STABILITY AND CALIBRATING METHODS OF SOME THIN FILM PLATINUM THERMOMETERS IN THE RANGE 13.8–273.16 K

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Resistance stability of thin film platinum resistance thermometers produced by PRIBOR Ltd., Koprivshtitza, Bulgaria was investigated 30 months after their thermal stabilization. A three–point calibration method for the range between 13.8 K and 273.16 K is proposed. The optimum conditions necessary for an adequate individual calibration of the investigated sensors are defined. Mathematical descriptions of the $T(R)$ and the $R(T)$ functions between 13.8 K and 273.16 K are suggested using a new weighted least squares method.
Key words: mathematical description, thermometers, calibration.

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I. INTRODUCTION

The interest in industrial platinum resistance thermometers (IPRTs) arose recently [1]. There are three types of IPRTs — wire, thin film and thick film thermometers. The thin film platinum resistance thermometers (TPRTs) have some indisputable advantages such as small dimensions, high response, easy fabrication and relatively low cost [2]. They can be used without any calibration in the range of 70–200 K following the $T(R)$ and $R(T)$ common descriptions with an accuracy of $\pm(1-2)$ K and $\pm(0.4-0.8)\Omega$, respectively [3]. A two–point calibration method allows to apply them in the range of 70–200 K with an accuracy of ± 0.1 K or $\pm 0.04\Omega$ [3]. Their main disadvantages are:

i) a shift of the $R(T)$ function after a thermal cycling below 200 K, and

ii) individual $R(T)$ and $T(R)$ functions for any particular sensor.

Therefore TPRTs can be used for precise measurements below 200 K after an adequate stabilization and an individual calibration [3], [4].

The task of the present work was to investigate the resistance stability of TPRTs, produced by PRIBOR Ltd., Koprivshtitza, Bulgaria under a technology described in [5] and their $T(R)$ and $R(T)$ functions in the temperature range 13.8 — 273.16 K in order to define:

i) period of stability of TPRTs after their thermal stabilization,

ii) the optimum conditions for an adequate individual calibration of the investigated sensors, and

iii) suitable mathematical descriptions in polynomial form for $T(R)$ and $R(T)$ functions.

For this reason 10 TPRTs selected randomly from different series and numbered from 031 to 042 save 037 and 039) have been investigated. Sensors 037 and 039 were [4] and became unusable. Investigations were carried out on a computerized experimental setup. The experimental method and the setup were fully described in [4] and [6].

II. STABILITY OF THE INVESTIGATED SENSORS

The problem of TPRTs characteristic stabilization was a subject of our previous paper [4]. It was found out that they remained stable at least 5 months after their stabilization. Now we carried out 6 new thermal cycles numbered from 38 to 43 — once between 10 and 300 K (helium cycle) and 5 times between 65 K and 300 K. The helium cycle included 12 cycles between 5 and 75 K as it was done through the 16^{th} cycle (see about it in [4]. The sensor 032 resistance variance at 6 fixed temperatures after every single thermal cycle including the described in [4] is shown in Fig. 1. The results for all other sensors are analogous. It is well seen that the resistance stabilizes after cycle No 27. The resistance variance from 28^{th} to 43^{th} cycles was less than TPRTs accuracy [3] and the experimental error [4].

III. MATHEMATICAL DESCRIPTION OF THE INDIVIDUAL $T(R)$ AND $R(T)$ FUNCTIONS

It was established in [3] that the TPRTs can be used as thermometers in the range 13.8–273.16 K with an accuracy of $\pm(5-6)$ mK. Having in mind those results we have searched for a mathematical description that could describe the calibration results with the respective accuracy. For this purpose a weighted least squares method was developed. It was based on the well known methodology described in Refs.7 and 8. The basic functions — $P_k(x)$ used by the Chi–square function [7] can be:

i) power functions

$$P_k(x) = x^{k-1}, \tag{1}$$

ii) power functions orthogonalized on variable interval -1.0–+1.0,

iii) Chebyshev Polynomials,

iv) Displaced Chebyshev Polynomials, and

v) Orthogonal Polynomials (normalized or not) made up for every data points set using a well known three–term recurrence procedure for generating orthogonal polynomials [8], [9], [10].

The fitted functions are ordinary polynomials of the type

$$Y(y) = \sum_{i=1}^{M} B_i \left[X(x)\right]^{i-1}, \tag{2}$$

where $Y(y)$ and $X(x)$ can be as follows:

$$Y = y, \tag{3a}$$

$$Y = 1/y, \tag{3b}$$

$$Y = \ln(y), \tag{3c}$$

$$Y = e^{y}, \tag{3d}$$

$$Y = y^{m}, \tag{3e}$$

and

$$X = x, \tag{4a}$$

$$X = 1/x, \tag{4b}$$

$$X = \ln(x), \tag{4c}$$

$$X = e^{x}, \tag{4d}$$

$$X = x/a + b, \tag{4e}$$

$$X = \left[\ln\left(x/a + b\right) + c\right]/d, \tag{4f}$$

$$X = \left[\left(x/a + b\right)^{m} + c\right]/d, \tag{4g}$$

Here a, b, c, d and m are free coefficients ($a \neq 0$ and $d \neq 0$) and M is polynomial number of terms. Eqs.(3) and Eqs.(4) allow to investigate a large number of polynomial types.

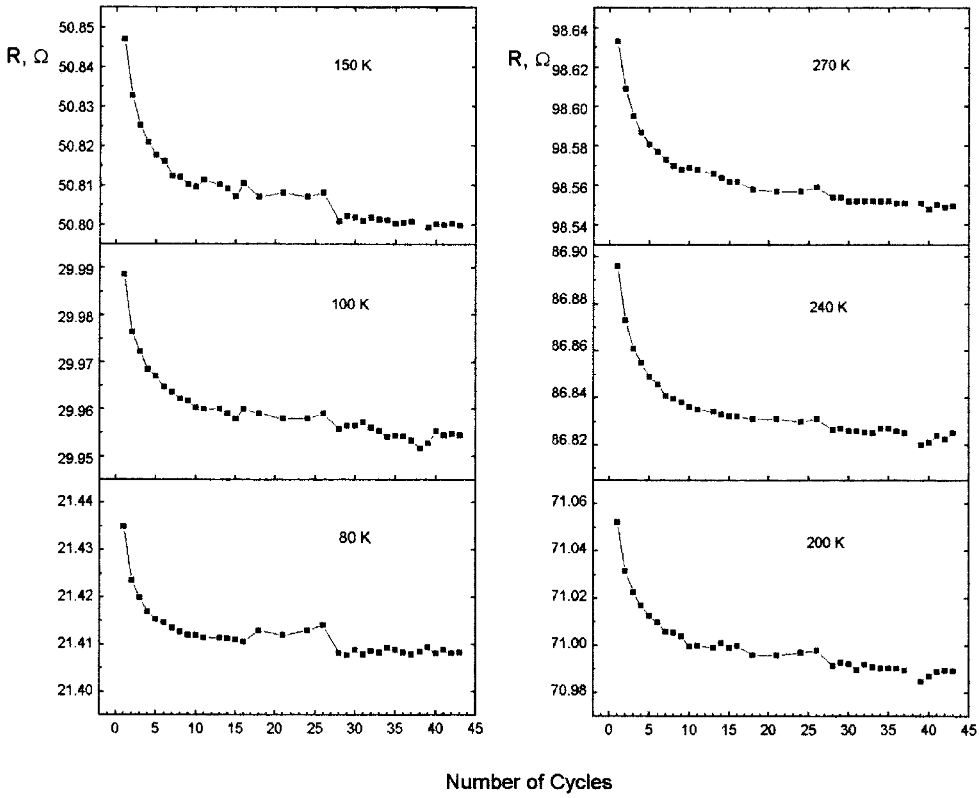

Fig. 1. Dependence of resistance on thermal cycle number for six temperatures for sensor 032.

Software realizing the method was written in Borland Pascal 7.0. It calculates:

i) the polynomial coefficients — B_i,

ii) the standard deviation — σ,

iii) the Chi–square — χ^2,

iv) the normalized value of Chi — χ_n,

v) the maximal deviation,

vi) the optimum polynomial number of terms, and

vii) the standard deviation dependence on number of terms — $\sigma(M)$, where

$$\sigma = \left[\sum_{I=1}^{N} \left[y_i - y(x_i) \right]^2 / (N - M) \right]^{1/2}, \qquad (5)$$

$$\chi_n = \left[\chi^2 / (n - M) \right]^{1/2}. \qquad (6)$$

Here y_i is a measured value of y (experimental data), $y(x_i)$ is the fitted value for the same data point and N is number of the experimental points.

The program allows also to present graphically the fitted function — $Y(X)$, its first and second derivatives — $Y'(X)$ and $Y''(X)$ and the dependence $\Delta Y(Y)$, where $\Delta Y = y_i - y(x_i)$. Thus, one can control visually if the fitted function is a smooth curve or any oscillations occur.

The main feature of the method is the possibility to does the fitting procedure using iteratively two criteria:

i) the minimization of funcion Chi–square — χ^2 and

ii) the fulfillment for all data points of the inequality

$$\rho(x_i)[y_i - y(x_i)]^2 \leq 1 \qquad (7)$$

where $\rho(x_i)$ is the weight of the i^{th} experimental point. This principle of minimization is proposed in [11] (see also [12] and [13]). Our program defines the suitable weight — $\rho(x_i)$ for all data points according to inequality Eq.(7) after every fit and then do the next fit. The iterative process stops when the normalized value of Chi — χ_n stabilizes. This method eliminates the "bad" points influence and improves the physical authenticity of the fitted curve. The new moment in the proposed method is that there is a possibility to apply a bigger initial weight ($\rho(x_i) \geq 3.33$) for some data points which remains constant through the fitting process.

This program routine was applied in our investigations to define the suitable polynomial type and the optimal number of terms for the fitted functions.

The experimental data between 11 K and 320 K from all the carried out investigations after the thermal stabilization (cycles 28–43) were fitted. At least 12 calibration points were available at every temperature level. All the fits were carried out with the initial data points weight $\rho(x_i) = 1$ for the most part of the experimental data because they were obtained with the same accuracy [4]. Exceptions were done in two cases:

i) for the data at the edges of the investigated temperature range according to Hamming's recommendations [8] (mainly the range 14–25 K because of the low accuracy of the data below 14 K), and

ii) when it was necessary to correct the fitted curve in some region. We had such problems in the range of 25–50 K for some $T(R)$ functions.

The fitting procedure was made applying all the above said basic functions — P_k and polynomial types (Eqs.(3) and Eqs.(4)). The best results were obtained with the orthogonal polynomials basic functions (normalized or not) orthogonalized on variable interval -1.0–+1.0. There was no difference between them for polynomial fits with the number of terms up to 21 but the calculations with Chebyshev polynomials and orthogonalized Power functions (Eq.(1)) were simpler because there was no necessity to calculate the coefficients of a three–term recurrence [8], [9], [10]. The normalization of orthogonal polynomials did not lead to better results but made the calculations more complicated. For this reason it is preferable to use in the least squares fits Chebyshev polynomials or Power functions orthogonalized on variable interval -1.0–+1.0 as they are the simplest systems of orthogonal polynomials.

The use of simple basic functions and the possibility to apply a bigger initial weight for any of the data points are the main advantages of this method in comparison with the OPEM method [11]. Moreover there is no necessity to estimate previously the absolute resistance resolution to determine the initial data points weight. These features make our method simpler and more universal. We fitted the commercially available test data obtained under specified operating conditions by Lake Shore Cryotronics for the Platinum thermometer (model Pt–103, Serial No P3170) investigated in Refs.12 and 13 to compare the proposed method, the standard Chebyshev and the OPEM method [11]. The results for $T(R)$ function obtained with our program (standard Chebyshev and weighted Chebyshev — new method) and those from Refs.12 and 13 for OPEM are shown in Table 1. It is well seen that the results obtained with our method and OPEM are very close and excel the standard Chebyshev fit.

The fitting procedure results allowed us to propose the following ordinary polynomials to describe the individual $T(R)$ and $R(T)$ functions of TPRTs in the temperature range 13.8–273.16 K

$$T = \sum_{I=1}^{M} A_i \left\{ \left[(R/R_o)^{1/6} - 0.65 \right] / 035 \right\}^{i-1}, \qquad (8)$$

$$W = R/R_o = \sum_{I=1}^{M} B_i \left\{ \left[(T/T_o)^{1/6} - 0.65 \right] / 035 \right\}^{i-1}, \qquad (9)$$

where A_i and B_i were polynomial coefficients, T_o = 273.16 K and R_o was the value of R at $T = T_o$. $R_o \approx 100\Omega$ for the investigated sensors and the fit could be made using the value R_o= 100 which leads to a negligible error. Eqs.(8) and (9) are similar to the inverse formula for platinum thermometer reference $T(R)$ function of $ITS-90$ [14].

Temperature range, K	Number of points	Number of terms	Fit type	Standard deviation, mK	Maximum deviation, mK
11 - 31	19	11	Chebyshev	7.4	−11.9
		11	OPEM	6.6	−19.8
		11	New method	7.0	−14.9
		12	New method	4.2	−11.1
31 −95	20	8	Chebyshev	9.7	−19.9
		8	OPEM	5.5	−15.2
		8	New method	3.8	−23.3
95 −325	27	9	Chebyshev	2.0	−3.5
		9	OPEM	1.6	3.9
		9	New method	2.0	−3.5
14 −325	56	18	Chebyshev	11.9	−27.3
		18	OPEM	5.8	20.9
		18	New method	4.2	−45.1

Table 1. Comparative results between OPEM, standard Chebyshev fit and new weighted method for $T(R)$ function of Platinum thermometer model PT-103 Ser.No P3170.

Function type	Temperature range, K	Polynomial number of terms	Standard deviation, mK or mΩ
$T(R)$ function — Eq.(8)	13.8–70.0	15–16	1.5–2.5 mK
$T(R)$ function — Eq.(8)	70.0–273.16	9–10	3.0–5.0 mK
$T(R)$ function — Eq.(8)	13.8–273.16	17–18	3.0–5.0 mK
$R(T)$ function — Eq.(10)	13.8–70.0	10–11	0.5–1.5 mΩ
$R(T)$ function — Eq.(9)	70.0–273.16	8–10	1.5–2.5 mΩ
$R(T)$ function — Eq.(9)	13.8–273.16	14–16	1.5–2.5 mΩ

Table 2. Recommended number of polynomial terms (coefficients) for the investigated temperature ranges.

It was found out that any equation of the types $T = F(\ln R)$ and $R = f(\ln T)$ could not be applied for to describe TPRTs produced by PRIBOR Ltd in most cases instead of Eqs.(8) and (9) unlike the wire Platinum resistors. No correction functions were necessary to Eqs.(8) and (9) as recommended in [15] because they described the experimental data with an adequate accuracy (see Table 1 and [3]).

Equations (8) and (9) can be applied to describe $T(R)$ and $R(T)$ functions in the "nitrogen" interval 70–273.16 K and $T(R)$ functions in the lower range 13.8–70 K. Conventional polynomials

$$W = R/R_o = \sum_{I=1}^{M} B_i T^{i-1}, \tag{10}$$

describe the $R(T)$ functions in this range (13.8–70 K) and up to 90 K better than Eq.(9).

The recommended numbers of polynomial terms for the investigated temperature ranges and the respective standard deviations are shown in Table 1. They were defined using:

i) the dependency of polynomial standard deviation on the number of terms for every sensor ($\sigma_T(M)$ and $\sigma_R(M)$),

ii) the derivatives dT/dR and dR/dT and

iii) the dependence of the residuals between the polynomial description and the experimental data on temperature — $\Delta T(T)$ or $\Delta R(T)$.

The $\sigma_T(M)$ and $\sigma_R(M)$ dependences allowed to define the values of M at which the σ_T and σ_R values corresponded to the data on TPRTs accuracy presented in [3]. The $\sigma_T(M)$ and $\sigma_R(M)$ dependences for five thermometers for the investigated temperature ranges (13.8–70 K, 70–273.16 K and 13.8–273.16 K) are shown in Figs. 2a, 2b and 2c.

The derivatives dT/dR and dR/dT were investigated after every fit to verify if oscillations occur. The derivatives dT/dR for sensor 032 and dR/dT for sensor 041 are shown in Fig. 3. The polynomial numbers of terms are 18 for the $T(R)$ function and 15 for the $R(T)$ function. It is well seen that they are smooth curves with the maximum or minimum at $T \approx$85–86 K. The second derivatives are also smooth curves and they are equal to 0 at $T \approx$ 85–86 K. The derivatives of lower or higher power polynomials in comprarison with those recommended in Table 2 (1–3 degrees less or 2–4 higher) are not smooth curves.

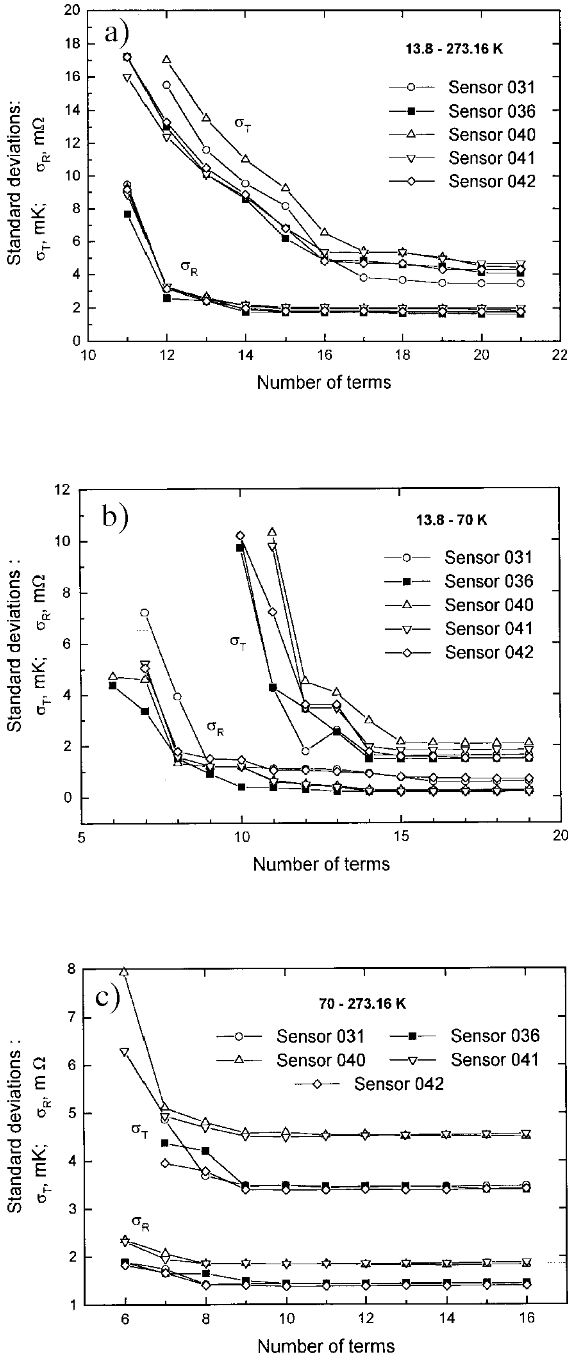

Fig. 2. Dependence of polynomial standard deviations σ_T and σ_R on number of polynomial terms for sensors 031, 036, 040, 041 and 042: 2a — Temperature range 13.8–273.16 K; 2b — Temperature range 13.8–70 K; 2c — Temperature range 70–273.16 K.

That is why, the application of higher or lower power polynomials than recommended in Table 2 is not admissible because oscillations of the fitted function can occur. So, one can assume that the proposed mathematical descriptions of $T(R)$ and $R(T)$ functions for the TPRTs can be applied in practice. This conclusion is confirmed also by the character of dependence of the residuals between the polynomial descriptions and the experimental data on the temperature — $\Delta T(T)$ and $\Delta R(T)$. The dependences $\Delta T(T)$ for sensor 033 and $\Delta R(T)$ for sensor 034 are shown in Figs. 4a and 4b. The recommended polynomial types and number of terms are applied to obtain them (see Table 1). It is well seen that the residuals — ΔT_i and ΔR_i are less than ± 6 mK and ± 2.5 m Ω, respectively, for the most part of the experimental data and the data points are situated regularly on both sides of the fitted curve and no oscillations are detected.

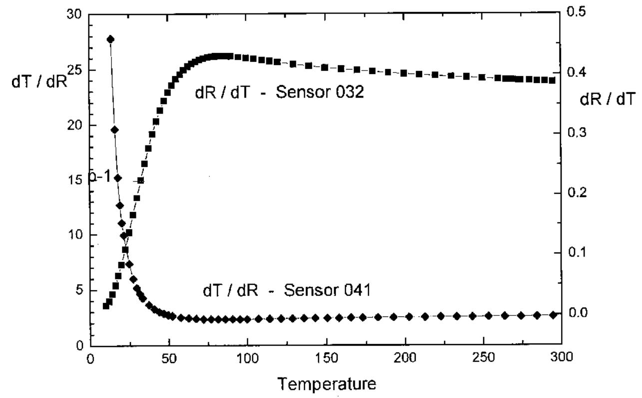

Fig. 3. Dependence on temperature of the derivatives dT/dR for sensor 032 and dR/dT for sensor 041.

IV. CALIBRATION METHODS

It is well known that thermometer calibration must be realized at a sufficient number of temperature levels but the number of calibration points depends on the applied method.

Two calibration points are enough to calibrate the TPRTs in the range of 70–273.16 K with an accuracy of ± 0.1 K or $\pm 0.04\Omega$ [3]. The two–point calibration method is based on the linearity of the dependences $\Delta W_j(T)$ and $\Delta T_j(W)$ above 70 K and on the description of the thermometer characteristics by the sum of reference and deviation functions as known for the standard platinum thermometers [14]. $\Delta W_j(T)$ is the difference between $W_j(T)$ and $W_c(T)$, and $\Delta T_j(W)$ is the difference between $T_j(W)$ and $T_c(W)$ of TPRTs above 70 K, respectively. $W_j(T)$ and $T_j(W)$ are functions of the j^{th} thermometer. The common mathematical descriptions $T_c(W)$ and $W_c(T$, defined in [3] are used as reference functions. The $T(W)$ and $W(T)$ functions of any TPRT can be used as $T_c(W)$ and $W_c(T)$ but it is preferable to select a TPRT whose parameters are in the vicinity of the statistically average for the investigated group of sensors [3].

The two–point calibration method can be developed to a three–point method to expand the temperature range of the calibration down to 13.8 K. The three–point calibration method is also based on the description of the thermometer characteristics by the sum of reference and deviation functions. It ensures the accuracy of $\pm 0.04\Omega$

(± 0.1 K above 45 K and ± 0.3 K below 45 K) in the range of 13.8–273.16 K. The calibration points have to be: $T_1 =$ 270–280 K, $T_2 =$ 75–80 K and $T_3 =$ 4.2–10 K. We recommend the first calibration point to be at 273.16 K (water triple point) because it is obligatory to determine R_o of any TPRT at this temperature. The recommended value of T_2 is 77.36 K (liquid Nitrogen boiling point). The liquid Nitrogen boiling point is preferable because it can be realized easier than any other temperature in the range (75–80 K). The proposed T_3 value is 10 K because the use of a lower value decreases slightly the accuracy of Eq.(11). W and R of TPRTs are almost constant below 10–11 K because this is the region of the residual resistance of the TPRTs. The deviation is less than 3. For this reason the curves of $W_j(T)$ functions are parallel and $\Delta W_j(T) \approx$ const. Therefore W_3 can be defined at any temperature below 10 K, including in liquid helium at 4.2 K, and T_3 can be accepted to be 10 K.

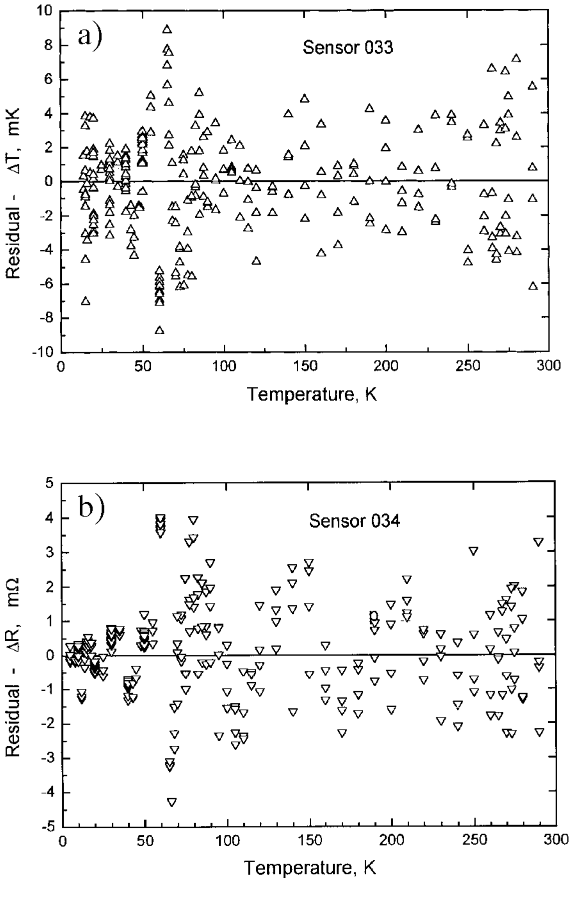

Fig. 4. Dependence of residuals between the polynomial descriptions and the experimental data on temperature: 3a — $\Delta T(T)$ for sensor 033; 3b — $\Delta R(T)$ for sensor 034.

The $W(T)$ and $T(W)$ functions are the same as for the two–point calibration method

$$W_j(T) = W_c(T) + \Delta W_j(T), \tag{11}$$

$$T_j(W) = T_c(W) + \Delta T_j(W). \tag{12}$$

The $W(T)$ and $T(W)$ functions (Eqs. (8) and (9)) of sensor 031 were used as reference functions because the common mathematical descriptions [3] could not be applied below 70 K and the parameters of sensor 031 were in the vicinity of statistical mean for the investigated group of sensors. It is well seen from Fig. 5 where the dependences on temperature of the differences ΔW_j between the experimental W_j data of all investigated TPRTs and the W of sensor 031 in the range 5–320 K are shown.

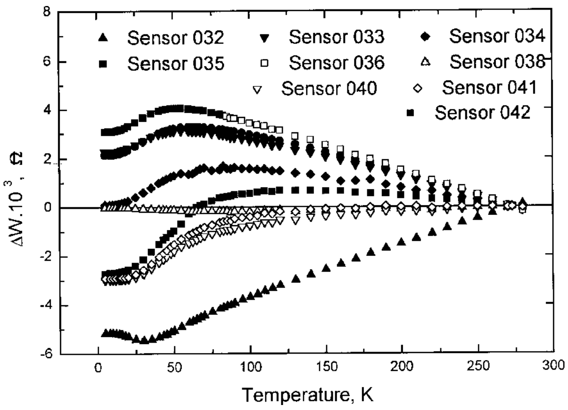

Fig. 5. Dependences on temperature of the differences ΔW_j between the experimental W_j data and W of sensor 031 in the range 5–320 K for all thermometers.

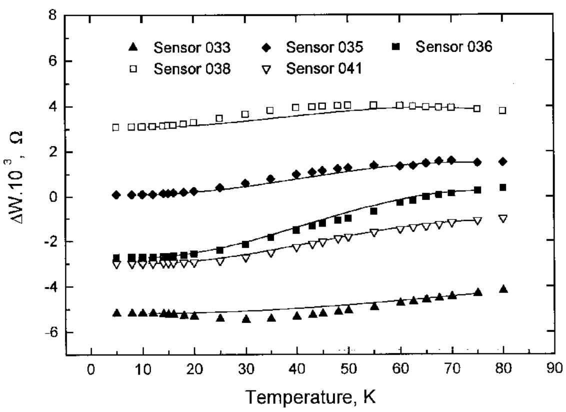

Fig. 6. The deviation functions $\Delta W_j(T)$ according to Eq.(15) for sensors 033, 035, 036, 038 and 041 in the range 10–77.4 K and the experimental data points.

The deviation functions $\Delta W_j(T)$ and $\Delta T_j(W)$ above T_2 are the same as for the two–point method [3]

$$\Delta W_j(T) = \Delta W_j(T_2)(T_0 - T)/(T_0 - T_2) = Z_W \Delta W_j(T_2), \tag{13}$$

$$\Delta T_j(W) = \Delta T_j(W_2)(1 - W)/(1 - W_2) = Z_T \Delta T_j(W_2). \tag{14}$$

Here T_0 and T_2 are the first and the second calibration temperature levels ($T_0 = T_1 = 273.16\,\mathrm{K}$) and Z_T and Z_W are the functions first suggested by Cragoe [16] and used later by Besley and Kemp for their two–points method [1]. The deviation functions $\Delta W_j(T)$ can be defined as 4 — terms polynomials below T_2

$$\Delta W_j(T) = \sum_{i=1}^{4} D_i T^{i-1}. \tag{15}$$

Unfortunately, the deviation functions $\Delta T_j(W)$ could not be defined properly below T_2. It is a consequence of the fact that the differences between $T_j(R)$ functions of these sensors below 25 K became too large ($\Delta T_j^{'}(W) \to \infty$ below 10 K) and an adequate mathematical description of $\Delta T_j(W)$ could not be found. For this reason Eqs. (11), (9) and (15) have to be used to define T below T_2. It is the main disadvantage of the three–point method. As a consequence the reference functions $W_c(T)$ and $T_c(W)$ were defined in the ranges 11–300 K and 70–300 K respectively using the proposed fitting method. The numbers of terms were $M = 15$ for the $W_c(T)$ function (Eq.(9)) and $M = 9$ for the $T_c(W)$ function (Eq.(8)).

The Eq.(15) coefficients D_i can be calculated as follows. The values of T and W at the second and third calibration points of the calibrated TPRT and Eqs. (11) and (13) allow to write the equations

$$D_4 T_2^3 + D_3 T_2^2 + D_2 T_2 + D_1 = W_2 - W_c(T_2), \tag{16}$$

$$D_4 T_3^3 + D_3 T_3^2 + D_2 T_3 + D_1 = W_3 - W_c(T_3), \tag{17}$$

where W_3 is the value of W_j at the temperature T_3.

It was shown that $\Delta W_j(T) \approx$ const below 10–11 K. For this reason the first derivative of Eq.(15) is equal to 0 at $T = T_3$

$$\Delta W_j^{'}(T_3) = 3D_4 T_3^2 + 2D_3 T_3 + D_2 = 0. \tag{18}$$

The first derivative of Eq.(15) is obviously equal to the derivative value of Eq.(13) at $T = T_2$ because this is the common point of the two functions that describe the $\Delta W_j(T)$ functions above and below T_2 and here their derivatives must be identical. For this reason one can write the following equation

$$3D_4 T_2^2 + 2D_3 T_2 + D_2 = -\Delta W_j(T_2)/(T_0 - T_2), \tag{19}$$

Now the coefficients D_i can be calculated from the system of equations (16), (17), (18) and (19).

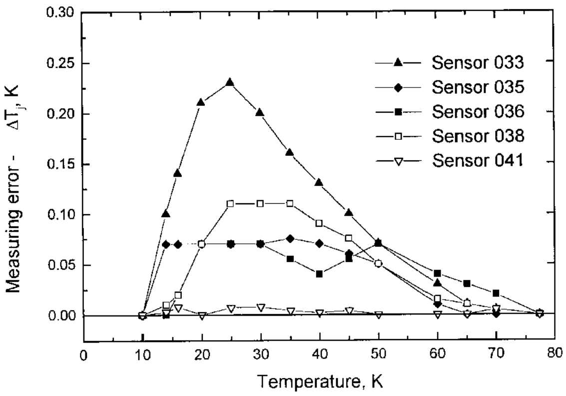

Fig. 7. The differences $\Delta T_j(T)$ between calculated Eq.(11) and experimental results for sensors 033, 035, 036, 038 and 041.

The adequacy of three–points calibration method in the region of 13.8 – 77.4 K (below T_2) is illustrated in Figs. 6 and 7 for 5 sensors using the above said experimental data (cycles 28–43). The values of $\Delta W_j(T)$ according to Eqs.(11) and (15) and the experimental data are shown in Fig. 6 for sensors 033, 035, 036, 038 and 041. The differences $\Delta T_j(T)$ between the calculated and experimental results for the same sensors in the same temperature region are shown in Fig.7. The analysis of these data allowed to ascertain that an accuracy better than $\pm 0.04\Omega$ (± 0.1 K above 45 K and ± 0.3 K below 45 K) was achieved. The accuracy below 45 K can be better than ± 0.1 K if the value of $W_3 - W_c(T_3)$ is small (sensors 035, 036 and 038 for example). For this reason it is preferable to classify the sensors into 2–3 groups in accordance with their residual resistance — W_3 when industrial quantities of sensors has to be calibrated. Every group of sensors must possess its own reference function — $W_c(T)$. As said above, $W(T)$ function of any sensor can be used as $W_c(T)$ but the most suitable for this purpose can be any sensor whose residual resistance — W_3

is in the vicinity of the statistically mean for the group. It has to be calibrated precisely in the range 13.8–300 K.

Temperature range, K	Temperature steps, K
Below 16	0.5 – 1.0
16 – 25	1.0 – 2.0
25 – 90	2.0 – 2.5
90 – 120	5.0
120 – 260	10.0
265 – 275	2.5
Above 280	10.0

Table 3. Recommended steps between the temperature levels for precise TPRTs calibration.

The precise calibration of TPRTs requires considerably more than 3 calibration points and an adequate mathematical description to ensure their maximum accuracy. It was ascertained that the steps between calibration temperature levels had to be as shown in Table 3. These results were based on the fitting results described in Section 3. The application of larger steps did not allow sometimes to obtain smooth curves for higher power polynomials.

V. CONCLUSIONS

The investigated thermometers were selected at random from different series. Therefore we consider that the sensor stability results, the proposed calibration methods and the mathematical descriptions of $T(R)$ and $R(T)$ functions are valid for all the TPRTs produced by PRIBOR Ltd as well as for the sensors manufactured under analogous technologies.

The results confirmed one more time that these sensors can be used in the temperature range of 13.8–273.16 K with the accuracy of $\pm(5-6)$ mK after thermal stabilization and precise calibration and allowed to realize this possibility using the proposed mathematical descriptions and calibration method. The calibration is valid for at least 30 months (2.5 years) after their thermal stabilization and probably more.

The proposed three–point calibration method allows to apply them with the accuracy of $\pm 0.04\Omega$ (± 0.1 K above 45 K and ± 0.3 K below 45 K) for small quantities of TPRTs. Their accuracy can be better (± 0.1 K in the whole range of 13.8–300 K) when industrial quantities of sensors have to be calibrated.

The new developed least squares method is not specialized and allows to make an adequate mathematical description of any experimental results in an ordinary polynomial form. It ensures a higher physical authenticity of the fitted curve in comparison with the conventional least squares methods and the same results as OPEM [11] but it possesses some indisputable advantages. For this reason it can be recommended for practice.

VI. ACKNOWLEDGMENTS

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СТІЙКІСТЬ ТА МЕТОДИ КАЛІБРУВАННЯ ДЕЯКИХ ТЕРМОМЕТРІВ НА ТОНКИХ ПЛАТИНОВИХ ПЛІВКАХ В ІНТЕРВАЛІ 13.8–273.16 K

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Стійкість опору термометрів на тонких металевих плівках, що виробляє фірма ПРІБОР, досліджували через 30 місяців після їх температурної стабілізації. Запропоновано триточковий метод калібрування для ділянки 18.8 K і 273.16 K. Визначено оптимальні умови, необхідні для адекватного індивідуального калібрування досліджених сенсорів. На підставі цього методу запропоновано математичні описи функцій $T(R)$ і $R(T)$ між 13.8 K та 273.16 K.