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THE EXCITATION OF ELECTROSTATIC WAVES IN DUSTY SELF–GRAVITATIONAL PLASMA BY MOVING PARTICLES

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Some electrodynamic effects connected with the motion of dust grains in self–gravitational plasma are considered. The investigation of energy losses of a probe particle moving through the dusty self–gravitational plasma demonstrates that electric disturbances can be excited not only by a charged particle but by neutral one as well which is impossible for the conventional ion–electron plasma. The energy losses of a charged massive particle associated with wave excitation can either increase (as compared with the conventional plasma) or drop down to zero for specific values of the charge–to–mass ratio, depending on the charge sign. Moreover, it is shown that the simultaneous action of electric and gravitational forces in a plasma medium can alter the traditional conditions for the development of beam instability or change its growth rate.
Key words: dusty plasma, self–gravitation, energy losses, electrostatic waves.

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I. INTRODUCTION

Charged dust grains are often encountered in space (e.g. in planetary rings, comet tails, interstellar dust clouds, etc.). If the particulate density is sufficiently high, these grains, along with electrons and ions, are involved in collective processes and form a mixture that is referred to as a dusty plasma [1]. The interaction of dust particles in a dusty plasma does not occur via electric fields alone but also via gravitation which is an equally long–range force. With the inclusion of self–gravitation effects, the longitudinal dielectric constant ε of such a medium is markedly different from that of the common plasma. In hydrodynamical approach it may be written in the form of [2]

$$\varepsilon(\omega, \mathbf{k}) = \varepsilon_p(\omega, \mathbf{k}) + \frac{K_T^2(\omega, \mathbf{k})}{\varepsilon_G(\omega, \mathbf{k})} \tag{1}$$

$$= \left(1 - \sum_{(\alpha)} \frac{\omega_{p,\alpha}^2}{\omega^2 - k^2 v_{T,\alpha}^2}\right) + \frac{K_T^2}{\left(1 + \sum_{(\alpha)} \frac{\omega_{G,\alpha}^2}{\omega^2 - k^2 v_{T,\alpha}^2}\right)},$$

where the frequencies $\omega_{p,\alpha}$ and $\omega_{G,\alpha}$ characterize the time scales of cooperative effects that are associated with the electric and gravitational interactions, i.e. $\omega_{p,\alpha} = (4\pi Q_\alpha^2 n_{o,\alpha} m_\alpha^{-1})^{1/2}$ and $\omega_{G,\alpha} = (4\pi G n_{o,\alpha} m_\alpha)^{1/2}$ are the plasma and Jeans frequency, respectively, for the particles of species α (plasma particles are characterized by the charge Q_α, mass m_α, the equilibrium density $n_{o,\alpha}$ and thermal velocity $v_{T,\alpha}$, G is the gravitational constant), $K_T = \sum_{(\alpha)} \omega_{p,\alpha}\omega_{G,\alpha}(\omega^2 - k^2 v_{T,\alpha}^2)^{-1}$.

According to (eq. 1) the structure of dispersion equation for the longitudinal waves $\varepsilon(\omega, \mathbf{k}) = 0$ is such, that the plasma processes ($\varepsilon_p(\omega, k) = 0$) and gravitational effects ($\varepsilon_G(\omega, k) = 0$) are involved in an equipotent manner. The term K_T is the thermal factor responsible for the coupling of gravitational and electrical processes in a hot medium ($v_{T,\alpha} \neq 0$).

Stationary magnetic, electric or gravitation fields are nearly always present in space, affecting the motion of different kinds of particles in a different way. Quite often, the result may be a relative motion of particles. Consider, for example, the motion of particles in planetary rings. For heavy particles gravity prevails independently of their electric charge, and hence such particles move through the gravitation field of the planet in accordance with Kepler's laws. Contrary to this, the motion of microparticles (i.e. electrons and ions) is governed by electromagnetic forces greatly exceeding the gravitation and the microparticles corotate with the planetary magnetic field. As for electrically charged dust grains, they "feel" both gravitation and electromagnetic forces. As a result, the macroscopic particles do not move around the planet at the Kepler velocity but rather at a somewhat different velocity $V_{0,\alpha}$, which is determined by the charge/mass ratio [1], hence it may vary for particles with different Q_α/m_α even at the same orbit. Apparently, similar multistream structures should exist not only in dusty plasma of planetary rings but in other astrophysical objects characterized by the presence of a dust component.

If particles of different species move at different regular velocities ($V_{0,\alpha} \neq 0$), then the electric and gravitational perturbations are coupled through the drift coupling factor K_V. In hydrodynamical approach the dielectric constant for a model of unbounded, mutually penetrating cold particle beams would be [2]

$$\varepsilon(\omega, \mathbf{k}) = \varepsilon_p(\omega, \mathbf{k}) + \frac{K_V^2(\omega, \mathbf{k})}{\varepsilon_G(\omega, \mathbf{k})}, \tag{2}$$

with

$$\varepsilon_p(\omega,\mathbf{k}) = 1 - \sum_\alpha \frac{\omega_{p,\alpha}^2}{(\omega - \mathbf{kV}_{0,\alpha})^2}, \tag{3}$$

$$\varepsilon_G(\omega,\mathbf{k}) = 1 + \sum_\alpha \frac{\omega_{G,\alpha}^2}{(\omega - \mathbf{kV}_{0,\alpha})^2}, \tag{4}$$

and

$$K_V(\omega,\mathbf{k}) = \sum_\alpha \frac{\omega_{p,\alpha}\omega_{G,\alpha}}{(\omega - \mathbf{kV}_{0,\alpha})^2}. \tag{5}$$

As a result of coupling between electric and gravitational disturbances in self–gravitation plasma, new branches of eigenwaves can appear in the spectrum of free oscillations, and the criteria for the stability of wave-like perturbations, as well as their propagation conditions, become altered. In this paper we concentrate on the electrodynamic processes that a moving dust particle causes in self–gravitational plasma. We consider two effects. One of them is the excitation of electrostatic waves by a probe particle moving through the hot dust self–gravitational plasma ($v_{T,\alpha} \neq 0$). The other effect is connected with different regular velocities of dust grains in space plasma ($V_{0,\alpha} \neq 0$) which may give rise to the beam instability of the waves.

II. ENERGY LOSSES OF MOVING PARTICLES

In the conventional plasma, electromagnetic waves are known to be excited by electric currents, i.e. charged particles moving through the medium. The situation may be different with the inclusion of self–gravitation effects. In particular, electromagnetic waves can be excited in a self–gravitational plasma by a stream of electrically neutral, as well as charged particles. Indeed, a heavy neutral particle moving through the dusty medium acts upon grains of the medium through its gravitation field. The dust grains start to move and produce, owing to their electrical charge, electric currents that excite an electromagnetic field. Contrary to this, a massive charged particle moving through such a medium may fail to excite any perturbations, should the gravitational perturbation be compensated by an electric disturbance. Such a compensation is only possible if the mass M_0 of the probe particle and its charge Q_0 are related in some special way. To analyze these unusual effects, let us consider energy losses of a particle moving through the self–gravitational plasma at a velocity $\mathbf{v}_0$.

In the conventional plasma, the energy $\mathcal{E}$ spent by a moving charged particle to excite plasma waves can be evaluated as the work done against the braking force owing to the electric field $\mathbf{E}$ at the point where the particle is at the time moment t. The change in energy per unit time is

$$d\mathcal{E}/dt = Q_0(\mathbf{v}_0\mathbf{E})_{\mathbf{r}=\mathbf{v}_0 t}.$$

Allowing for the gravitation effects, this becomes

$$\frac{d\mathcal{E}}{dt} = Q_0(\mathbf{v}_0\mathbf{E})_{\mathbf{r}=\mathbf{v}_0 t} + M_0(\mathbf{v}_0\mathbf{\Gamma})_{\mathbf{r}=\mathbf{v}_0 t}, \tag{6}$$

where $\mathbf{E} = -\nabla\psi_E$ and $\mathbf{\Gamma} = -\nabla\psi_G$. The electrical ψ_E and gravitational ψ_G potentials can be found from the equations of motion and continuity plus Poisson's equations

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} = \frac{Q_\alpha}{m_\alpha}\nabla\psi_E - \nabla\psi_G - \frac{v_{T\alpha}^2}{n_\alpha}\nabla n_\alpha, \tag{7}$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla(n_\alpha \mathbf{v}_\alpha) = 0, \tag{8}$$

$$\Delta\psi_E = -4\pi\left(\sum_\alpha Q_\alpha n_\alpha + Q_0\delta(\mathbf{r} - \mathbf{v}_0 t)\right), \tag{9}$$

$$\Delta\psi_G = 4\pi G\left(\sum_\alpha m_\alpha n_\alpha + M_0\delta(\mathbf{r} - \mathbf{v}_0 t)\right), \tag{10}$$

with δ being Dirac's delta. The Fourier components of $\mathbf{E}$ and $\mathbf{\Gamma}$ following from ψ_E and ψ_G are

$$\mathbf{E}_\mathbf{k} = -i\mathbf{k}\frac{4\pi e^{-i\mathbf{kv}_0 t}}{k^2\varepsilon(\omega,\mathbf{k})}\left[Q_0 - 4\pi G M_0 \frac{K_T^{1/2}(\omega,\mathbf{k})}{1 + \sum_{(\alpha)}\omega_{G,\alpha}^2/(\omega^2 - k^2 v_{T,\alpha}^2)}\right], \tag{11}$$

$$\mathbf{\Gamma}_\mathbf{k} = -i\mathbf{k}\frac{4\pi e^{-i\mathbf{kv}_0 t}}{k^2\varepsilon(\omega,\mathbf{k})}\left\{[M_0\varepsilon_p(\omega,\mathbf{k}) + 4\pi Q_0 K_T^{1/2}(\omega,\mathbf{k})]\varepsilon_G^{-1}(\omega,\mathbf{k})\right\}. \tag{12}$$

Applying the inverse Fourier transformation and substituting the result into equation (6) we can arrive, with an account of $d^3k = dk_\parallel d^2k_\perp \to 2\pi k_\perp\, dk_\perp\, d\omega/v_0|_{\omega=\mathbf{kv}_0}$ and $k_\parallel = v_0(\mathbf{kv}_0)/v_0^2$, at

$$\frac{d\mathcal{E}}{dt} = -\frac{i}{\pi v_0}\int_0^{k_0} dk_\perp\, k_\perp \int_{-\infty}^{\infty} d\omega \left\{\omega\left[Q_0^2\left(1 + \sum_\alpha \frac{\omega_{G,\alpha}^2}{\omega^2(1 - v_{T,\alpha}^2/v_0^2) - k_\perp^2 v_{T,\alpha}^2}\right)\right.\right.$$

$$-2Q_0M_0G^{1/2}K_T^{1/2}(\omega,\mathbf{k}) - GM_0^2\left(1-\sum_\alpha \frac{\omega_{p,\alpha}^2}{\omega^2(1-v_{T,\alpha}^2/v_0^2)-k_\perp^2 v_{T,\alpha}^2}\right)\Bigg]\Bigg\}$$

$$\times\left\{(k_\perp^2+\omega^2/v_0^2)\varepsilon(\omega,\mathbf{k})\left(1+\sum_\alpha \frac{\omega_{G,\alpha}^2}{\omega^2(1-v_{T,\alpha}^2/v_0^2)-k_\perp^2 v_{T,\alpha}^2}\right)\right\}^{-1}. \qquad (13)$$

Here k_0 is the highest value of $k_\perp$ allowable in the classical approach to the description of collisions between the probe particle and particles of the medium. The integration along the real ω–axis can be performed unambiguously with the use of the standard substitution [3]

$$\frac{1}{\varepsilon(\omega)} = \frac{P}{\varepsilon(\omega)} - i\pi\delta\{\varepsilon(\omega)\},$$

where P denotes the Cauchy principal value and $\delta\{\varepsilon(\omega)\} = \sum_s \delta(\omega-\omega_s)/\varepsilon'(\omega_s)$ with $\varepsilon(\omega_s)=0$. The latter equations is the dispersion relation for longitudinal waves in the self–gravitational plasma. In general case solutions $\omega=\omega_s(\mathbf{k})$ can be represented as complex combinations of the Jeans and plasma frequencies for individual components and show a marked dependence on the thermal velocities [2]. The analysis of these solutions is sufficiently complex. We shall, therefore, consider some simpler particular cases.

For a cold single–component medium, accordingly to eq. (1) $\varepsilon(\omega_s)=0$ is characterized by two real roots (with $\omega_p^2 > \omega_G^2$)

$$\omega = \pm(\omega_p^2-\omega_G^2)^{1/2}. \qquad (14)$$

The rate of energy losses in this case is

$$\frac{d\mathcal{E}}{dt} \simeq -\frac{4\pi n_0(Q_0Q - M_0Gm)^2}{mv_0} \times \log\left(k_0v_0(\omega_p^2-\omega_G^2)^{-1/2}\right) \qquad (15)$$

with $\omega_p^2>\omega_G^2$ and $k_0^2v_0^2 \gg \omega_p^2-\omega_G^2$. The roots of the characteristic equation (14) are imaginary if $\omega_p^2<\omega_G^2$, in which case the integrand in equation (13) has no poles on the real axis. As a result,

$$\frac{d\mathcal{E}}{dt} = 0, \quad (\omega_p^2 < \omega_p^2). \qquad (16)$$

The $|d\mathcal{E}/dt|$ *vs* ω_p^2 dependence for $M_0=0$ (this condition should be understood in the sense that the mass of moving particle is so small, that $M_0 \ll Q_0\omega_p/\sqrt{G}\omega_G$) is shown schematically in fig. 1 (curve 2). At $\omega_p^2=\omega_G^2$ the function shows a singularity, namely $|d\mathcal{E}/dt|=0$ at $\omega_p^2=\omega_G^2-0$ and $|d\mathcal{E}/dt|\to\infty$ at $\omega_p^2=\omega_G^2+0$. A singular behavior can be removed by taking into account the wave absorption in the medium into account. The most straightforward way is to introduce the frictional force caused by collisions between particles into the equation of motion which is used for the obtaining of dielectric constant tensor in accordance with the standard procedure [3]. If ν is a collision frequency, then the dielectric constant ε of the single–component medium becomes more complicated: $\varepsilon = 1-\omega_p^2/(\omega^2+i\nu\omega+\omega_G^2)$. Substituting it into equation (13), we obtain $d\mathcal{E}/dt$ under $\nu\neq 0$. In the case of $\omega_p^2=\omega_G^2$, this equation can be reduced to the simple relation

$$\frac{d\mathcal{E}}{dt} \simeq -\frac{(Q_0\omega_p - G^{1/2}M_0\omega_G)^2}{v_0}\log\frac{k_0v_0}{\nu}; \qquad (17)$$

$$k_0v_0 \gg \nu.$$

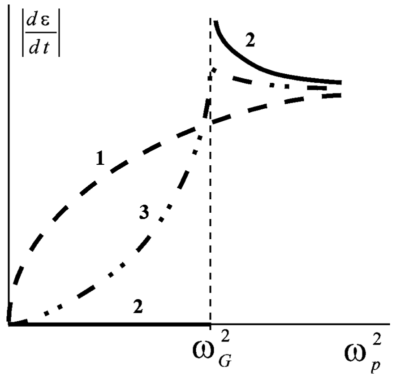

Fig. 1. Energy losses of a charged particle moving through a plasma: curve 1 is for the conventional plasma; curve 2 for a collisionless self–gravitational plasma, and curve 3 for a self–gravitational plasma with collisions.

Curve 3 in fig. 1 shows the $|d\mathcal{E}/dt|$ *vs* ω_p^2 dependence for $\nu\neq 0$, $M_0=0$. It can be compared with a similar dependence for the common plasma (curve 1) given by [3]

$$\left(\frac{d\mathcal{E}}{dt}\right)_p \simeq -\frac{Q_0^2\omega_p^2}{v_0}\log\frac{k_0v_0}{\omega_p}. \qquad (18)$$

As can be seen from the figure, the three curves are markedly different even with $M_0 = 0$. The differences are more pronounced with $M_0 \neq 0$ (i.e. $M_0 \gg Q_0\omega_p/\sqrt{G}\omega_G$). Of the greatest interest is the effect shown by self–gravitational plasmas, namely excitation by a neutral particle of longitudinal oscillations involving an electric field component ($\mathbf{E}_k \neq 0$ with $Q_0 = 0$). The energy lost by the neutral particle is given by

$$\frac{d\mathcal{E}}{dt} \simeq -GM_0^2\frac{\omega_G^2}{v_0}\log\frac{k_0v_0}{(\omega_p^2-\omega_G^2)^{1/2}}, \qquad (19)$$

$$\omega_p^2 > \omega_G^2.$$

The rate of the energy losses, equation (15), happens to depend on the sign of the charges Q_0 and Q, specifically it can be either higher than in equation (18) (Q_0 and Q are of opposite signs) or lower than that (Q_0 and Q are of the same sign). Moreover, the polarization losses can vanish if some special relation between Q_0, M_0 and parameters of the medium holds, *viz.* $Q_0/M_0 = Gm/Q$. This can only occur if the moving probe particle has a charge of the same sign as the ambient plasma particles, and their electrostatic repulsion is balanced by gravitational attraction (fig. 2).

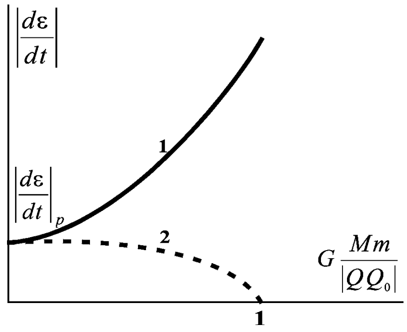

Fig. 2. The polarization losses of a massive charged particle in a self–gravitational plasma: curve 1 corresponds to the opposite signs of Q and Q_0; curve 2 for the same signs of Q and Q_0.

The pattern described will be somewhat different in the case of a bicomponent medium. The rate of energy losses will show a considerably more complex dependence upon $\omega_{p,1,2}^2$, $\omega_{G,1,2}^2$ and $k^2v_{T,1,2}^2$. We shall consider a simpler case of sharply different masses of the two kinds of particles ($m_1 \ll m_2$), assuming $\omega_{p,1}^2 \gg \omega_{G,1}^2$; $\omega_{p,2}^2 \ll \omega_{G,2}^2$ and $v_0^2 \gg v_{T,1}^2 \gg v_{T,2}^2 \to 0$. In such a case the real solutions of the equation $\varepsilon(\omega_s) = 0$ are $\omega = \pm(\omega_{p,1}^2 + k^2v_{T,1}^2)^{1/2}$.

Integrating equation (13) we arrive at

$$\frac{d\mathcal{E}}{dt} \simeq -\frac{\omega_{p,1}^2}{v_0}\left[Q_0^2 - 2G^{1/2}M_0Q_0\frac{\omega_{p,1}\omega_{G,1}+\omega_{p,2}\omega_{G,2}}{\omega_{p,1}^2+\omega_{G,2}^2} - GM_0^2\frac{\omega_{p,2}^2v_{T,1}^2}{(\omega_{p,1}^2+\omega_{G,2}^2)v_0^2}\right]\log(k_0v_0/\omega_{p,1}), \qquad (20)$$

where we have assumed $k_0^2v_0^2 \gg \omega_{p,1}^2$, $\omega_{G,2}^2$ and $k_0^2v_{T,1}^2 \ll \omega_{p,1}^2$, $\omega_{G,2}^2$. As can be seen, the energy losses in a bicomponent medium cannot turn to zero for any relation between $\omega_{p,1}$, $\omega_{G,1,2}$ and $k^2v_{T,1,2}^2$. Indeed, the dispersion relation has at least two real solutions. However, the particle will not be decelerated if its charge and mass are related so as to nullify the square brackets in equation (20).

Generally, both charged and neutral particles moving through a multicomponent self–gravitational plasma are capable of exciting waves similar to the electrostatic waves of conventional plasmas.

III. TWO–BEAM INSTABILITY OF THE SELF–GRAVITATIONAL DUSTY PLASMA

Until now we have considered the effect, connected with the motion of one probe particle. Along with randomly moving particles, space plasmas often involve particle streams with ordered velocities[1,2]. The regular speeds of particles characterized by different masses and charges are different, which may give rise to beam instabilities. Unstable states accompanied by generation of noise or individual waves can arise in the case of purely gravitational as well as purely electrostatic interaction between the streams [4]. The simultaneous action of electric and gravitational forces in a plasma medium can alter the traditional conditions for the development of an instability or change its growth rate. To analyze the instability of self–gravitational plasma flows, let us return to the dispersion relation equation (2) with ε_p, ε_G and K_V given by equations (3), (4) and (5), respectively. As a result, the dispersion relation becomes

$$1 - \sum_\alpha \frac{\omega_{p,\alpha}^2 - \omega_{G,\alpha}^2}{(\omega - kV_{0,\alpha})^2} - \frac{1}{2}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{(\omega_{p,\alpha}\omega_{G,\beta}-\omega_{p,\beta}\omega_{G,\alpha})^2}{(\omega-kV_{0,\alpha})^2(\omega-kV_{0,\beta})^2} = 0. \qquad (21)$$

Once again, we will consider the simplest case of two particle streams, $\alpha;\beta = 1,2$. Then in the frame of a reference moving at the mean velocity $V_0 = (1/2)(V_{0,1}+V_{0,2})$ equation (21) turns into a fourth-order equation in $\widetilde{\omega} = \omega - kV_0$, *viz.*

$$1 = \frac{\omega_{p,1}^2 - \omega_{G,1}^2}{(\widetilde{\omega} - k\Delta V)^2} + \frac{\omega_{p,2}^2-\omega_{G,2}^2}{(\widetilde{\omega}+k\Delta V)^2} + \frac{\omega_{p,G}^4}{(\widetilde{\omega}^2 - k^2\Delta V^2)^2}, \qquad (22)$$

where $\omega_{p,G}^2$ denotes $|\omega_{p,1}\omega_{G,2} - \omega_{p,2}\omega_{G,1}|$. The dispersion relation can be analyzed graphically. To do so, let us consider the right–hand side of equation (22) as a function of frequency, $\eta = f(\tilde{\omega})$. The solutions of equation (22) are given by intersections of this graph with the straight line $\eta = 1$. To analyze the relative importance of the two kinds of particle interaction, let us introduce the dimensionless parameters $y_\alpha = \omega_{p,\alpha}/\omega_{G,\alpha} = Q_\alpha/(m_\alpha G^{1/2})$ and re–write $f(\tilde{\omega})$ as

$$f(\tilde{\omega}) = \frac{\omega_{G,1}^2(y_1^2 - 1)}{(\tilde{\omega} - k\Delta V)^2} + \frac{\omega_{G,2}^2(y_2^2 - 1)}{(\tilde{\omega} + k\Delta V)^2} + \frac{\omega_{G,1}^2\omega_{G,2}^2(y_1 - y_2)^2}{(\tilde{\omega}^2 - k^2\Delta V^2)^2}. \tag{23}$$

If y_1 or y_2 happen to equal 1, then the corresponding term in equation (23) vanishes but parameters of that beam are still represented in the dispersion relation owing to the third term. With any y_α, the graph of $f(\tilde{\omega})$ is characterized by two vertical asymptotes at $\tilde{\omega} = \pm k\Delta V$, and four different types of the curves are possible (see fig. 3):

(a) $y_\alpha > 1$. Electrical interaction prevails in both beams. The appearance of four intersection points with $\eta = 1$ suggests the existence of four eigenmodes, two of which propagate at phase velocities close to the speed $V_{0,1}$ of one beam and the other two at the velocities close to $V_{0,2}$. As the loop in the area $\tilde{\omega} < |k\Delta V|$ moves above the level $\eta = 1$, intersection points 2 and 3 merge and then disappear, giving rise to two complex–conjugate roots of equation (22). The system becomes unstable with respect to electrical interaction of the beam particles (beam instability in conventional plasma [3]).

(b) $y_\alpha < 1$. Gravitation interaction prevails in both beams. The system is unstable against excitation of four wavelike disturbances propagating in two pairs at phase velocities close to $V_{0,1}$ and $V_{0,2}$(the beam instability of gravitating particles develops against the background of the Jeans–unstable ambient medium [4]). In this case the beam parameters are such that the loop lying in the domain $\tilde{\omega} < |k\Delta V|$ moves above $\eta = 1$, then the two waves with the phase velocities close to the mean velocity of the two beams are stable, however the system as a whole remains unstable.

(c) $y_2 < 1$ and $y_1 > 1$. Gravitation prevails in the second beam with $V_{0,2} = V_0 - \Delta V$. Owing to that beam, the system is unstable against gravitational interactions. The waves with $v_{ph} \simeq V_{0,1} = V_0 + \Delta V$ remain stable.

(d) $y_1 < 1$ and $y_2 > 1$. The system is unstable like in c), however the growing wave is controlled by gravitation forces in the first beam. Besides, constant–amplitude waves are possible in the system, traveling at a velocity close to $V_{0,2} = V_0 - \Delta V$.

Strictly speaking, the run of $f(\tilde{\omega})$ is not determined by the parameters y_1 and y_2 alone (altogether, there are five parameters of the frequency dimension, i.e. $\omega_{p,1}$; $\omega_{p,2}$; $\omega_{G,1}$; $\omega_{G,2}$; and $k\Delta V$). Therefore, the inequalities like $y_{1,2} > 1$ or $y_{1,2} < 1$ should be complemented by other conditions, namely the sign of $f(\tilde{\omega})$ should be determined by that of the first term in the vicinity of $\tilde{\omega} = k\Delta V$ and by the second term of equation (23) in the vicinity of $\tilde{\omega} = -k\Delta V$.

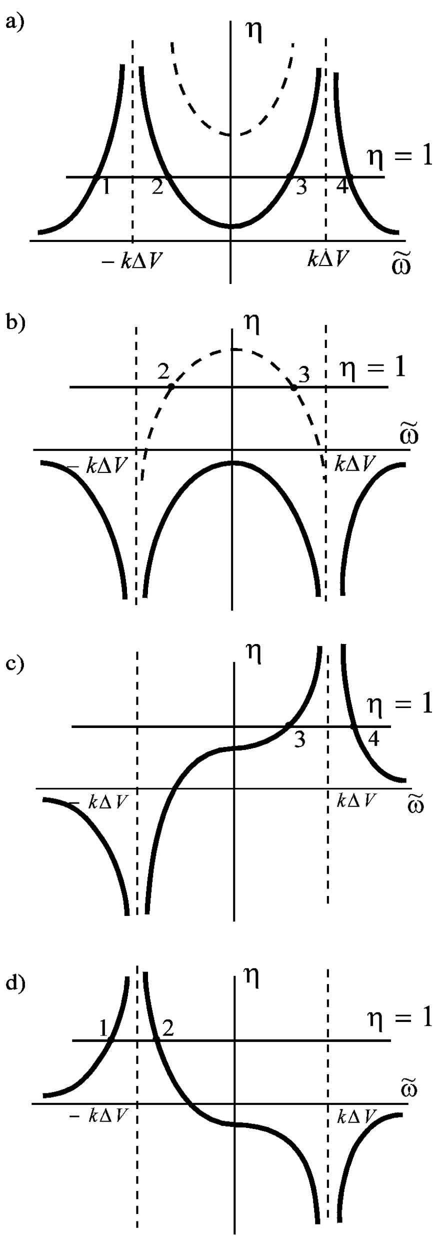

Fig. 3. Four possible variants of the $\eta = f(v_{ph})$ dependence: a) $y_1 > 1$ and $y_2 > 1$, the electric interaction prevails in both beams; b) $y_1 < 1$ and $y_2 < 1$, the gravitational interaction prevails in both beams; c) $y_1 > 1$ and $y_2 < 1$, the electric interaction is dominant in the first beam, while the gravitational in the second; d) $y_1 < 1$ and $y_2 > 1$, the gravitational interaction is dominant in the first beam and the electric in the second.

Investigating the system stability analytically is quite difficult as it involves the analysis of a fourth–order algebraic equation. By way of example, we will just consider a restricted model. Let the beam of a heavy particles ($\omega_{G,2} \gg \omega_{p,2}$) move through an ensemble of fine dust grains $\omega_{G,1} \ll \omega_{p,1}$. To further simplify the analysis, we assume $\omega_{p,1}^2 - \omega_{G,1}^2 = \omega_{G,2}^2 - \omega_{p,2}^2 = \omega_0^2$. The general run of $f(\widetilde{\omega})$ is determined by the magnitude of k. For short wave disturbances, $k\Delta V > \omega_{p,G}^2/2\omega_0$, the growth rate is $\nu \approx 2k\Delta V\omega_0/(4k^2\Delta V^2 - \omega_0^2)^{1/2}$. For the disturbances satisfying the opposite inequality, i.e. $k\Delta V < \omega_{p,G}^2/2\omega_0$, $f(\widetilde{\omega})$ is symmetrical at $\omega > 0$ and $\omega < 0$ and tends to infinity at $\widetilde{\omega} \to \pm k\Delta V$. It reaches the minimum of $\omega_{p,G}^4/k^4\Delta V^4 = f(0)$ at $\widetilde{\omega} = 0$. In the case of $f(0)$ lying below the unit level, which is possible with $\omega_{p,G} < k\Delta V < \omega_{p,G}^2/2\omega_0$, the self–gravitating plasma streams are stable with respect to such disturbances.

In the case of long wave disturbances, $k\Delta V < \omega_{p,G}$, such that $f(0) = \omega_{p,G}^4/k^4\Delta V^4 > 1$ there are two unstable branches with the respective growth rates $\nu_{1,2} \approx \omega_{p,G}^2/(\omega_0^2 \pm 4k^2\Delta V^2)^{1/2}$.

The dispersion relation equation (22) allowing for both kinds of the particle interaction is capable of predicting one of the four possible versions (see fig. 3) for instability development in a two–component plasma–beam system. We consider the linear theory, which describes just the initial stage of the growing disturbance. A further development of the process is essentially nonlinear, and it is not described by the dispersion relation equation (23). Such a process can be analyzed through numerical modeling of the particle dynamics. Apparently, the first step in this direction is the numerical modeling of the particle dynamics in a two–component system with regular drifts [5].

IV. SUMMARY

Summarizing this paper, we have tried to demonstrate some of the interesting peculiarities of dusty self–gravitational plasma. Calculations of energy losses of a probe particle moving through the self–gravitational plasma showed, that electric disturbances can be excited even by a neutral particle, which is impossible for the conventional ion–electron plasma. Meanwhile, the energy losses of a charged massive particle associated with wave excitation can either increase (as compared with the conventional plasma) or drop down to zero for specific values of the charge–to–mass ratio, depending on the charge sign. The beam instability can arise not only in charged particle beams or in neutral beams but in the streams of self–gravitational plasmas as well where particles interact through electric and gravitation forces. The simultaneous action of electric and gravitational forces in a plasma medium can alter the traditional conditions for the development of the beam instability or change their growth rate. In the general case of a self–gravitational plasma, the instability growth rates can be higher or lower as compared with the “pure” modes. The growth rate can even change its sign to become an attenuation owing to the combined interaction, i.e. an unstable stream–controlled state can turn stable and vice versa.

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ЗБУДЖЕННЯ ЕЛЕКТРОСТАТИЧНИХ ХВИЛЬ ЧАСТИНКАМИ, ЩО РУХАЮТЬСЯ В ПИЛОВІЙ САМОҐРАВІТУЮЧІЙ ПЛАЗМІ

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У роботі розглянуті деякі електродинамічні ефекти, пов'язані з рухом заряджених пилових частинок у пиловій самоґравітуючій плазмі. Показано, що врахування самоґравітації в плазмі змінює величину втрат енерґії частинок, що рухаються, бо поряд з електричною необхідно враховувати і ґравітаційну взаємодію пробної частинки з плазмою. Навіть у найпростішому випадку однокомпонентної самоґравітуючої плазми для дуже легкої зарядженої пилинки характер залежности поляризаційних утрат істотно відрізняється від утрат енерґії у звичайній плазмі. Найцікавішою особливістю пилової плазми є ґенерація електричних полів нейтральною масивною частинкою. У випадку багатокомпонентної пилової плазми при визначеному співвідношенні між параметрами середовища і пробної частинки поляризаційні втрати можуть згорнутися до нуля. Крім того, у роботі досліджені умови збудження електромагнетних хвиль у потоках самоґравітуючої плазми. Одночасне врахування електричної та ґравітаційної взаємодій компонентів плазми приводить до істотної трансформації критеріїв звичайної пучкової нестійкости. При цьому змінюються порогові значення та інкременти нестійкости; більш того, з'являється вузька область стійких розв'язків, властива тільки самоґравітуючим плазмовим потокам.