

MODEL FOR SYNCHROTRON X-RAYS FROM SHELL SUPERNOVA REMNANTS IN NONUNIFORM INTERSTELLAR MEDIUM AND NONUNIFORM MAGNETIC FIELD

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A possibility to model the high energy synchrotron emission (in X- and γ -rays) from supernova remnants (SNRs) is an important task for modern astronomy and astrophysics because it may be responsible for the nonthermal X-rays and TeV γ -rays observed recently from a number of SNRs. This emission allows us to observe the processes of particle acceleration on SNR shocks and generation of cosmic rays. In this paper a model for the synchrotron emission from shell SNR in nonuniform interstellar medium and nonuniform magnetic field is presented. This model is a generalization of the model of Reynolds & Chevalier developed for a spherical SNR in the uniform medium and uniform magnetic field. The model will be used for studies on the thermal and nonthermal images and spectra from nonspherical SNRs in different interstellar magnetic field configurations.

Key words: interstellar medium, supernova remnants, hydrodynamics, shock wave, X-rays, synchrotron emission.

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I. INTRODUCTION

For the last few years, a considerable interest has grown in the investigation of the synchrotron X-ray emission from supernova remnants (SNRs), caused by electrons accelerated on SNR shocks up to the energy $\sim 10^{13\div 14}$ eV. This emission process is believed to be responsible for the nonthermal X-ray (SN 1006, Cas A, Tycho, G347.3-0.5, IC443, G266.2-1.2, RCW86: G156.+5.7, Kepler, W49B, W50) and TeV γ -ray (SN 1006, Cas A, G347.3-0.5) emission observed recently from a number of SNRs.

The model for synchrotron X-rays from shell SNRs developed up till now [1,2] works with uniform magnetic field and is based on the simple spherical Sedov model [3] for SNR hydrodynamics. The most essential simplifications of the Sedov model are the uniform interstellar medium and spherical SN explosion.

Observations certify that SNRs are nonspherical objects with complicated distributions of surface brightness. The nonuniform interstellar medium are the most important factor in modification of the structure and evolution of SNRs. We present here a model for the synchrotron emission from shell SNR in nonuniform medium and nonuniform magnetic field. This model is a generalization of the model [1,2].

side the volume of SNR depend on the ambient density distribution $\rho^\circ(r, \theta, \varphi)$ and anisotropy of the supernova explosion energy $E_{\text{sn}}(\theta, \varphi)$. It is enough to have parameters of gas within $R(\theta, \varphi)$ calculated to model the thermal X-ray emission of SNR. Besides $\rho^\circ(r, \theta, \varphi)$ and $E_{\text{sn}}(\theta, \varphi)$, the synchrotron image and spectrum of SNR are also affected by the distribution of interstellar magnetic field $\mathbf{B}_o(r, \theta, \varphi)$ which may also be nonuniform.

It is assumed hereafter that SNR are on the adiabatic phase of evolution. The dynamics of the shock front of nonspherical adiabatic SNR and the distribution of gas parameters inside whole shocked volume can be described with the hydrodynamic method presented in [4]. In this method, a three-dimensional (3-D) object is divided on a number of 1-D sectors and the distributions of parameters are found separately for each of them. Note, that the velocity of flow is radial in the hydrodynamic method [4] and the direction of a sector and electron velocity in it are given by the same angles (θ, φ) of a spherical coordinate system with the origin in the place of explosion.

We assume also that $\gamma = 5/3$, shock compression ratio $\sigma = 4$, gyrofactor $\eta = 1$, $B_{\text{CMB}}/B_o \rightarrow 0$, maximum energy attained before the adiabatic stage $E_{\text{max},0} = 0$ (see [2] for details). We do not consider the effect of the cosmic ray pressure that can change σ .

II. MODEL FOR SYNCHROTRON EMISSION FROM NONSPHERICAL SNR IN NONUNIFORM MAGNETIC FIELD

A. Hydrodynamics

The position $R(\theta, \varphi)$ and velocity $D(\theta, \varphi)$ of the shock front as well as the distribution of gas parameters in-

B. Emission

The synchrotron volume emissivity of a fluid element (r, θ, φ) is [5]

$$S_\nu = C(\alpha) K B_\perp^{(\alpha+1)/2} \nu^{-(\alpha-1)/2}, \quad (1)$$

where $C(\alpha)$ is a constant, B_{\perp} is the tangential component of the magnetic field (perpendicular to the electron velocity in the fluid element), ν is frequency, K is the normalization of the sharply truncated electron distribution

$$N(E)dE = \begin{cases} KE^{-\alpha}dE, & E \leq E_{\max} \\ 0, & E > E_{\max} \end{cases} \quad (2)$$

Let us consider the distribution of the emission parameters *over the shell*, and their evolution *downstream*.

1. Spectral index

The power index α is constant *downstream* because electrons lose energy proportionally to their energy (Eq. (10)) and remain essentially confined to the fluid element in which they were produced. The power index α is also constant *over the surface* of a nonspherical SNR because the shock is strong. Namely, in the first order Fermi acceleration mechanism, $\alpha = (\sigma + 2)/(\sigma - 1)$, where the shock compression ratio σ does not depend on the ambient density distribution in the strong shock limit (Mach number $\gg 1$) realized in SNRs: $\sigma = \rho_s/\rho_s^o = (\gamma + 1)/(\gamma - 1)$, the subscripts "s" and "o" refer to the values at the shock and to the surrounding medium, respectively.

2. Magnetic field components

The components of the magnetic field evolve differently behind the shock: B_{\perp} rises everywhere *at the shock* by the same factor $\sigma = \rho_s/\rho_s^o$ and B_{\parallel} is not modified by the shock [1]:

$$B_{\perp,s} = B_{\perp,s}^o \cdot \frac{\rho_s}{\rho_s^o}, \quad B_{\parallel,s} = B_{\parallel,s}^o. \quad (3)$$

No further turbulent amplification of the magnetic field is assumed.

Downstream variation of the components follows from the flux-freezing condition $B_{\perp}(r)r dr = B_{\perp}^o(a)a da$, continuity equation $\rho(r)r^2 dr = \rho^o(a)a^2 da$ and magnetic flux conservation $r^2 B_{\parallel}(r) = a^2 B_{\parallel}^o(a)$, where a is Lagrangian and $r = r(a, t)$ is Eulerian coordinates. Thus, if both B_o and ρ^o are nonuniform, the components of magnetic field B are (in a sector)

$$B_{\perp}(a, t) = B_{\perp}^o(a) \frac{\rho(a, t)}{\rho^o(a)} \frac{r(a, t)}{a},$$

$$B_{\parallel}(a, t) = B_{\parallel}^o(a) \left(\frac{a}{r(a, t)} \right)^2. \quad (4)$$

The components of the ambient magnetic field \mathbf{B}_o in the point (r, θ, φ) are

$$B_{\perp}^o = B_o |\sin \Theta_o|, \quad B_{\parallel}^o = B_o |\cos \Theta_o|, \quad (5)$$

where the obliquity angle Θ_o is the angle between the directions of the magnetic field \mathbf{B}_o in this point and the electron velocity in the sector.

3. Normalization K

The assumption that the energy density ω of relativistic particles is proportional to the energy density of the magnetic field in each fluid element

$$\omega \equiv \int_{E_{\min}}^{E_{\max}} EN(E)dE = K \int_{E_{\min}}^{E_{\max}} E^{1-\alpha} dE \propto B^2 \quad (6)$$

gives for the variation of K *over the shell*, and *downstream*

$$K \propto \begin{cases} B^2 (E_{\max}^{2-\alpha} - E_{\min}^{2-\alpha})^{-1}, & \alpha \neq 2 \\ B^2 \ln^{-1} (E_{\max}/E_{\min}), & \alpha = 2. \end{cases} \quad (7)$$

4. Injection energy

Particles start to accelerate on the shock when their energy E become more than the injection energy E_{\min} . The first order Fermi acceleration mechanism suggests for E_{\min} : $E_{\min} = 1.68 \cdot 10^{-4} \ln \Lambda^{2/3} (\lambda_o n_{\text{tot}}/D)^{2/3}$ eV, where $\ln \Lambda$ is Coulomb logarithm, $\lambda_o = 1.0 \cdot 10^{18} T_6^2/n_{\text{tot}}$ cm is the mean free path of electron, n_{tot} is the total number density (electrons plus ions) [6]. This E_{\min} is $14.65 \ln \Lambda^{2/3}$ times the average kinetic energy of electrons.

Variation of E_{\min} *over the shell* of nonspherical SNR found for each sector independently:

$$E_{\min}(R, \theta, \varphi) = 0.026 \ln \Lambda^{2/3} D_3(\theta, \varphi)^2 \text{ MeV}, \quad (8)$$

where D_3 is the shock velocity in 10^3 km/s.

If particles move *downstream* leaving the region of acceleration, the energy varies as in [2]

$$E(a) = E_o \bar{\rho}(a)^{1/3}, \quad (9)$$

if electron loses its energy due to the adiabatic expansion only

$$\dot{E} = E \dot{\bar{\rho}}/(3\bar{\rho}) \quad (10)$$

where $\bar{\rho}(r) = \rho(r)/\rho_s$, E_o is the energy of the fluid element a at the time t_o when the shock crosses it.

5. Maximum energy

The maximum energy E_{\max} which electrons are accelerated to varies *over the shell* of nonspherical SNR. Therefore, E_{\max} are also found independently in each sector, with the expressions given in [2]: $E_{\max}(R, \theta, \varphi) = \min(E_{\max,1}, E_{\max,2}, E_{\max,3})$ where the radiative losses, finite time of acceleration and escaping of electrons allow for the maximum energies:

$$E_{\max,1} = 7.8 \cdot 10^{13} F_1 B_{o,\mu}^{-1/2} D_3 \text{ eV}, \quad (11)$$

$$E_{\max,2} = 1.9 \cdot 10^{14} \int_{t_{\text{ad}}}^t F_2 B_{o,\mu} D_3^2 dt_3 \text{ eV}, \quad (12)$$

$$E_{\max,3} = 7.7 \cdot 10^{12} \lambda_{\max,17} B_{o,\mu} \text{ eV}. \quad (13)$$

Here the age of SNR t and the time of transition into the adiabatic stage $t_{\text{ad}}(\theta, \varphi)$ are in 10^3 years, $B_{o,\mu}$ is B_o in μG , $\lambda_{\max,17}$ is λ_{\max} in 10^{17} cm. This $\lambda_{\max} \sim 10^{16 \div 18}$ cm is a free parameter.

The geometric factors are [2]

$$F_1 = \sqrt{G/R_j}, \quad F_2 = 1/R_j, \quad (14)$$

with

$$G = \frac{Z + 4/\sigma_B}{Z + 4\sigma_B}, \quad Z = \frac{\cos^2 \Theta_o + 1}{\cos^2 \Theta + 1},$$

$$R_j = \frac{1}{2} \frac{\sigma_B (\cos^2 \Theta_o + 1) + 4(\cos^2 \Theta + 1)}{4 + \sigma_B}, \quad (15)$$

$$\sigma_B \equiv \frac{B_s}{B_s^o} = \sqrt{\frac{1 + 16 \tan^2 \Theta_o}{1 + \tan^2 \Theta_o}},$$

where the obliquity angles are Θ_o for upstream and Θ for downstream; $\tan \Theta = 4 \tan \Theta_o$.

The maximum energy changes *downstream* in accordance with (9).

6. Obliquity angle

Synchrotron emission of the fluid element (r, θ, φ) depends on the values $B_o(r, \theta, \phi)$ and $\Theta_o(r, \theta, \phi)$. The calculation of Θ_o depends on the way how the distribution $\mathbf{B}_o(r, \theta, \phi)$ is given.

If we know distributions of $B_o(r, \theta, \phi)$, $\theta_H(r, \theta, \phi)$ and $\varphi_H(r, \theta, \phi)$, where (θ_H, φ_H) are the spherical angles of the vector \mathbf{B}_o in the point (r, θ, ϕ) , then

$$\cos \Theta_o = \sin \theta \sin \theta_H \cos(\varphi - \varphi_H) + |\cos \theta \cos \theta_H|. \quad (16)$$

There are three independent of a coordinate system angles in our task: the inclination angle δ between the line of sight and the density gradient, the aspect angle ϕ between the line of sight and the ambient magnetic field, and the third angle ξ between the magnetic field and the density gradient.

Let us consider two interesting particular cases.

i). If the direction of the ambient magnetic field is uniform, i. e. $\phi(r, \theta, \phi) = \text{const}$, then let us accept that the origin of the coordinates coincides with the place of explosion, the axis x of Cartesian coordinate system is oriented opposite the direction of the line of sight (towards the observer) and that \mathbf{B}_o lies in the plane (xz) . In such a case $\cos \Theta_o$ is given by (16) with $\theta_H = \pi/2 - \phi$ and $\varphi_H = 0$.

ii). If direction of the density gradient is uniform, i. e. $\delta(r, \theta, \phi) = \text{const}$, then let us accept that the origin of the coordinates coincides with the place of explosion, axis z of Cartesian coordinate system is oriented opposite the density gradient and the line of sight lies in the plane (xz) . Geometrical consideration yields for $\delta \neq 0$ and $\xi \neq 0$

$$\cos \Theta_o = \sin \theta \sin \xi \cos(\varphi - \varphi_H) + |\cos \theta \cos \xi|,$$

$$\text{where } \cos \varphi_H = \frac{\cos \phi - |\cos \delta \cos \xi|}{\sin \delta \sin \xi}. \quad (17)$$

If $\delta = 0$ or π ($\text{grad } \rho^o$ is parallel to the line of sight) but $\xi \neq 0$, then $\theta_H = \pi - \xi$ and $-\xi$ respectively, and additionally $\varphi_H(r, \theta, \varphi)$ should be known. If $\xi = 0$ or π (direction of \mathbf{B}_o coincides with $\text{grad } \rho^o$), then $\Theta_o = \pi - \theta$ or θ respectively.

III. CONCLUSIONS

The model for the synchrotron emission of nonspherical SNR in nonuniform interstellar magnetic field is presented. Being applied to studies of high energy emission of SNRs, in particular to X-rays, it allows us to compare the thermal and synchrotron X-ray images and spectra of the objects, to make conclusions about the SNR itself, supernova explosion, structure and properties of the interstellar medium, shock wave physics, acceleration processes on the shocks, γ -ray and cosmic ray generation.

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**МОДЕЛЬ СИНХРОТРОННОГО РЕНТГЕНІВСЬКОГО ВИПРОМІНЮВАННЯ
ВІД ОБОЛОНКОПОДІБНИХ ЗАЛИШКІВ НАДНОВИХ ЗІР У НЕОДНОРІДНОМУ
СЕРЕДОВИЩІ**

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Моделювання високоенергетичного синхротронного випромінювання (в рентгенівському та гамма-діапазоні) залишків наднових зір (ЗН) є важливим завданням сучасної астрономії й астрофізики, оскільки синхротронне випромінювання електронів, прискорених на фронті ударної хвилі до енергій $\sim 10^{13-14}$ eV, може бути відповідальним за нетеплове рентгенівське та гамма-випромінювання, спостережене нещодавно від низки ЗН. Цей механізм випромінювання дозволяє вивчати процеси прискорення електронів на фронтах ударних хвиль та процеси генерації галактичних космічних променів.

У цій статті вперше подано модель для розрахунку синхротронного випромінювання від оболонкоподібних ЗН, які еволюціонують у неоднорідному середовищі та неоднорідному магнетному полі. Вона є узагальненням моделі Рейнольдса та Шевальє, розробленої для сферичного ЗН в однорідному середовищі та магнетному полі. Модель буде використано для аналізу й порівняння теплового та нетеплового випромінювання несферичних ЗН у міжзоряних магнетних полях різної конфігурації.