excitations tends, at $W \gg \epsilon_l$, to the value of $\nu_l = 1/2$, then as in the Bose case an indefinite accumulation by the law $\nu_l \simeq W/2\epsilon_l$ takes place.

In addition to the safety reasons, the arguments of a fundamental character can be advanced in favour of the quasi–Gibbs distribution (A.1). The exponential distribution of such a type is truly realized not only for the thermodynamic systems but for the nonequilibrium ones also (including the strong nonequilibrium systems), provided that they are in the steady state (i.e., the distribution function does not depend on time) [45]. For the equilibrium case under the exponent curve, there is an energy divided by temperature. For the steady–state system, there is a synergetic potential divided by the intensity of the noise determining the degree of nonequilibrium.

Since the Anderson model is compatible with the zero temperature [1], the level distribution fixed as a result of the disordered system quenching remains unchanged, and the system under discussion is stationary. Consequently, its distribution function is described by the exponential dependence (A.1) where the exponent numerator approximates the synergetic potential, and the denominator defines the intensity of the noise in the nonequilibrium system fixed due to the quenching (quenched disorder).

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[46] We were convinced of that while numerically solving the equations (26), (28) in Ref. [3]. The square–root singularities cited by Götze can be obtained from the (26), (28) only after a number of additional assumptions. Such a situation is caused by the absence of interaction (nonlinear coupling) between the “coupled modes” [2,3]. It should be noted that, in the kinetic theory of glass transition developed within the framework of the same mode–coupling approximation, the phase transition appears just due to the nonlinear coupling [2].

[47] This circumstance reflects the presence of the quantum interference [19].

[48] In other words, the coupling realized in the system is the linear feedback which, as is known, cannot cause the critical behaviour. The given fact explains the cause of Götze scheme inconsistency [2,3] (see [46] as well).

[49] It is interested to note that, in the single–level term H_0, the sites l, m are permuted by the fixed levels i, and, in the interstitial term U_0, the i, j levels are permuted by the l, m sites.

[50] In principle, such a situation is not new: so, the spontaneous symmetry breaking in the Higgs model [31] leads to origination of vector fields. In our case, reduction in the permutation symmetry results in the interaction of the exchange type.

[51] Since the potential V also depends on the values of W and E, the density of extended states is proportional to the order parameter η but not to the gap width $\Delta = |V|\eta$.

[52] At the double logarithmic accuracy, substitution of $\ln(1-n)^{-1}$ by $\ln\left[(1-n)^{-1}\ln(1-n)^{-1}\right]$ should be performed in the formulae (37) and (45).

[53] It is convenient here to use conventional (dimensional) values.

САМОУЗГОДЖЕНА ТЕОРІЯ ДЕЛОКАЛІЗАЦІЇ КВАНТОВОЇ ЧАСТКИ У ВИПАДКОВОМУ ПОЛІ

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На підставі аналізу конфіґураційного простору одночасткової системи показано, що наявність квантової інтерференції приводить до ефективної взаємодії обмінного типу. Загартований безлад подано за аналогією з термодинамічними флюктуаціями, для яких роль температури відіграє півширина розкиду рівнів частки. Для ферміонів у рамках локаторного підходу Андерсона показано, що щілина в спектрі одночасткових збуджень, зумовлена конденсацією пар зв'язаних вузлів, визначає густину делокалізованих станів. Знайдена її залежність від ширини розкиду рівнів і зміщення хемпотенціялу з центру зони. Визначено закон дисперсії колективної моди. Для заряджених ферміонів вона має звичний дифузійний вигляд, а для нейтральних (квантові кристали) у довгохвильовій межі з'являється звукова мода нульових коливань густини делокалізованих ферміонів. Досліджена залежність швидкости нульового звуку і характерних значень його частоти та хвильового числа від температури й параметра квантової дилатації. Для бозевського випадку визначено залежність густини конденсату делокалізованих станів й ефективної взаємодії від ширини розкиду рівнів W. Показано, що колективні збудження бозонів зводяться до першого звуку, який зі зменшенням W трансформується в чисто дисипативну моду, і другого звуку, швидкість якого критичним чином залежить від W.