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CALCULATION OF THREE–KINK STATES IN ϕ^4–THEORY WITH DAMPING

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Some new solutions for the equation of motion in ϕ^4–theory with damping are constructed. These solutions describe the field configurations corresponding to the coupled states of three kinks and/or antikinks. To obtain them a new direct method for nonlinear partial differential equations which generalises the Hirota method for the a degeneracy case is applied.
Key words: coupled state, kink, scalar field.

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I. INTRODUCTION

The scalar field ϕ^4–theory is one of the most investigated and widely used for the description of physical phenomena both in classical and quantum physics. Up till to recent time for the equation of motion in this theory

$$\phi_{tt} - \phi_{xx} = \phi - \phi^3 \tag{1}$$

only such dynamic and static soliton–like solutions as one kink (antikink) were considered for the self–similarity case. Here $\phi_{tt} = \partial^2\phi/\partial t^2$ and so on. To study a coupled state of kink and antikink one should introduce an external force [1] or consider using perturbative approach such a field configuration which corresponds to the sum of kink and antikink plus a small additional term [2]–[4]. In such a case the problem can be solved numerically or by considering a linearized version of equation for the small term. A numerical investigation has demonstrated the existence of a quasistable approximate coupled state which decays in time [5]– [7].

In [8] it was proved by considering the symmetry properties that a mathematically stable exact solution of eq. (1) which corresponds to the coupled state of kink and antikink does not exist. The formation and propagation of nonlinear waves is determined by the processes connected with energy transformation and characterized by dispersion, nonlinearity and dissipation. In eq. (1) just dispersion and nonlinearity are accounted for. But, opposite to the Korteweg–de Vries equation where an expansion of wave packet due to dispersion is compensated by its compression due to nonlinearity, for eq. (1) there is no such balance. As a result, this equation does not have the exact and stable solutions corresponding to the coupled states of any number of kinks and/or antikinks. The mentioned above balance for eq. (1) can be obtained by taking into account additionally the physical mechanism that is always present practically in any process, namely, that of dissipation. It may be caused by different reasons (friction, damping, etc.) and changes the shape and propagation of nonlinear wave. The equation of motion in this case has the form:

$$\phi_{tt} - \phi_{xx} + \alpha\phi_t = \phi - \phi^3, \tag{2}$$

where α is a damping coefficient. Without loss of generality, all other coefficients in eq. (2) may be taken equal to unity. For this equation the exact and stable solutions corresponding to the coupled states of any number of kinks and/or antikinks can be constructed.

For the self–similarity case eq. (2) reduces to the ordinary differential equation possessing the Painlevé property [9,10]. This equation is of integrable type. We shall use the next definition of integrability. The non–linear partial differential equation will be called integrable if it has N–soliton solutions, $N = 1, 2, 3, \ldots$ [11]. Hence, one can construct the solutions of eq. (2) which correspond to the coupled states. As we consider the boundary problem one can use the direct methods to construct the coupled state solutions. Applying the well–known Hirota method [13] or its generalization using gauge invariance as a determining property [14] to eq. (2) does not allow to do this because there is a problem of degeneracy of parameters for these solutions. One–kink and one–antikink solutions for eq. (2) are well–known [9,10,12] and for them the values of wave numbers and velocities are fixed and depend on α only. It means that, for instance, in the case of two–kink solution both a the kinks have the same parameters.

This problem may be solved by a recently developed direct method for nonlinear partial differential equations which generalises the Hirota method for a degeneracy case [15,16]. By this method the two–kink coupled states for eq. (2) have been constructed [15,17]. In this paper the method is applied to construct the solutions for eq. (2) which correspond to the coupled states of three kinks and/or antikinks. The nonlinear character of the problem under consideration leads to some peculiarities in a three–kink case. In particular, due to increasing the number of the terms in an expansion for the function that determines the solution, the number of the nonlinear algebraic equations for parameters of this solution exceeds that of these parameters. Now, to choose appropriate values of the parameters the using of the symmetry properties as for a two–kink solution is not enough. It is necessary to make an additional assumption about the independence of one–particle contributions to the solution (for nonlinear problems it is not obviously).

II. FORMULATION OF THE PROBLEM

To construct the self–similarity solutions for eq. (2) corresponding to the coupled states of various combinations of three kinks and/or antikinks, one should transform this equation to an infinite system of linear partial differential equations. To do this let introduce a new unknown function $F(x,t)$ by the Cole–Hopf transformation

$$\phi(x,t) = \sigma F_x(x,t)/F(x,t), \tag{3}$$

where σ is a constant determined below. The arguments of functions will be dropped where it is possible. By using eq. (3), eq. (2) may be written as

$$\begin{aligned} &F_{xtt}F^2 - 2F_{xt}F_tF + 2F_xF_t^2 - F_xF_{tt}F \\ &-F_{xxx}F^2 + 3F_xF_{xx}F - (\sigma^2-2)F_x^3 \\ &+\alpha F_{xt}F^2 - \alpha F_xF_tF - F_xF^2 = 0. \end{aligned} \tag{4}$$

According to the Hirota method, now the parameter σ should be determined from eq. (4). Let $\sigma^2 = 2$. Eq. (4) reduced to

$$\begin{aligned} &F_{xtt}F^2 - 2F_{xt}F_tF + 2F_xF_t^2 - F_xF_{tt}F - F_{xxx}F^2 \\ &+3F_xF_{xx}F + \alpha F_{xt}F^2 - \alpha F_xF_tF - F_xF^2 = 0. \end{aligned} \tag{5}$$

For one–kink solution eq. (4) and eq. (5) lead to the same result. But for the coupled states eq. (5) leads to certain difficulties in calculating some coefficients are determined such solutions. To avoid these difficulties one uses eq. (4). In the case of a one–kink solution σ can be determined at the last stage of calculations. In this paper the same value of σ will be used to construct the three–kink coupled states.

The next step is usual for direct methods [13,14]. Let us represent $F(x,t)$ as a formal series:

$$F(x,t) = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3 + \ldots, \tag{6}$$

where $f_i(x,t)$ are new unknown functions and ε, generally speaking, is not a small constant. By substituting eq. (6) into eq. (4) and equating to zero coefficients for each degree of ε, one obtains the infinite system of linear partial differential equations for the functions f_i. For three–kink solutions one need only three functions f_i $(i = 1,2,3)$ of series (6). To determine them one should use the first four equations of the infinite system. They have the form:

$$\varepsilon^1 : f_{1,xtt} - f_{1,xxx} + \alpha f_{1,xt} - f_{1,x} = 0, \tag{7}$$

$$\varepsilon^2 : f_{2,xtt} - f_{2,xxx} + \alpha f_{2,xt} - f_{2,x} = 2f_{1,xt}f_{1,t} + f_{1,x}f_{1,tt} - 3f_{1,x}f_{1,xx} + \alpha f_{1,x}f_{1,t}, \tag{8}$$

$$\begin{aligned} &\varepsilon^3 : f_{3,xtt} - f_{3,xxx} + \alpha f_{3,xt} - f_{3,x} = 2f_{1,xt}f_{2,t} + 2f_{1,t}f_{2,xt} - f_{1,x}f_{2,tt} + f_{2,x}f_{1,tt} \\ &-2f_{1,x}f1,t^2 - 3f_{1,xx}f_{2,x} - 3f_{1,x}f_{2,xx} - (\sigma^2-2)f_{1,x}^3 + \alpha f_{1,x}f_{2,t} + \alpha f_{1,t}f_{2,x} \\ &-f_{2,xtt}f_1 + f_{2,xxx}f_1 + \alpha f_{2,xt}f_1 + f_{2,x}f_1, \end{aligned} \tag{9}$$

$$\begin{aligned} &\varepsilon^4 : f_{4,xtt} - f_{4,xxx} + \alpha f_{4,xt} - f_{4,x} = 2f_{1,xt}f_{3,t} + 2f_{2,xt}f_{2,t} + 2f_{1,t}f_{3,xt} - 4f_{1,x}f_{1,t}f_{2,t} \\ &-2f_{2,x}f_{1,t}^2 + f_{1,x}f_{3,tt} + f_{2,x}f_{2,tt} + f_{1,tt}f_{3,x} - 3f_{1,x}f_{3,xx} - 3f_{1,x}f_{1,xxx}f_2 - 3f_{2,x}f_{2,xx} \\ &-3f_{1,xx}f_{3,x} - 3(\sigma^2-2)f_{1,x}^2f_{2,x} + \alpha f_{1,x}f_{3,t} + \alpha f_{2,x}f_{2,t} + \alpha f_{1,t}f_{3,x} - 2f_{1,xt}f_{1,t}f_2 \\ &-f_{1,x}f_{1,tt}f_2 + 3f_{1,x}f_{1,xx}f_2 - \alpha f_{1,x}f_{1,t}f_2 + 2f_{1,xt}f_{1,t}f_1^2 + f_{1,x}f_{1,tt}f_1^2 - 3f_{1,x}f_{1,xx}f_1^2 \\ &+\alpha f_{1,x}f_{1,t}f_1^2 - 2f_{1,xt}f_{2,t}f_1 - 2f_{1,t}f_{2,xt}f_1 + 4f_{1,x}f_{1,t}^2f_1 - f_{1,x}f_{2,tt}f_1 - f_{1,tt}f_{2,x}f_1 \\ &+3f_{1,x}f_{2,xx}f_1 + 3f_{1,xx}f_{2,x}f_1 - \alpha f_{1,x}f_{2,t}f_1 + 2(\sigma^2-2)f_{1,x}^3 - \alpha f_{1,t}f_{2,x}f_1. \end{aligned} \tag{10}$$

The Hirota method possesses a remarkble property. If for some nonlinear equation a N–soliton solution exists, a formal series (6) is broken and all $f_i = 0$ for $i = N+1, \ldots$ [13]. But, contrary to the Hirota method, the method used in this paper does not require a special form for the coefficients of the functions f_i. That is why for a N–soliton solution one should use the right hand side of the $(N+1)$–equation to break a series (6) [16].

III. THREE–KINK STATES

Let us construct the explicit expressions for the field configurations corresponding to the coupled states of various combinations of three kinks and/or antikinks. To do this, one needs the explicit expressions for the functions f_i $(i = 1, 2, 3)$. Each of these three functions describes an appropriate contribution to a three–kink solution. But one should note that functions the f_1, f_2 and f_3 are not the same as a one–kink state and coupled states of two– and three kinks. They may be called as one–, two– and three–particle functions. It is clear from eq. (7)–eq. (10) that for every value of i the function f_i is determined by the previous functions and the damping coefficient only.

Let us represent a one–particle function in the form

$$f_1(x,t) = \exp(\eta_1) + \exp(\eta_2) + \exp(\eta_3), \qquad (11)$$

where $\eta_i = k_i x - \omega_i t + \eta_i^{(0)}$, $i = 1, 2, 3$. Here k_i, ω_i and $\eta_i^{(0)}$ are a wave number, velocity and initial phase shift, respectively. It is known [9] that in the ϕ^4–theory with damping the wave number and velocity for a one–kink solution are fixed and equal to

$$k_1 = \pm(9 + 2\alpha^2)^{1/2}/2\alpha, \qquad \omega_1 = -3/2\alpha, \qquad (12)$$

(-/+ corresponds to kink/antikink). These values are used in this paper. According to the Hirota method, to avoid a degeneracy of the coefficients in the expansion of the functions f_i, the condition $k_i \neq k_j$ for the absolute values of wave numbers should be fulfilled. This condition is not required here, because no special form for the coefficients mentioned above are demanded.Only at a final stage of calculations the various explicit relations between these parameters are considered.

For the different values of i one should use the same relations between k_i and ω_i as for a one–kink case. The first three of these relations can be obtained by substituting eq. (11) into eq. (7) and equating to zero the coefficients for each of the exponential functions. They have the form:

$$\omega_i^2 - k_i^2 - \alpha\omega_i - 1 = 0, \qquad i = 1, 2, 3. \qquad (13)$$

The second three relations can be obtained by substitution eq. (11) into the right hand side of eq. (8) and equaling to zero the coefficients for the functions $\exp(2\eta_i)$, $i = 1, 2, 3$:

$$3\omega_i^2 - 3k_i^2 - \alpha\omega_i = 0, \qquad i = 1, 2, 3. \qquad (14)$$

Eq. (13) and eq. (14) determine the parameters k_i and ω_i $(i = 1, 2, 3)$ by a single–value way. They have the same values (12) as in a one–kink case. It supposes the independent contributions to the solution from one–, two– and three–particle functions. The additional relations for k_i and ω_i, $(i = 1, 2, 3)$ can be obtained from the right hand side of eq. (8) by equating to zero the coefficients for $\exp(\eta_i + \eta_j), i \neq j$, but they are not linearly independent and may be made consistent with the relations mentioned above.

By definition for a one–particle function all the coefficients equal unity. Let us represent two–particle function in the form:

$$\begin{aligned} f_2(x,t) &= M\exp(2\eta_1) + N\exp(2\eta_2) \\ &+ O\exp(2\eta_3) + P\exp(\eta_1 + \eta_2) \\ &+ R\exp(\eta_2 + \eta_3) + S\exp(\eta_2 + \eta_3). \end{aligned} \qquad (15)$$

Here, M, N, O, P, R and S are the coefficients to be determined. If one substitutes eq. (15) into eq. (8) and tries to calculate these coefficients, the uncertainties of the type 0/0 for all the physically admissible relations between k_i and ω_i $(i = 1, 2, 3)$ will be obtained.

To avoid this difficulty one should use the right hand side of eq. (9). By substituting into it eq. (11) and eq. (15) and equating to zero coeffcients for the exponential functions all the coefficients for the function f_2 can be determined in a single–valued way. From the calculations it results that $M = N = O = 0$. It means that the contribution of the two–particle function f_2 into a three–kink state does not take into account self–interactive terms.

Let us determine the three–particle function. In the general case, it should be written in the form:

$$\begin{aligned} f_3(x,t) &= A_1\exp(3\eta_1) + A_2\exp(3\eta_2) + A_3\exp(3\eta_3) \\ &+ A_4\exp(\eta_1 + 2\eta_2) + A_5\exp(\eta_1 + 2\eta_3) \\ &+ A_6\exp(2\eta_1 + \eta_2) + A_7\exp(2\eta_1 + \eta_3) \\ &+ A_8\exp(2\eta_2 + \eta_3) + A_9\exp(\eta_2 + 2\eta_3) \\ &+ A_{10}\exp(\eta_1 + \eta_2 + \eta_3). \end{aligned} \qquad (16)$$

For simplicity, $\eta_i^{(0)} = 0$, $i = 1, 2, 3$.

In the general case explicit expressions for A_i $(i = 1, \ldots, 10)$ are very cumbrous. To simplify this problem without loss of generality one can consider the special cases when the wave numbers of kinks and/or antikinks are determined by the first of eqs. (12) and all the initial phase shifts equal zero.

Let $k_1 = k_2 = k_3 = k$. This case corresponds to a coupled state of three kinks. A one–particle function f_1 takes the form:

$$f_1 = 3\exp(\eta_1), \qquad \eta_1 = k_1 x - \omega_1 t. \tag{17}$$

The technique of determinating the coefficients M, N, O, P, R and S for a two–particle function have been described above. By applying this technique to the case under consideration one obtains that all of these coefficients are equal to zero and as a result $f_2 = 0$.

Let us calculate the coefficients A_i $(i = 1, \ldots, 10)$ for a three–particle function. Eq. (9) now takes the form:

$$\begin{aligned} &f_{3,xtt} - f_{3,xxx} + \alpha f_{3,xt} - f_{3,x} \\ &\quad = -2f_{1,x}f_{1,t}^2 - (\sigma^2 - 2)f_{1,x}^3 - 2f_1 f_{1,xt} f_{1,t} \\ &\quad - f_1 f_{1,x} f_{1,tt} + 3f_1 f_{1,x} f_{1,xx} - \alpha f_1 f_{1,x} f_{1,t}. \end{aligned} \tag{18}$$

By substituting eq. (16) and eq. (17) into eq. (18) and equaling to zero the coefficients for every exponential function one obtains that all A_i $(i = 1, \ldots, 10)$ are equal to zero. So, $f_3 = 0$.

Now, it is possible to write down the field configuration which corresponds to a coupled state of three kinks:

$$\begin{aligned} \phi(x,t) &= \sigma F_x(x,t)/F(x,t) = (1/k_1)f_{1,x}/(1 + f_1) \\ &= 3\exp(\eta_1)/[1 + 3\exp(\eta_1)]. \end{aligned} \tag{19}$$

In the case of $\eta_i^{(0)} = 0$ $(i = 1, 2, 3)$, the solution of eq. (2) can be written down in the form:

$$\phi(x,t) = \{1 + \tanh[(kx - \omega t + \ln 3)/2]\}/2. \tag{20}$$

Let $k_1 = k_2 = -k_3 = k$. This case corresponds to a coupled state of two kinks and antikink. One–particle function takes the form:

$$f_1 = 2\exp(k_1 x - \omega_1 t) + \exp(-k_1 x - \omega_1 t). \tag{21}$$

For the coefficients of a two–particle function one obtains $M = N = O = P = 0$, $R = S = 6/(6 + \alpha^2)$. The two–particle function may be written in the form:

$$\begin{aligned} f_2 &= 6\exp(\eta_1 + \eta_3)/(6 + \alpha^2) + 6\exp(\eta_2 + \eta_3)/(6 + \alpha^2) \\ &= 12\exp(3t/\alpha)/(6 + \alpha^2). \end{aligned} \tag{22}$$

In this case eq. (9) takes the form:

$$\begin{aligned} &f_{3,xtt} - f_{3,xxx} + \alpha f_{3,xt} - f_{3,x} \\ &= 2f_{1,xt}f_{2,t} - 2f_{1,x}f_{1,t}^2 + f_{1,x}f_{2,tt} \\ &-(\sigma^2 - 2)f_{1,x}^2 + \alpha f_{1,x}f_{2,t} - 2f_1 f_{1,xt} f_{1,t} \\ &-f_1 f_{1,x} f_{1,tt} + 3f_1 f_{1,x} f_{1,xx} - \alpha f_1 f_{1,x} f_{1,t}. \end{aligned} \tag{23}$$

By substituting into eq. (23) eqs. (16), (21) and (22) and equating to zero coefficients for every exponential function one obtains that all $A_i = 0$ $(i = 1, \ldots, 10)$ as for coupled state of three kinks. So, $f_3 = 0$.

Now, a field configuration which corresponds to a coupled state of two kinks and antikink can be written in a form:

$$\begin{aligned} \phi(x,t) &= \sigma F_x(x,t)/F(x,t) = (1/k_1)f_{1,x}/(1 + f_1 + f_2) \\ &= (6 + \alpha^2)[2\exp(kx - \omega t) \\ &- \exp(-kx - \omega t)]/\{(6 + \alpha^2)[1 + 2\exp(kx - \omega t) \\ &+ \exp(-kx - \omega t)] + 12\exp(-2\omega t)\}. \end{aligned}$$

Let $k_1 = -k_2 = -k_3 = k$. This case corresponds to a coupled state of a kink and two antikinks. One–particle function takes the form:

$$f_1 = \exp(k_1 x - \omega_1 t) + 2\exp(-k_1 x - \omega_1 t). \tag{24}$$

By analogy to the previous case one can obtain that $M = N = O = S = 0$, $P = R = 6/(6 + \alpha^2)$. Consequently, f_2 has the same form as for two kinks and an antikink:

$$\begin{aligned} f_2 &= 6\exp(\eta_1 + \eta_2)/(6 + \alpha^2) + 6\exp(\eta_1 + \eta_3)/(6 + \alpha^2) \\ &= 12\exp(3t/\alpha)/(6 + \alpha^2). \end{aligned} \tag{25}$$

To determine the coefficients A_i $(i = 1, \ldots, 10)$ for a three–particle function one should use the procedure which has been described above. This leads to the following values for the coefficients: $A_1 = A_2 = A_3 = A_6 = A_7 = A_8 = A_9 = 0$, $A_4 = A_5 = A_{10} = -6/(6 + \alpha^2)$. Then f_3 takes the form:

$$\begin{aligned} f_3 &= -6\exp(\eta_1 + 2\eta_2)/(6 + \alpha^2) - 6\exp(\eta_1 + 2\eta_3)/(6 + \alpha^2) \\ &- 6\exp(\eta_1 + \eta_2 + \eta_3)/(6 = \alpha^2) \\ &= -18\exp(-kx - \omega t)/(6 + \alpha^2). \end{aligned} \tag{26}$$

By using eqs. (24)–(26) a coupled state for one kink and two antikinks may be written in the form:

$$\phi(x,t) = \sigma F_x(x,t)/F(x,t) = (1/k_1)(f_{1,x} + f_{2,x} + f_{3,x})/(1 + f_1 + f_2 + f_3)$$

$$= \Big[(6+\alpha^2)\exp(kx - \omega t) - 2(6+\alpha^2)\exp(-kx-\omega t) + 18\exp(-kx - 3\omega t)]/[1 + (6+\alpha^2)\exp(kx-\omega t)$$

$$-2(6+\alpha^2)\exp(-kx-\omega t) + 18\exp(-kx-\omega t) + 12\exp(-2\omega t)\Big].$$

Comparing this function $\phi(x,t)$ with that describing a coupled state of two kinks and an antikink one can see that in both the cases the self–similarity is violated (there is a term with a temporal coordinate only).

Let $k_1 = k_2 = k_3 = -k$. This case corresponds to a coupled state of three antikinks. It is obviousl that the solutions of eq. (2) which describes this state differs from solutions (19) and (20) by the sign of spatial coordinate only. So, for a three–antikink coupled state one obtains

$$\phi(x,t) = \{1 + \tanh[(-kx - \omega t + \ln 3)/2]\}/2.$$

IV. CONCLUSION

In this paper the explicit expressions for coupled states of three kinks and/or antikinks in the ϕ^4–theory with damping are constructed. By direct substitution one can verify that all of them are solutions of eq. (2).

It is known that the ϕ^4–theory without damping is not integrable. This theory has become integrable and admits the solutions that describe the coupled states of any number of kinks and/or antikinks only in the case when the energy dissipation is taken into account, for example, due to damping. This result allows to introduce a physical criterion of integrability for nonlinear equations. The nonlinear equation will be integrable (possesses the N–soliton solution) if there is a balance between an expansion and compression of the wave packet due to the processes of energy transformation (dispersion, dissipation, non–linearity and so on). For eq. (1) such a balance is absent. To achieve it one should introduce into the term $\alpha\phi_t$ the equation. This criterion is fulfilled for the system of two scalar fields [18] where there is an exchange of energy between both the fields and as a result the coupled states can be constructed.

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РОЗРАХУНОК ТРИКІНКОВИХ СТАНІВ У ТЕОРІЇ ϕ^4 ІЗ ЗАГАСАННЯМ

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Побудовано нові розв'язки рівняння руху в теорії ϕ^4 із загасанням. Ці розв'язки описують польові конфіґурації, які відповідають зв'язаним станам різних комбінацій трьох кінків і/або антикінків. Розв'язки отримано за допомогою нового прямого методу для нелінійних рівнянь у частинних похідних, який узагальнює метод Гіроти для випадку виродження.