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TWO DIFFERENT SCALING REGIMES IN GINZBURG–LANDAU MODEL WITH CHERN–SIMONS TERM

H. Kleinert, F. S. Nogueira
Institut für Theoretische Physik, Freie Universität Berlin
Arnimallee 14, D–14195 Berlin, Germany
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The Ginzburg–Landau model with a Chern–Simons term is shown to possess two different scaling regimes depending on whether the mass of the scalar field is zero or not. In contrast to pure ϕ^4 theories, the Ginzburg–Landau model with a topologically generated mass exhibits quite different properties in perturbation theory. Our analysis suggests that the two scalings could coincide at a non-perturbative level. This view is supported by a $1/N$-expansion in the massive scalar field regime.

Key words: Scaling, Ginzburg-Landau model, Chern-Simons term

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I. INTRODUCTION

The Ginzburg–Landau (GL) model [1] with the Lagrange density

$$\mathcal{L}_{\rm GL} = \frac{1}{2}(\nabla \times \mathbf{a})^2 + |(\nabla - iq\mathbf{a})\phi|^2 + r|\phi|^2 + \frac{u}{2}|\phi|^4 \quad (1)$$

was set up more than 50 years ago to describe superconductivity. Since then it has been used to describe a variety of other physical systems [2], where complex order field $\psi(\mathbf{x})$ and vector potential $\mathbf{a}(\mathbf{x})$ are not related to Cooper pairs and magnetic fields. An important application deals with smectic liquid crystals where $\psi(\mathbf{x})$ describes the smectic order and $\mathbf{A}(\mathbf{x})$ the transverse displacement of the nematic director [3]. Other fascinating applications arise by adding a topological Chern–Simons (CS) term [4] for the vector potential to the GL model:

$$\mathcal{L}_{\rm CS} = \frac{i\theta}{2}\mathbf{a}\cdot(\nabla \times \mathbf{a}). \quad (2)$$

In this case one speaks of a Chern–Simons–Ginzburg–Landau (CSGL) model. This model possesses properties found in the famous fractional quantum Hall effect, where the coupling parameter of the CS term determines a nontrivial phase factor for the exchange of two complex fields and thus the statistics of Laughlin quasi-particles [5]. Without an initial Maxwell term $(\nabla \times \mathbf{a})^2/2$, this interpretation has been advanced by Zhang [6]. All gradient terms in his effective action are caused by fluctuations of a vector potential with only a CS term. But also the theory with a Maxwell term has physical significance since it emerges naturally by when constructing a dual disorder model of CSGL model without the Maxwell term [7]. In addition, such a model has been found when bosonizing theories of strongly interacting fermions in three dimensions [8,9].

Another interesting application of the CSGL model without a Maxwell term arises in the field theoretical approach to polymers, where the degree of entanglement is controlled by the parameter θ [10]. Detailed results have been obtained recently [11,12].

In this paper we shall discuss the fixed point structure of the the CSGL model with a Maxwell term. The Lagrangian of the model is

$$\mathcal{L}_{\rm CSGL} = \mathcal{L}_{\rm GL} + \mathcal{L}_{\rm CS}. \quad (3)$$

The fixed-point structure of a standard GL Lagrangian has been investigated at various places [13,14,16]. In the presence of a CS term, it has been discussed in Refs. [17] and [18]. It must be emphasized that in these references *the Maxwell term is explicitly included* in contrast to earlier work where it was ignored [19]. This makes important differences in the fixed-point structure since in the presence Maxwell term, the charge is no longer dimensionless, and there is the generation of another mass called the topological mass. An important result of Ref. [18] was that, although the CS term is not renormalized, the β-function of the topological coupling *is not* zero due to the presence of the Maxwell term.

It was noted by Semenoff [19] that the renormalization of the CSCL model depends on the mass of the scalar field. Kleinert and Schakel [17] considered the CSGL and derived scaling laws as a function of the renormalized mass of the scalar field. Later, de Calan *et al.* [18] considered the same model, but within renormalization group (RG) approach at the critical point. They obtained considerably more involved RG functions than those of Ref. [17].

In this paper we shall improve considerably the discussions of Refs. [17] and [18] and exhibit the relation between both scaling behaviours at the one-loop level. The plan of the paper is the following. In Section II we discuss an effective mean-field theory which only includes fluctuations of the gauge field. From this we can already observe a particular feature coming from the CS term: for large θ, there is no tricritical point and therefore no first-order phase transition, in contrast to the pure GL model [20]. This approximation, however, does not distinguish reliably critical from tricritical behaviour. For this we employ in Section III the RG to obtain information on the phase transition. At the one-loop level we are then able to distinguish *two different scaling behaviours with quite different physical properties.* The relation between them is illuminated in Section IV by comparing a non-perturbative $1/N$ expansion in the nonzero-mass regime with the one-loop approximation in the massless regime. The qualitative behaviour of the non-perturbative result shows a remarkable agreement with the one-loop approximation. In Section V we obtain the exponents for the "fermionic" fixed point, which is reached for a specific value of the CS coupling parameter obtained from the bosonization scheme [9,6]. A final discussion is given in Section VI.

II. EFFECTIVE MEAN-FIELD THEORY

Let us integrate out the vector potential in the effective action to derive a lowest-order effective mean-field theory for model (3), by analogy with the procedure on the pure GL theory by Halperin *et al.* [21]. For a uniform order field $\phi \equiv \phi_0/\sqrt{2}$, where ϕ_0 is a real constant, this operation can be done exactly. The result is a a free energy density

$$\mathcal{F} = -\frac{1}{12\pi}\{[M_+^2(\phi_0^2)]^{3/2} + [M_-^2(\phi_0^2)]^{3/2}\} + \frac{r}{2}\phi_0^2 + \frac{u}{8}\phi_0^4, \tag{4}$$

where

$$M_\pm^2(\phi_0^2) \equiv q^2\phi_0^2 + \frac{\theta^2}{2} \pm \frac{|\theta|}{2}\sqrt{\theta^2 + 4q^2\phi_0^2}. \tag{5}$$

In (4) we dropped field-independent infinite term and absorbed a term proportional to the ultraviolet cutoff in r. For $\theta = 0$, our result reduces to the usual Halperin–Lubensky–Ma (HLM) expression [21] which displays a first-order phase transition. This remains true for sufficiently small $\theta \neq 0$. At larger values of θ, however, the transition is of second-order. This change of order is quite subtle: if we expand $\mathcal{L}^{\rm eff}$ à la Landau up to the power ϕ_0^4, we obtain for a constant order field

$$\mathcal{F}_{\rm L} \simeq -\frac{|\theta|^3}{12\pi} + \left(\frac{r}{2} - \frac{q^2|\theta|}{4\pi}\right)\phi_0^2 + \frac{u}{8}\phi_0^4, \tag{6}$$

and we see that the above equation does not have the correct $\theta \to 0$ limit, being valid only for large θ. The free energy (6) has only a second-order phase transition, with a critical point at $r_c = q^2|\theta|/2\pi$. Note that in contrast to the GL case there is no cubic term in ϕ_0, and that the ϕ_0^4-term receives no contribution from the CS coupling — both properties would have generated a tricritical point in this approximation. The latter property implies that the one-loop gauge field graph with four external scalar field lines vanishes if the external momenta are set to zero, as a peculiar feature of the CSGL model noted earlier in Ref. [17]. The same graph is, however, non-vanishing at nonzero external momenta. This is the origin of the two different scaling regimes in this model which we want to dicuss in this paper.

III. RENORMALIZATION GROUP FUNCTIONS

We now calculate the RG functions of the model. As discussed in Section II, the scaling with a finite scalar field mass looks different from the one where such a mass is absent, as noted by Semenoff [19]. Let us study these scaling behaviours separately and see how the are related to each other. Many of the results of this section have been obtained before in Refs. [17,18]. However, a discussion on the relation between the scaling behaviours of Ref. [17] and Ref. [18] is new. This relation will require an extension of the CSGL model (3) to $N/2$ complex scalar field.

A. Massive Scaling Regime

In the scaling regime with a massive scalar field the propagators are given by

$$G(\mathbf{p}) = \frac{1}{\mathbf{p}^2 + r}, \quad r \neq 0. \tag{7}$$

for the scalar and

$$D_{\mu\nu}(\mathbf{p}) = \frac{1}{\mathbf{p}^2 + \theta^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{\mathbf{p}^2} - \theta\epsilon_{\mu\nu\lambda}\frac{p_\lambda}{\mathbf{p}^2}\right) \tag{8}$$

for the vector field in the Landau gauge. The Lagrangian is written in terms of renormalized quantities as

$$L = \frac{Z_a}{2}(\nabla \times \mathbf{a_r})^2 + \frac{i\theta_r}{2}\mathbf{a}_r \cdot (\nabla \times \mathbf{a_r}) + Z_\phi|(\nabla - iq_r\mathbf{a}_r)\phi_r|^2 + Z_\phi^{(2)}m^2|\phi_r|^2 + \frac{Z_g m g}{2}|\phi_r|^4. \tag{9}$$

The renormalized fields are given by $\phi_r = Z_\phi^{-1/2}\phi$ and $\mathbf{a}_r = Z_a^{-1/2}\mathbf{a}$, and we have set $u_r \equiv mg$ to have a dimensionless coupling constant g. We also have introduced a mass of the scalar field m by $m^2 \equiv Z_\phi^{(2)^{-1}} Z_\phi r$. Note that the CS term is not renormalized [19], implying that $\theta_r = Z_a\theta$. The renormalized charge is $q_r = Z_a^{1/2}q$. We introduce two dimensionless gauge coupling constants by $t \equiv \theta_r/m$ and $f \equiv q_r^2/m$. The renormalization constants are fixed by imposing normalization conditions for the one-particle irreducible two- and four-point functions:

$$\Gamma^{(2)}_{r,11}(0) = m^2, \tag{10}$$

$$\left.\frac{\partial \Gamma^{(2)}_{r,11}}{\partial \mathbf{p}^2}\right|_{\mathbf{p}=0} = 1, \tag{11}$$

$$\Gamma^{(4)}_{r,1111}(0,0,0,0) = 3mg, \tag{12}$$

$$\Gamma^{(1,2)}_{r,11}(0,0,0) = 1, \tag{13}$$

$$\left.\frac{\Gamma_{r,\mu\mu}}{\partial \mathbf{p}^2}\right|_{\mathbf{p}=0} = 2. \tag{14}$$

Let us define the RG functions:

$$\gamma_\phi \equiv m\frac{\partial \ln Z_\phi}{\partial m},$$

$$\gamma_a \equiv m\frac{\partial \ln Z_a}{\partial m},$$

$$\gamma^{(2)}_\phi \equiv m\frac{\partial \ln Z^{(2)}_\phi}{\partial m}. \tag{15}$$

Within the present renormalization scheme, these functions are given explicitly in the one-loop approximation by

$$\gamma_\phi = -\frac{2}{3\pi}\frac{f}{(1+|t|)^2},$$

$$\gamma_a = \frac{Nf}{48\pi},$$

$$\gamma^{(2)}_\phi = -\frac{(N+2)g}{16\pi}. \tag{16}$$

The β-functions are given by

$$\beta_f \equiv m\frac{\partial f}{\partial m} = (\gamma_a - 1)f,$$

$$\beta_t \equiv m\frac{\partial t}{\partial m} = (\gamma_a - 1)t,$$

$$\beta_g \equiv m\frac{\partial g}{\partial m} = (2\gamma_\phi - 1)g + \frac{N+8}{16\pi}g^2. \tag{17}$$

Note the absence of a term proportional to f^2 in Eq. (17). This generalizes the observation in the previous approximation that the ϕ_0^4-term in the Landau expansion (6) is θ-independent. This is in contrast to the pure GL model where a f^2-term is present in β_g [21–23,13,14]. The present absence of the f^2-term is the reason for the existence of a charged fixed point (which remains true for all values of N, if the model is extended to $N/2$ complex fields, in contract to the pure GL model where $N > 365$ is needed as pointed out by Halperin *et al.* [21].

The anomalous dimension of the gauge field $\eta_a \equiv \gamma_a^*$ has an interesting property. From Eq. (17) we see that $\eta_a = 1$, implying a fixed point also in (17) for any t, which means that the critical exponents can vary continuously. The charged fixed point is given by

$$f_* = 48\pi/N, \tag{18}$$

$$g_* = \frac{16\pi}{N+8}\left[\frac{64}{N(1+|t_*|)^2} + 1\right].$$

Note that β_t does not vanish identically as in the absence of a Maxwell term. It does, however, vanish at the fixed point where $\gamma_a = 1$ for all values of t, which has the same effect as $\beta_t \equiv 0$, thus allowing for arbitrary fixed-point values t_*.

The critical exponent η is given by the fixed point value of the RG function γ_ϕ:

$$\eta = -\frac{32}{N(1+|t_*|)^2}. \tag{19}$$

Thus, although a charged fixed point exists for all values of N, the fixed points with $N \le 32/[(1+|t_*|)^2]$ are unphysical since the inequality $\eta > -1$ is not fulfilled. Thus, we still have a critical value of N, but considerably smaller than the value $N_c = 365$ of Ref. [21]. For any fixed $N \le N_c = 32$, there are physical critical exponents provided that

$$|t_*| > t_c(N) = 4\sqrt{\frac{2}{N}} - 1. \tag{20}$$

In Fig. 1 we plot $N\eta$ as a function of t_* for positive values of t_*.

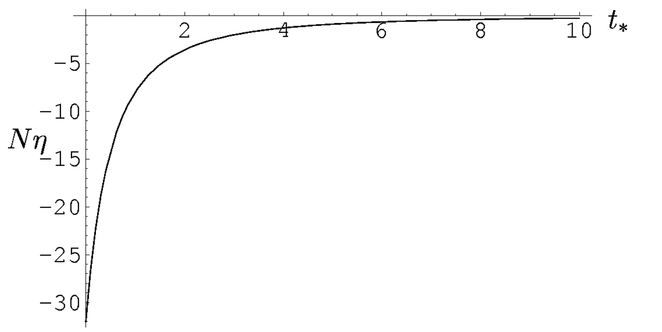

Fig. 1. Plot of $N\eta$, where η is given by Eq. (19), as a function of $t_* > 0$.

The critical exponent ν is obtained as the fixed-point value of the RG function $\nu_\phi = 1/[2 + \gamma^{(2)}_\phi - \gamma_\phi]$,which is

$$\nu = \left[2 - \frac{N+2}{N+8} - \frac{N-4}{N+8}\frac{32/N}{(1+|t_*|)^2}\right]^{-1}. \qquad (21)$$

This is plotted in Fig. 2 as a function of t_* for $N = 68$.

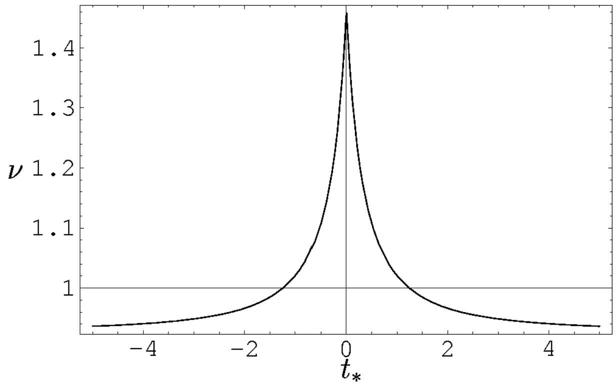

Fig. 2. Plot of the critical exponent ν as a function of t_* for $N = 68$.

B. Massless Scaling Regime

We now derive the scaling behaviour at the critical point, where the mass of the scalar field vanishes. Then the coupling constants must be defined at nonzero external momenta of the vertex functions. For g we choose the normalization condition

$$\Gamma^{(4)}_{1111}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)|_{SP} = 3\mu g, \qquad (22)$$

where the symbol SP stands for the symmetry point [15]

$$\mathbf{p}_i \cdot \mathbf{p}_j = \frac{\mu^2}{4}(4\delta_{ij} - 1). \qquad (23)$$

We shall distinguish the RG functions of the massive from those of the massless scaling regime by adding a tilde over the latter. The β-functions of the gauge couplings have the same form as before. The anomalous dimension of the vector field changes, however, being now

$$\tilde{\gamma}_a = \frac{Nf}{32}. \qquad (24)$$

Also the beta function β_g is now different, since the one-loop gauge field graph with four external legs is now nonzero. This leads to an f^2-term in β_g:

$$\tilde{\beta}_g \equiv \mu\frac{\partial g}{\partial \mu} = (2\tilde{\gamma}_\phi - 1)g + \frac{N+8}{16}g^2 + \frac{\delta}{4\pi}f^2, \qquad (25)$$

where

$$\tilde{\gamma}_\phi = -\frac{f}{4\pi}\left[\frac{3\pi}{4t^2} + \frac{\pi}{2} - \frac{3\pi t^2}{4} + 3|t| - \frac{3}{|t|} - \left(\frac{3}{2t^2} - 1 + \frac{3t^2}{2}\right)\arctan\left(\frac{1-t^2}{2|t|}\right)\right], \qquad (26)$$

$$\delta = \frac{\pi}{2t^2} + \frac{1}{|t|} - \frac{5\pi}{4} + \left(-\frac{3}{2t^4} - \frac{4}{t^2} + 8\right)\arctan\left(\frac{1}{2|t|}\right) + \left(\frac{3}{2t^4} + \frac{3}{t^2} - \frac{5}{2}\right)\arctan\left(\frac{1-t^2}{2|t|}\right). \qquad (27)$$

In the limit $t \to 0$ we have $\tilde{\gamma}_\phi \to -f/4$ and $\delta \to 3\pi/2$, corresponding to the GL limit.

In contrast with the massive scaling regime, the present equations yield a charged fixed point only for a limited range of N. The beta functions vanish at

$$f_* = 32/N,$$

$$g^*_\pm = \frac{8}{N+8}\left[1 - 2\tilde{\eta} \pm \sqrt{(1-2\tilde{\eta})^2 - \frac{160\delta_*}{\pi}}\right], \qquad (28)$$

where $\tilde{\eta} \equiv \tilde{\gamma}_\phi(f_*, t_*)$ is the anomalous dimension of the complex field in the massless scaling regime. Remarkably, there we find a tricritical fixed point g^*_-, which is absent in the massive regime. The two regimes are similar for $\delta_* = 0$, in which case there will be no tricritical fixed point in both regimes. This happens for $t_* = t_0 \simeq 0.802693$. For $t_* > t_0$ we find $\delta_* < 0$, in which case the tricritical point becomes unstable, since it corresponds to $g^*_- < 0$. In Fig. 3 we plot δ as a function of t.

The charged fixed points are accessible only if $t_* \geq \tilde{t}_c(N)$, where $\tilde{t}_c(N)$ is the value of t_* that vanishes the discriminant in Eq. (28). For example, if $N = 10$ we have $\tilde{t}_c(10) \simeq 0.752751$. From Eq. (20) we find $t_c(10) \simeq 0.788854$, and therefore $\tilde{t}_c(10) < t_c(10)$.

In order to calculate the $\tilde{\nu}$-exponent, we need the RG function $\tilde{\gamma}^{(2)}_\phi$. This function is much more complicate in the massless scaling regime and is given explicitly by

$$\tilde{\gamma}_\phi^{(2)} = -\frac{(N+2)g}{16} - \frac{f}{4\pi}\left\{\pi\frac{4t^2-3}{4\sqrt{3}t^2} + \frac{4t^2+3}{2\sqrt{3}t^2}\arctan\left(\frac{3-4t^2}{4\sqrt{3}|t|}\right) + \frac{(3-4t^2)(3+4t^2)}{8|t|\Delta}\left[1+\frac{|t|}{\sqrt{\Delta}}\arctan\left(\frac{\sqrt{\Delta}}{|t|}\right)\right]\right\}, \tag{29}$$

where

$$\Delta \equiv t^4 + \frac{t^2}{2} + \frac{9}{16}. \tag{30}$$

In Fig. 4 we plot $\tilde{\nu}$ as a function of $t_* \geq t_c(10)$. The curve has a maximum for $t_* \simeq 1.631$, where $\tilde{\nu}_{\max} = 1.7$.

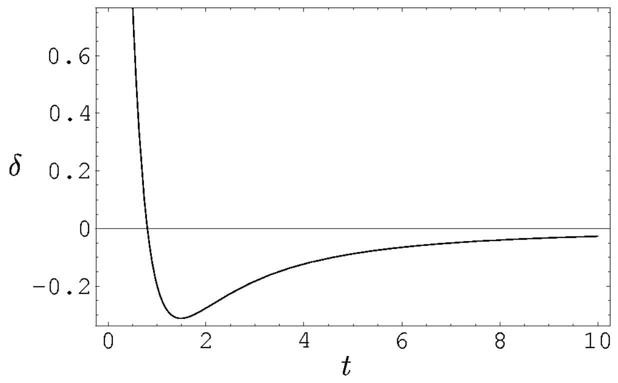

Fig. 3. Plot of δ as a function of t.

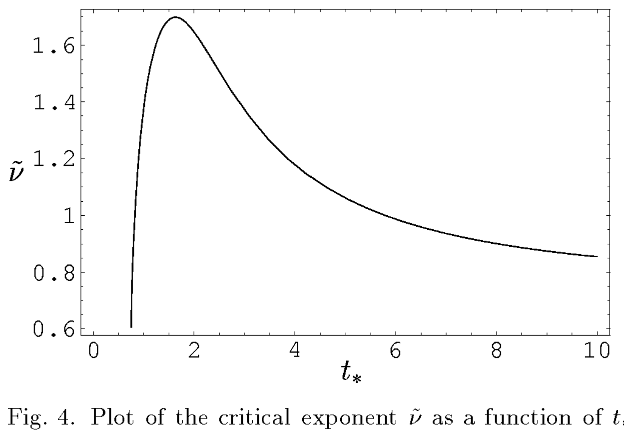

Fig. 4. Plot of the critical exponent $\tilde{\nu}$ as a function of t_* for $N = 10$.

IV. SIGN OF ANOMALOUS DIMENSION η AND LARGE-N LIMIT

A much debated topic in the GL theory is the physical meaning of a negative sign of the η-exponent found in analytic calculations [24,25,16] and computer simulations [26]. In early discussions of the subject it had been argued that a negative η would be unphysical since it would violate the Källen–Lehmann spectral representation [24,27]. However, it is now being understood that a negative sign of η in the GL model makes sense. In fact, there are several physical systems where negative ηs have been found before, most prominently magnetic systems, which show strong momentum space instabilities [28]. These can produce a non-uniform phase with a modulated order parameter. The point where the modulated phase sets in is called a Lifshitz point [29]. Physical systems with a Lifshitz point have a negative η-exponent. It has recently been argued by us [25,16] that such momentum space instabilities occur also in superconductors, implying the existence of a Lifshitz point in the phase diagram, and thus explaining the negative sign of η. A different explanation has been given recently in Ref. [30] focusing on the geometric properties of the critical fluctuations. There the anomalous dimension is related to the Hausdorff dimensions of the critical fluctuations.

The CS term in the GL model is expected to affect this picture, since for infinite t_*, the gauge field decouples from the scalar field. In this limit, the critical exponents are those of a pure scalar field theory which has $\eta > 0$. Thus we may wonder at which finite value of t_* the sign change of η occurs.

In Fig. 5 we plot $N\tilde{\eta}$ as a function of t_*. We see that at one-loop order, $\tilde{\eta}$ is always negative and approaches zero for $t_* \to \infty$. It would be desirable to know the two-loop corrections in the massless scaling regime and check if there exists a finite value of t_* where the sign of $\tilde{\eta}$ changes. We have not yet done this calculation due to its complexity for arbitrary t. We can, however, easily write down η in the limit of large N for all coupling strengths in the massive scaling regime. Then the CSGL model is in the same universality class as the $CP^{N/2-1}$ model with a CS term, and here the critical exponents have been computed by Rajeev and Ferretti [31]. The result for η is

$$\eta = -\frac{40}{\pi^2 N}\left(1 - \frac{16}{15}\frac{\bar{t}^2}{1+\bar{t}^2}\right), \tag{31}$$

where $\bar{t} = 4t/\pi$. In the limit $\bar{t} \to \infty$ this reduces correctly to η of the $O(N)$-symmetric scalar model [15,32] to order $1/N$.

From Eq. (31) we see that η changes sign at $\bar{t} = \sqrt{15}$. In Fig. 6 we plot $N\eta$ as given in Eq. (31) as a function of t. Interestingly, Figs. 5 and 6 look very similar for low t, up to a factor of two in the vertical scale, although the two curves come from two completely different approximations. In addition, the comparison teaches us that the sign of η may easily change in perturbation theory by including higher-order corrections.

It is also interesting to consider the critical exponent ν to the leading order in $1/N$. We have

$$\nu = 1 - \frac{96}{\pi^2 N}\left[1 - \frac{8}{9}\frac{\bar{t}^2(\bar{t}^2+4)}{(1+\bar{t}^2)^2}\right]. \tag{32}$$

We have plotted the ν-exponent given above in Fig. 7 for $N = 10$, for the sake of comparison with the one-loop result for the massless scaling regime. There is again a remarkable similarity between this curve and the one for $\tilde{\nu}$ in Fig. 4. This striking resemblance between the massive scaling regime at the order $1/N$ and the massless scaling regime at one-loop seems to indicate that perturbation theory within the massive scaling regime is worse than perturbation theory within the massless scaling regime, and that the exact curves in the two regimes may ultimately coincide.

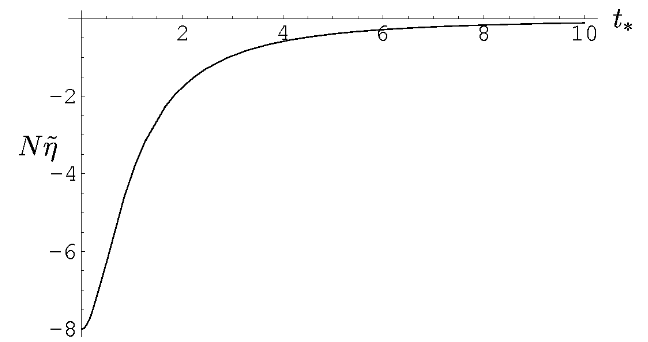

Fig. 5. Plot of $N\tilde{\eta}$ as a function of t_*.

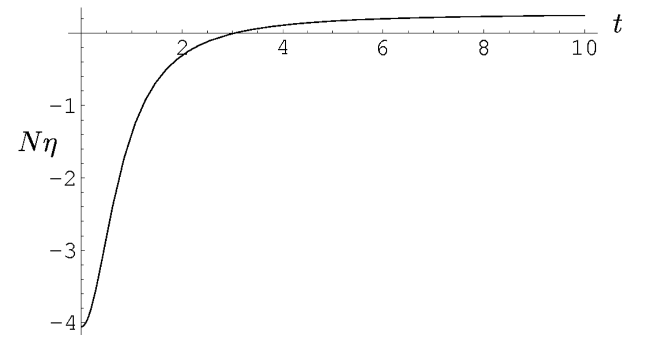

Fig. 6. Plot of $N\eta$, where η is given by Eq. (31), as a function of t.

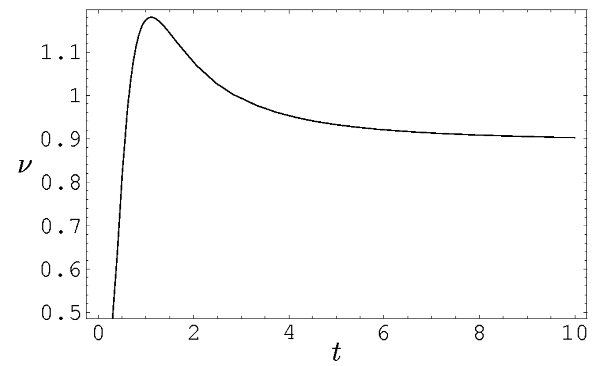

Fig. 7. Plot of ν, where ν is given by Eq. (32) with $N = 10$, as a function of t.

V. THE "FERMIONIC" FIXED POINT

Let us see what we can learn from our scaling study about strongly interacting fermions. General arguments involving bosonization [9] and duality transformations [7] indicate that the fixed points corresponding to fermions lie, in the massless scaling regime, at $t_* = 1/2\pi$ and $f_* = 32/N$. Then there are no fixed points $g_\pm^*$ for the physical case $N = 2$. We know, however, from duality arguments applied to the GL model [2,20,33] that this one-loop result is not trustworthy for $N = 2$. Within the massless scaling regime it is possible to obtain a charged fixed point for all values of N at the one-loop level by introducing a new arbitrary parameter c which corresponds to the ratio between the two renormalization scales defining the gauge and scalar couplings, respectively [14]. This procedure was also applied to the CSGL model in Ref. [18]. A charged fixed point is found for $N = 2$ if c is chosen large enough. This happens since γ_a is modified to

$$\gamma_a = \frac{cNf}{32}. \tag{33}$$

Since $\gamma_a = 1$ at the fixed point, a large c makes f_* sufficiently small to reach the fixed point for $N = 2$. The main drawback of this technique is the fact that c is not determined by the formalism. In Ref. [14] it was chosen to reproduce the tricritical point determined in Ref. [20] by a disorder field theory of the GL model.

Recently, we have succeeded in obtaining a charged fixed point at $N = 2$ by defining a new RG approach in the ordered phase [16], where the two length scales of the GL model are well defined by the correlation length ξ and the penetration depth λ. This makes the parameter c in Refs. [14] and [18] superfluous. The application of Sothis calculation procedure to the Lagrangian (3) is complicated due to the CS term. This creates a gauge field propagator in the ordered phase with two different masses, the CS mass and another one generated by the Higgs mechanism. To avoid this complication we shall restrict ourselves here to the c-approach. The constant c will be fixed by demanding that in the $\theta = 0$-model the critical exponent ν has a XY value, as found in the duality approach [34]. For the XY value $\nu \simeq 0.67$, this fixes $c \simeq 82.7$. In Ref. [18], a smaller value of c was used by approximating the RG function $\tilde{\nu}_\phi$ by $\tilde{\nu}_\phi \simeq (1 - \tilde{\gamma}_\phi^{(2)}/2 + \tilde{\gamma}_\phi/2)/2$. However, this approximation gives $\tilde{\nu} = 0.6$ for $f_* = t_* = 0$, while if we do not use such an approximation we obtain a much better value in this limit, $\tilde{\nu} = 0.625$, which is just the one-loop value for the $O(2)$-symmetric ϕ^4 theory [15].

Therefore we obtain, with $c = 82.7$ and $t_* = 1/2/\pi$:

$$\tilde{\eta} \simeq -0.05, \tag{34}$$

$$\tilde{\nu} \simeq 0.66. \tag{35}$$

We see that the "fermionic" fixed point is not much different from the GL fixed point for the value of c under consideration.

VI. CONCLUSION

We have studied and compared two scaling regimes in the CSGL model. In the massive scaling regime, charged fixed points exist for all values of N. However, not all of them lead to physical values of the critical exponents which restrict the range of allowed values of N. By restricting the values of t_* we have been able to obtain physical exponents to all N in the massive scaling regime. The interval of admissible t_* is obtained from inequality (20).

In the massless scaling regime we find a similar restriction through a more involved inequality, since $\tilde{\eta}$ has a far more complicated expression.

The discussion in the massive scaling regime yields no tricritical point, a result consistent with the Landau expansion of the mean-field free energy in Eq. (6). However, from the non-expanded mean-field free energy (4) it is seen that for sufficiently small θ we obtain a first-order phase transition. In particular, we recover the usual HLM result for $\theta = 0$, in contrast to Eq. (6) which does not have the correct $\theta \to 0$-limit.

The behaviour in the massless scaling regime is more consistent with Eq. (4) since it exhibits a tricritical fixed point for $t_* < t_0$. In the region $t_* \geq t_0$, the two scaling regimes look quite similar, at least qualitatively. The $1/N$-expansion applied to the massive scaling regime makes this similarity even greater and suggests that perturbation theory applied in the massless scaling regime is better behaved than in the massive regime.

An interesting point with respect to the $1/N$-expansion in the massive scaling regime is the sign change in η for $\bar{t} = \sqrt{15}$. This never happens for a GL model [16,25,26]. Inspired by our recent work suggesting that the sign of η is related to momentum space instabilities [16,25], we may conjecture that when an external magnetic field is included in the CSGL model, vortex lattices should not exist above a certain critical value of the topological mass.

We have discussed briefly what we called "fermionic" fixed point, that is, the fixed point where the CSGL model corresponds to bosonized three-dimensional interacting fermions. At this fixed point $t_* = 1/2\pi$. Unfortunately, $g_{\pm}^*$ is not real in this case if $N = 2$. In order to reach a charged fixed point for $N = 2$ we introduced an arbitrary parameter c corresponding to the ratio between the renormalization points of the gauge couplings and scalar coupling. The value of c has been fixed in the $t = 0$ model. As t is turned on to $t_* = 1/2\pi$ the values of the critical exponents do not show an appreciable change with respect to the $t = 0$ case.

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ДВА РІЗНІ СКЕЙЛІНҐОВІ РЕЖИМИ В МОДЕЛІ ҐІНЗБУРҐА–ЛАНДАУ З ДОДАНКОМ ЧЕРНА–САЙМОНСА

Г. Кляйнерт, Ф. Ноґуейра
Інститут теоретичної фізики. Вільний університет у Берліні
вул. Арнімале 14, D–14195, Берлін, Німеччина
kleinert@physik.fu-berlin.de; URL: http://www.physik.fu-berlin.de/kleinert/
nogueira@physik.fu-berlin.de

Показано, що модель Ґінзбурґа–Ландау з доданком Черна–Саймонса має два скейлінґових режими, які залежать від того, чи маса скалярного поля рівна нулю, чи ні. У теорії збурень модель Ґінзбурга–Ландау з топологічною масою демонструє досить відмінні властивості від чистої теорії ϕ^4. Наш аналіз дозволяє припустити, що два скейлінґові режими можуть збігатися на незбуреному рівні. Ця точка зору підтверджується $1/N$-розкладом у межах масивного скалярного поля.