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ELASTIC SCATTERING OF LIGHT α-CLUSTER NUCLEI BY ^{12}C NUCLEI AT INTERMEDIATE ENERGIES

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The differential cross-sections for the elastic ^{12}C–^{12}C at 1016 MeV and ^{16}O–^{12}C at 1503 MeV scattering are calculated on the basis of the multiple diffraction scattering theory and α-cluster model with dispersion. For ^{12}C–^{12}C scattering the calculations were performed by means of the "effective" α–α amplitude. The differential cross-section for the elastic ^{16}O–^{12}C scattering was calculated without any papameter fit. The results obtained are in agreement with experimental data.

Key words: elastic scattering, light α-cluster nuclei, high energies, α-cluster model with dispersion, multiple diffraction scattering theory.

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I. INTRODUCTION

A theoretical investigation of light nuclei interaction is a useful method to obtain some information about the structure of colliding nuclei. The α-cluster structure is manifested in light nuclei with the number of nucleons divisible by 4 (^{12}C, ^{16}O, ^{20}Ne, etc).

For ^{12}C and ^{16}O nuclei the α-cluster model with dispersion has been proposed [1, 2]. According to this model, the carbon and oxygen nuclei are considered as made up of three and four α-clusters arranged at the vertices of equilateral triangle and tetrahedron, respectively. These α-clusters can be displaced from their most probable equilibrium positions.

By means of the α-cluster model with dispersion and Glauber–Sitenko multiple diffraction scattering theory (MDST) [3, 4] we have described the observables in elastic and inelastic scattering of intermediate energy elementary particles on ^{12}C and ^{16}O nuclei. The results of the calculations were in agreement with the experimental data. We show that taking into account four-nucleon correlations of the α-cluster type and the correlations between α-clusters allow us to obtain a better agreement with the experimental data as compared with the free-nucleon model [2]. Moreover, spin-rotation functions for the elastic scattering of protons on ^{12}C and ^{16}O nuclei in these approaches differ qualitatively.

Notice that to calculate the p–^{12}C and p–^{16}O elastic scattering observables, "free" p–α amplitudes have been used as an elementary "brick" of the model. In other words, the parameters of the "elementary" p–α amplitude have been obtained from the comparison of the calculated and measured p–^{4}He elastic scattering observables.

The model proposed in [1, 2] was developed for the case of the elastic scattering of "composite" particles (deuterons, α-particles) on ^{12}C and ^{16}O nuclei [5, 6]. We show that the "effective" d–α and α–α amplitudes must be used to agree the differential cross-sections for the elastic d–^{12}C and α–^{12}C scattering with the experimental data. This result is in agreement with the result obtained in [7, 8], where the authors show that elastic scattering of light nuclei on nuclei in the energy region $E \sim 100$ MeV/nucleon cannot be satisfactorily described in the multiple scattering theory with "elementary" free nucleon–nucleon amplitudes. In other words, complex nuclei change their properties during the collisions and "free" amplitudes should be changed to "effective".

Note that in [5, 6] the parameters of the "effective" d–α and α–α amplitudes were obtained from the comparison of the calculated and measured differential cross-sections for the elastic d–^{12}C and α–^{12}C scattering. These sets of parameters were used to calculate the differential cross-sections for the elastic d–^{16}O and α–^{16}O scattering. The results obtained are in agreement with experimental data.

Therefore, it would be interesting to investigate the scattering processes of light nuclei in which cluster structure is manifested. A number of approaches to investigate the elastic scattering of light nuclei by nuclei at intermediate energies was used by many authors (see, for example, [9, 10] and references therein).

So, in [9] the differential cross-section for the elastic ^{12}C–^{12}C scattering at 2400 MeV was calculated by means of the optical limit approximation of MDST and "effective" nucleon–target amplitude. The density of ^{12}C nucleus was approximated by the sum of Gaussians.

Recently we have introduced a method of calculation of the observables for the elastic scattering of intermediate energy particles (deuterons, ^{6}Li and ^{6}He) on ^{12}C nuclei [11]. The calculations have been performed under the assumption of two-cluster (for deuterons and ^{6}Li) and three-cluster (for ^{6}He) structure of incident particle. In the calculations the "effective" cluster–cluster (two or three for ^{6}Li–^{12}C and ^{6}He–^{12}C scattering, respectively) amplitudes were used and the Coulomb interaction was not taken into account. For ^{6}Li–^{12}C and ^{6}He–^{12}C elastic scattering it was shown that there are quantitative distinctions in the behaviour of the observables calculated in the above approach [11]. Note that in this approach the differential cross-section for the elastic d–^{12}C scattering was in agreement with experimental data.

In this paper the elastic ^{12}C–^{12}C and ^{16}O–^{12}C scattering at intermediate energy is studied on the basis of this model.

II. ELASTIC ^{12}C–^{12}C AND ^{16}O–^{12}C SCATTERING

In accordance to MDST the elastic scattering amplitude of three cluster system on target can be presented in the form

$$F(\mathbf{q}) = 3F_1(\mathbf{q}) + 3F_2(\mathbf{q}) - F_3(\mathbf{q}), \tag{1}$$

$$F_1(\mathbf{q}) = \frac{k}{k_\alpha} f(\mathbf{q}) S_1(\mathbf{q}), \tag{2}$$

$$F_2(\mathbf{q}) = \frac{ik}{2\pi k_\alpha^2} \int d^2q'\, f(\mathbf{q}') f(\mathbf{q}-\mathbf{q}') S_2(\mathbf{q},\mathbf{q}'), \tag{3}$$

$$F_3(\mathbf{q}) = \frac{k}{(2\pi)^2 k_\alpha^3} \int d^2q'\, d^2q''\, f(\mathbf{q}-\mathbf{q}'-\mathbf{q}'') f(\mathbf{q}') f(\mathbf{q}'') S_5(\mathbf{q},\mathbf{q}',\mathbf{q}''), \tag{4}$$

where $F_1(\mathbf{q})$, $F_2(\mathbf{q})$ and $F_3(\mathbf{q})$ is one-, two- and three-order amplitudes of incident α-clusters on the target nucleus, $f(\mathbf{q})$=$f_{\mathrm{C,O}}(\mathbf{q})$ is the elastic scattering amplitude of α-clusters of incident nucleus on α-clusters of the target, $\mathbf{q}$ is the transferred momentum, k is the wavevector of the incident nucleus, k_α is the wavevector of the α-cluster of incident nucleus.

In these formulae structure form-factors $S_1(\mathbf{q})$, $S_2(\mathbf{q})$ and $S_3(\mathbf{q})$ are

$$S_1(\mathbf{q}) = \int d^3\gamma_1\, d^3\gamma_2\, \rho_\Delta(\boldsymbol{\gamma_1}, \boldsymbol{\gamma_2})\, e^{-i\frac{2}{3}\mathbf{q}\mathbf{w}}, \tag{5}$$

$$S_2(\mathbf{q}) = \int d^3\gamma_1\, d^3\gamma_2\, \rho_\Delta(\boldsymbol{\gamma_1}, \boldsymbol{\gamma_2})\, e^{i\frac{1}{3}\mathbf{q}\mathbf{w}-\mathbf{q_1}\mathbf{s}}, \tag{6}$$

$$S_3(\mathbf{q}) = \int d^3\gamma_1\, d^3\gamma_2\, \rho_\Delta(\boldsymbol{\gamma_1}, \boldsymbol{\gamma_2})\, e^{-i\mathbf{q_2}\mathbf{w}-\mathbf{q_3}\mathbf{s}}, \tag{7}$$

where $\boldsymbol{\gamma_1}$ and $\boldsymbol{\gamma_2}$ are the Jacobi coordinates of the α-clusters of incident nucleus, $\mathbf{s}$ and $\mathbf{w}$ are the projections of the vectors $\boldsymbol{\gamma_1}$ and $\boldsymbol{\gamma_2}$ onto the plane perpendicular to the incident beam, $\mathbf{q}_1$=$\frac{1}{2}\mathbf{q}-\mathbf{q}'$, $\mathbf{q}_2$=$\frac{2}{3}\mathbf{q}-\mathbf{q}'-\mathbf{q}''$, $\mathbf{q}_3$=$\mathbf{q}'-\mathbf{q}''$.

The amplitudes $f_{\mathrm{C}}(\mathbf{q})$ and $f_{\mathrm{O}}(\mathbf{q})$ of the elastic α–^{12}C and α–^{16}O scattering are

$$f_{\mathrm{C}}(\mathbf{q}) = \frac{ik_\alpha}{2\pi} \int d^2b\, d^3\xi\, d^3\eta\, e^{i\mathbf{q}\mathbf{b}} \rho_\Delta(\boldsymbol{\xi}, \boldsymbol{\eta})\, \Omega(\mathbf{b}, \mathbf{s}_j), \tag{8}$$

$$f_{\mathrm{O}}(\mathbf{q}) = \frac{ik_\alpha}{2\pi} \int d^2b\, d^3\xi\, d^3\eta\, d^3\zeta\, e^{i\mathbf{q}\mathbf{b}} \rho_\Delta(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta})\, \Omega(\mathbf{b}, \mathbf{s}_j), \tag{9}$$

$$\Omega(\mathbf{b}, \mathbf{s}_j) = 1 - \prod_{j=1}^{N} \left[1 - \frac{1}{2\pi i k_\alpha} \int d^2q\, e^{-i\mathbf{q}(\mathbf{b}-\mathbf{s}_j)} \tilde{f}(\mathbf{q})\right], \tag{10}$$

where $\mathbf{b}$ is the impact parameter, $\mathbf{s}_j$ are the α-cluster coordinates of the target nucleus, $\tilde{f}_{\alpha\alpha}(\mathbf{q})$ is the elastic scattering α–α amplitude, N is equal 3 or 4 for ^{12}C and ^{16}O nuclei, respectively.

We have chosen the amplitude $\tilde{f}_{\alpha\alpha}(\mathbf{q})$ in the form

$$\tilde{f}_{\alpha\alpha}(\mathbf{q}) = k_\alpha \sum_{i=1}^{2} G_{ci} \exp(-\beta_{ci} q^2). \tag{11}$$

The parameters G_{c1} and β_{c1} are the fitting ones, and the parameters G_{c2} and β_{c2} are related with G_{c1} and β_{c1} by relations

$$G_{c2} = \frac{3iG_{c1}^2}{32\beta_{c1}}, \qquad \beta_{c2} = \frac{1}{2}\beta_{c1}. \tag{12}$$

The densities of the ^{12}C and ^{16}O nuclei are determined by

$$\rho_\Delta^{\mathrm{C}}(\boldsymbol{\xi}, \boldsymbol{\eta}) = \int d^3\xi'\, d^3\eta'\, \rho_0^C(\boldsymbol{\xi'}, \boldsymbol{\eta'})\Phi_\Delta^{\mathrm{C}}(\boldsymbol{\xi} - \boldsymbol{\xi'}, \boldsymbol{\eta} - \boldsymbol{\eta'}), \tag{13}$$

$$\rho_0^{\mathrm{C}}(\boldsymbol{\xi}, \boldsymbol{\eta}) = \frac{1}{4\sqrt{3}\pi^2 d^2}\delta(\xi - d)\delta\left(\eta - \frac{\sqrt{3}}{2}d\right)\delta(\boldsymbol{\xi\eta}), \tag{14}$$

$$\Phi_\Delta^{\mathrm{C}}(\xi, \eta) = \frac{1}{(\sqrt{3}\pi\Delta^2)^3}\exp\left(-\frac{\xi^2 + \frac{4}{3}\eta^2}{2\Delta^2}\right), \tag{15}$$

$$\rho_\Delta^{(\mathrm{O})}(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = \int d^3\xi'\, d^3\eta'\, d^3\zeta'\, \rho_0^{(O)}(\boldsymbol{\xi'}, \boldsymbol{\eta'}, \boldsymbol{\zeta'})\Phi_\Delta^{(\mathrm{O})}(\boldsymbol{\xi} - \boldsymbol{\xi'}, \boldsymbol{\eta} - \boldsymbol{\eta'}, \boldsymbol{\zeta} - \boldsymbol{\zeta'}), \tag{16}$$

$$\rho_0^{(\mathrm{O})}(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = \frac{1}{(4\pi)^2}\delta(\xi - d)\delta\left(\eta - \frac{\sqrt{3}}{2}d\right)\delta\left(\zeta - \sqrt{\frac{2}{3}}d\right)\delta(\boldsymbol{\xi\eta})\delta(\boldsymbol{\xi\zeta})\delta(\boldsymbol{\eta\zeta}), \tag{17}$$

$$\Phi_\Delta^{(\mathrm{O})}(\xi, \eta, \zeta) = \frac{1}{8(\pi\Delta^2)^9}\exp\left(-\frac{\xi^2 + \frac{4}{3}\eta^2 + \frac{3}{2}\zeta^2}{2\Delta^2}\right), \tag{18}$$

where $\boldsymbol{\xi}$, $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ are the Jacobi coordinates of the α-clusters of ^{12}C and ^{16}O nuclei. Parameters d and Δ characterize the distance between α-clusters and the probability of the α-cluster displacement from its most probable position at a vertex of equilateral triangle and tetrahedron, respectively. The values of parameters d and Δ obtained in [1, 2] allow us to describe the measured form factors of ^{12}C and ^{16}O nuclei up to the values of transferred momenta $q \leq 3$ Fm^{-1}.

On the basis of the above approach we have calculated the differential cross-sections $\sigma/\sigma_{\mathrm{Ruth}}$ for the elastic ^{12}C–^{12}C scattering at 1016 MeV and ^{16}O–^{12}C scattering at 1503 MeV energy. The results obtained are presented in the Fig 1.

For the elastic ^{12}C–^{12}C scattering the calculations have been carried out using the "effective" α–α amplitude with the parameters obtained from the comparision of the calculated and measured ^{12}C–^{12}C elastic scattering differential cross-sections. The values of the parameters are: $G_{c1} = -0.909 + i\ 3.342$ and $\beta_{c1} = 0.865 + i\ 0.992$.

This set of parameters has been used to calulate the differential cross-section for the elastic ^{16}O–^{12}C scattering at 1503 MeV, i. e., this differential cross-section has been calculated without any parameter fit.

As can be seen from Fig. 1, the results obtained are in agreement with experimental data. The distinctions between the calculated and measured cross-sections

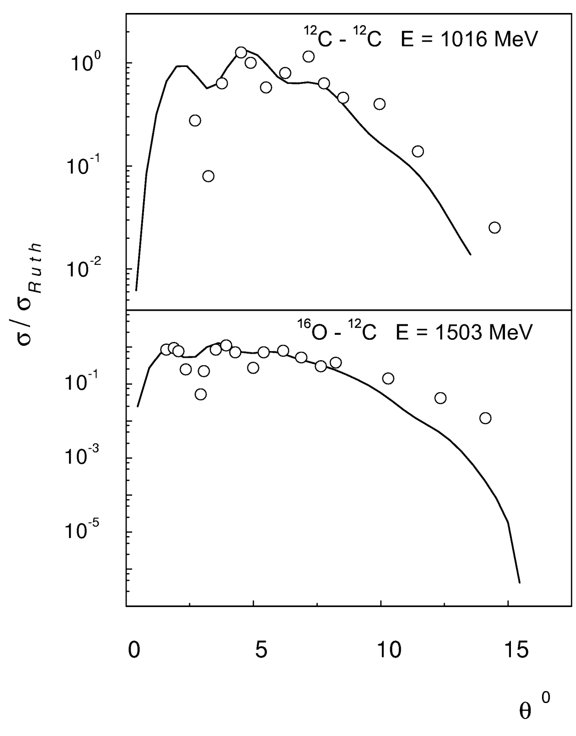

Fig. 1. Differential cross-section $\sigma/\sigma_{\mathrm{Ruth}}$ for the elastic scatttering of ^{12}C and ^{16}O nuclei by ^{12}C nuclei at 1016 and 1503 MeV energies, respectively. Experimental data from [12, 13].

in the region of the first minimum is due to the neglecting of the Coulomb interacion, and in the region of $\theta \geq 8^\circ$ is due to the fact that in this angular region the deviation of cluster model is pronounced, i. e., the influence of nucleon–nucleon interaction is significant.

III. CONCLUSION

In [11] the method of calculations of the differential cross-section for the elastic scattering of intermediate energy weekly-bound particles on α-cluster nuclei has been proposed. The calculations have been fulfilled by means of the α-cluster model with dispersion and MDST with "effective" cluster–cluster amplitude as an elementary "brick" of the model. These calculations have been performed under the assumption of two-cluster (for deuterons and ^{6}Li) and three-cluster (for ^{6}He) structure of incident particle. For ^{6}Li–^{12}C and ^{6}He–^{12}C elastic scattering it is shown that there are quantitative distinctions in the behaviour of the observables calculated in the above approach [11], and the differential cross-section for the elastic d–^{12}C scattering was in agreement with experimental data. Unfortunately, the lack of experimental data does not allow us to make conclusions about the manifestation of two- and three-cluster mode in ^{6}He nuclei.

The development of this approach allows us to agree the calculated and measured data for the elastic ^{12}C–^{12}C and ^{16}O–^{12}C scattering. The differential cross-section for the elastic ^{16}O–^{12}C scattering was calculated without any parameter fit.

We realise that ^{6}He and ^{12}C nuclei differ significantly due to structure distinctions and binding energies. However, the agreement between the calculated and measured elastic scatterig observables both for weekly-bound deuterons and strongly bound ^{4}He, ^{12}C and ^{16}O nuclei on α-cluster ^{12}C nuclei allows us to hope that the approach proposed can give information about the structure both of weekly-bound and strongly-bound particles. Moreover, experimental measurements of the observables for the elastic scattering of light α-cluster nuclei by similar nuclei at higher energies would give information about the structure of these nuclei and mechanisms of cluster–cluster interaction inside nuclear matter.

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ПРУЖНЕ РОЗСІЯННЯ ЛЕГКИХ α-КЛАСТЕРНИХ ЯДЕР ЯДРАМИ ^{12}C ПРИ СЕРЕДНІХ ЕНЕРҐІЯХ

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Диференційні перерізи пружного розсіяння ядер ^{12}C при енерґії 1016 МеВ та ядер ^{16}O при енерґії 1503 МеВ ядрами ^{12}C розраховано на основі теорії багатократного дифракційного розсіяння та α-кластерної моделі з дисперсією. Для пружного ^{12}C–^{12}C розсіяння розрахунки було виконано з використанням "ефективної" α–α амплітуди. Диференційний переріз пружного ^{16}O–^{12}C розсіяння розраховано без параметрів, що підганяються. Отримані результати дозволяють описати наявні експериментальні дані.