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ESTIMATION OF THE TRIANGULATION MEASUREMENT ACCURACY IN THE PROJECT THE "INTERPLANETARY SOLAR STEREOSCOPIC OBSERVATORY"

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The simulation of the observational process has been made with the purpose of evaluating efficiency of stereoscopic measurements of the Solar system body's positions with the application of the Interplanetary Solar Stereoscopic Observatory (ISSO) instruments in the stereoscopic mode. Pluto is adopted as the model object. The orbit obtained by integrating equations of motion using a series of nine observations with a one day interval allows to predict the object's position up to the period of five years with an error not exceeding $20''$.

Key words: space triangulation, stereoscopic space project.

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I. THE INTRODUCTION

Triangulation measurements of the positions within the limits of a Solar system and determination of the nearest stars parallaxes from synchronous observations in the stereoscopic mode with instruments of the Interplanetary Solar Stereoscopic Observatory (ISSO) [1–3] is a part of the scientific program of the project. The medium size astrographs are designed for observation from both spacecraft the direct exposures of star fields. The required minimum aperture is 50–70 cm, attainable observable magnitude $V_{\rm lim}$ is approximately 21^m. An additional small astrometric instrument is planned as the star sensor for the purposes of autonomous navigation and for making auxiliary observations (the aperture up to 30 cm, limiting stellar magnitude is $\sim 17^m$). Further modelling estimations of the expected accuracy are presented. It is supposed that the measurements are made in the coordinate system of a high-precision astrometric catalogue and the following software is placed onboard the both spacecraft, working in the autonomous mode:

1) The software package for calculation of the ephemeris positions of the Earth, Moon, major planets and their satellites, asteroids having orbits in a system of the theories DE200/LE200 or one of the later versions;

2) Catalogue and software for calculation of the astrographic positions of star and their brightness over the whole sky;

3) The numerical theory of the motion of zero mass bodies (both space vehicles are considered as the third body in the three body problem of Celestial Mechanics) in the vicinity of the triangular libration centers in the system "Sun — barycenter of Earth+Moon";

4) An onboard processor with conforming operating system and timing system.

Components (1) and (2) define the onboard ephemeris as the basis of the project coordinate system. It is known that the navigation of space vehicles in the far space is possible only relatively to the Solar system bodies' directions projected onto celestial sphere, and defined by a high-precision catalogue. Evidently, the application of the CCD-registration will demand a modern catalogue with the star density of about 1 star per 4 square minutes.

II. GENERAL ESTIMATION OF THE STEREOSCOPIC METHOD ACCURACY

In Fig. 1 the simple stereoscope scheme is shown as designed to evaluate the accuracy estimation of the measured distance to a planet using the stereoscopic base B.

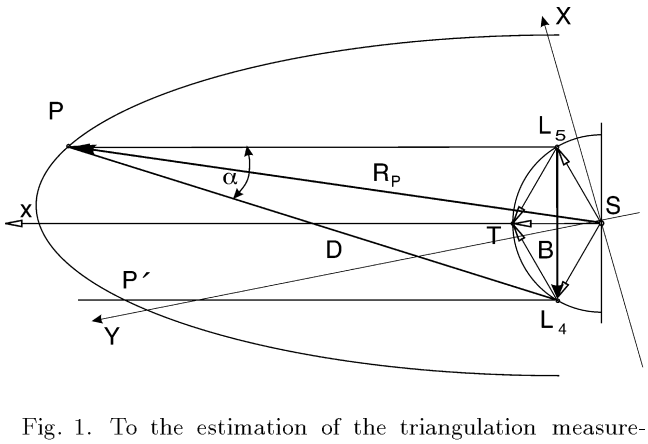

Fig. 1. To the estimation of the triangulation measurements accuracy.

The observations are made simultaneously aboard the spacecrafts set in the vicinity of the Lagrange circular libration centres in the system "the Sun (S) — Earth — Moon barycenter (T)". The base vector $B = L_5L_4$ lies exactly in the plane of the ecliptic SXY, and it is monitored by the on-board equipment [3] by measuring the directions of the vectors SL_5, SL_4, L_5T, L_4T in the barycentric frame of reference $SXYZ$. The position vector of the barycenter T at any required moment is calculated on the basis of theories of motion derived in the Jet Propulsion Laboratory [4], thus its value (the modulus) and direction are known. Obviously, when the vector ST is known, the moduli of the vectors SL_5, SL_4, L_5T, L_4T become known, as well as the vector $\mathbf{R}_p$ of the position of the planet in question, through a simple triangulation solution.

Let Sx be the line of crossing of the plane of ecliptic with a plane perpendicular to the base vector B. Then other two planes, which are also perpendicular to B and go through its ends L_5 and L_4, respectively, would form the boundaries of the area, where the stereoscopic measurement is carried out with optimum precision.

Let us suppose, that the planet P is observed stereoscopically, so that the perpendicular from its center on the base vector B is L_5P. We have the rectangular triangle PL_5L_4, where the side B is known and the angle $P = \alpha$ has been measured, and the angle L_5 is the right one.

We have the elementary formula fitting the sides and the angle α in this triangle (the reduction for the aberration and the light-time has yet to be made):

$$\sin\alpha = \frac{B}{D}, \tag{1}$$

where D is the length of hypotenuse or the distance of a planet from point L_4.

Differentiation of right-hand and left-hand members of equation (1) gives

$$\cos\alpha \cdot d\alpha = \frac{D \cdot dB - B \cdot dD}{D^2}. \tag{2}$$

The equation for the increment dD as a function of increments $d\alpha$ and dB and of the measured distance D becomes

$$dD = \frac{D}{B} \cdot dB - \frac{D^2}{B}\cos\alpha \cdot d\alpha, \tag{3}$$

which may be represented by the equivalent dispersion variations of the measured values (neglecting the high order terms):

$$(\delta D)^2 = \left(\frac{D}{B}\right)^2 \cdot (\delta B)^2 + \left(\frac{D^2}{B}\cos\alpha\right)^2 \cdot (\delta\alpha)^2. \tag{4}$$

From equation (4) it is evident, that the contribution to the error of the unknown distance D, made by an error of determination of the base B, is proportionate to the first power of D/B. The contribution to this error made by the inaccuracy of angular measurements, is proportionate to D^2/B. Hence the increase of the angular measurements accuracy is essentially significant for increase of the accuracy of triangulation measurements. Evidently, the error decreases with increase of base B. At the same time it's monitoring must be performed.

III. DEFINITION OF THE POSITIONAL VECTOR OF THE SOLAR SYSTEM OBJECT

Let us measure now in the mode of the stereoscope the position of the planet P. In Fig. 2 the orbit of the planet $NP_1P_2P_3$ is shown together with its projection to ecliptic plane $NP_1'P_2'$. The vector of base $B = L_4L_5$ and two angles α and β at the base in the triangle $L_4L_5P_1$ are known. The vectors $\overrightarrow{OL_5}, \overrightarrow{OL_4}, \overrightarrow{OT_3}, \overrightarrow{L_4T_3}, \overrightarrow{L_5T_3}$ are known as well. The directions of the vectors $\mathbf{R}_{4p_1} = \overrightarrow{L_4P_1}$, $\mathbf{R}_{5p_1} = \overrightarrow{L_5P_1}$ are measured also. The formulas for calculation of moduli of these vectors are simply derived as the solution of the plane triangle $L_4L_5P_1$.

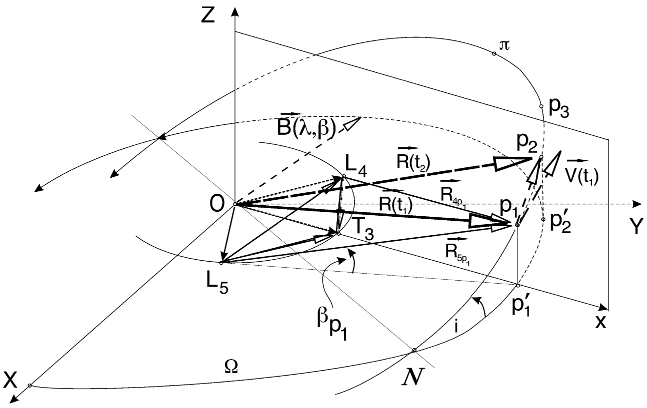

Fig. 2. A model of triangulation measurements of the positional vector of the object P in Solar system used for simulation.

The final formulas for calculation of vector moduli $\mathbf{R}_{4_{p1}}$ and $\mathbf{R}_{5_{p1}}$ are:

$$\left.\begin{aligned} \left|\mathbf{R}_{4_{p1}}\right| &= |\mathbf{B}|\tfrac{\sin\beta}{\sin p} \\ \left|\mathbf{R}_{5_{p1}}\right| &= |\mathbf{B}|\tfrac{\sin\alpha}{\sin p} \end{aligned}\right\}. \tag{5}$$

Here α is the angle $P_1L_4L_5$, β is the angle $L_4L_5P_1$, p is parallax angle at the vertex P_1, $\mathbf{B}$ is the vector of base.

The proper barycentric position vector of the planet at the moment t_1 is calculated as the sum of vectors:

$$\mathbf{R}(t_1) = \rho_4(t_1) + \mathbf{R}_{4_{p1}} = \rho_5(t_1) + \mathbf{R}_{5_{p1}}, \tag{6}$$

where $\rho_4(t_1)$, $\rho_5(t_1)$ are the radius-vectors of space vehicles at the moment t_1. Thus

$$\rho_4(t_1) - \rho_5(t_1) = \mathbf{B}(t_1). \tag{7}$$

The repeated observation of the same planet at another (near) moment t_2 is followed by the determination of barycentric radius-vector $\mathbf{R}\,(t_2)$ and determination of the vectors difference

$$\Delta\mathbf{R}\left(t_1+\frac{t_2-t_1}{2}\right)=\mathbf{R}\,(t_2)-\mathbf{R}\,(t_1)\,. \qquad (8)$$

If the object has been observed for the first time, then, in order to find it again in the future, it is necessary to construct its orbit; this is possible at least from two barycentric observations [5].

To increase the accuracy of such measurements it is proposed to determine more precisely the velocity vector of the body observed for the first time. For this purpose it is proposed to perform a series of observations of this body with approximately diurnal interval between observations which can be easy realized in orbital conditions. Further, the series of barycentric position vectors $\{\mathbf{R}\,(t_i)\,,\;\; i=0,1,2,\ldots,n\}$ is representable as the sum of a smooth vector function and a random vector-function of noises. The statistical smoothing allows to receive a mean weighted barycentric position vector (the filtered value of a smooth vector function at the mean moment of a series) and the velocity vector of a planet. The accuracy of a mean weighted values from the series of observations will be approximately $\sqrt{n}$ times higher than the accuracy of a single observation. The orbit as constructed on the basis of the results of the series observations will be more precise and veritable.

IV. OUTCOME OF SIMULATION

To show the correctness of the previous statement and to evaluate the efficiency of such series we make a simulation of the problem making some simplifications.

Pluto is taken as a model object. Its orbit in the system DE200/LE200 has been taken as the accurate one.

It is supposed that the observation of a series was made near the plane perpendicular to the base of the stereoscope with the symmetrical distribution of the time moments relatively to the time moment of intersection of this plane by Pluto. The modelled orbit will have this time moment as the time moment of osculation. The general arrangement of the simulation is shown in Fig. 2. In Fig. 3 the position of Pluto in the orbit is given at the epoch 2005, May, 19, 0h UT. The ephemeris positions of Pluto and the ephemeris positions of the "observers" (the spacecraft in points L_4 and L_5) have been calculated for 9 moments of modelled observations with an interval of 1 day, — all in the vertices of the triangle $L_4T_3L_5$. The directions $\mathbf{R}_{4p_1}$ and $\mathbf{R}_{5_{p1}}$ for each of the time moments, considered in the presented simulation as the true ones, have been computed. To receive their "observed" analogues, "the true" values have been noised by the random error of the angular observations, chosen with the help of the generator of the random numbers, in the supposition of the plane distribution of probability in a circle of radius 3σ, where $\sigma=\pm 0''.03$ — the expected error of a single direction determination from one exposure with the CCD-micrometer.

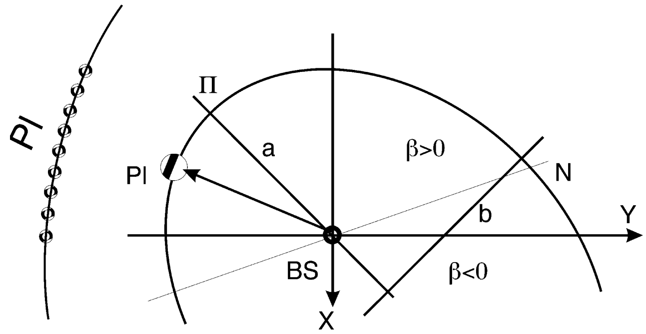

Fig. 3. The scheme of the positions of a modelled object on the example of Pluto's orbit.

Further "observations" have been smoothed as it was said above. The values of the radius-vector and the velocity are obtained at the epoch of oscilation. The orbit for the "observed" planet has been computed after the Everhart's method. The estimation of accuracy is made in terms of "observation minus calculation". The results are shown in Fig. 4.

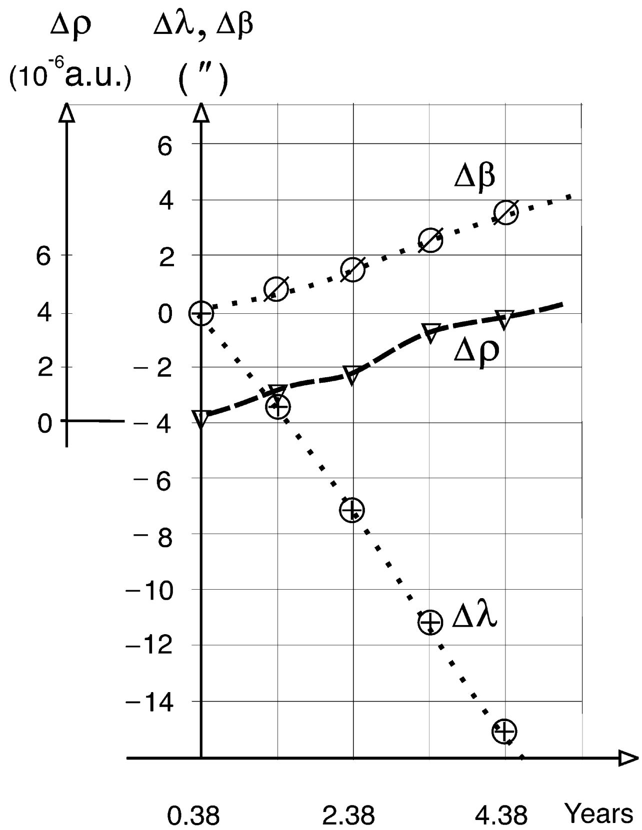

Fig. 4. The prediction of errors of the model orbit obtainned from a 9-point series of observations. Time zero point is 2005, January, 0, 0h UT.

V. CONCLUSION

The model orbit obtained from a 9-point series of stereoscopic observations performed in 8–10 days, allows to predict the position of a new (unknown) object with the accuracy of 20 arc seconds within the interval of 5–6 years. The repetition observations next year when the planet will again appear near the plane perpendicular to the stereoscope base vector should essentially improve the orbit.

It is very essential to supply the highest angular measurements accuracy in the on-board navigational instrumental complex of the project. Continuous monitoring of each spacecraft location is possible with using the angular observation of the planets and must be performed.

VI. ACKNOWLEDGEMENT

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ОЦІНКА ТОЧНОСТИ ТРІАНҐУЛЯЦІЙНИХ ВИМІРЮВАНЬ У ПРОЄКТІ “МІЖПЛАНЕТНА СОНЯЧНА СТЕРЕОСКОПІЧНА ОБСЕРВАТОРІЯ”

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Здійснено моделювання процесу спостережень з метою оцінки ефективности стереоскопічних вимірювань розташування тіл Сонячної системи за допомогою стереоскопічного режиму роботи інструментів Міжпланетної сонячної стереоскопічної обсерваторії. Як модельний об'єкт використано Плутон. Орбіта, отримана інтеґруванням рівнянь руху з використанням ряду з дев'яти спостережень з одноденним інтервалом, дає змогу передбачити положення об'єкта на період часу до п'яти років з похибкою, яка не перевищує $20''$.