ЖУРНАЛ ФІЗИЧНИХ ДОСЛІДЖЕНЬ
т. 9, № 4 (2005) с. 364-373

JOURNAL OF PHYSICAL STUDIES
v. 9, No. 4 (2005) p. 364-373

ON THE TRANSITION OF THE ADIABATIC SUPERNOVA REMNANT TO THE RADIATIVE STAGE IN A NONUNIFORM INTERSTELLAR MEDIUM

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(Received October 3, 2005; in final form February 28, 2006)

Methods for estimating of different reference times which appear in the description of transition of a strong adiabatic shock into the radiative era are reviewed. The need for consideration of an additional transition subphase in between the end of the adiabatic era and the beginning of the radiative "pressure-driven snowplow" stage for a shock running in the uniform or nonuniform medium is emphasized. This could be of importance in particular for studying the interaction of supernova remnants (SNRs) with molecular clouds and therefore for understanding the processes of the cosmic ray production in such systems. The duration of this subphase — about 70% of the SNR age at its beginning — is almost independent on the density gradient for media with increasing density and is longer for higher supernova explosion energy and for smaller density in the place of explosion. It is shown as well that if the density of the ambient medium decreases then the cooling processes could differ from the commonly accepted scenario of the "thin dense radiative shell" formation. This property should be studied in the future because it is important for models of nonspherical SNRs which could be only partially radiative.
Key words: supernova remnants, hydrodynamics, nonuniform medium, radiative shock

PACS number(s): 98.38.Mz, 95.30.Lz

I. INTRODUCTION

Physical processes accompanying the evolution of supernova remnants (SNRs) is a complex system. It is almost impossible to account for all of them in a single model of SNR. Therefore, the whole evolution of SNR from a supernova explosion until the mixing of a very old object with the interstellar matter is divided on a number of the model phases (e. g. [1–3]): the free-expansion, adiabatic, radiative and dissipation stages. There are some physical processes important during a given stage, some others could be neglected. Such an approach allows for a rather simple analytical description of SNR evolution during each phase.

The role of radiative losses, which is negligible in the adiabatic phase of SNR evolution, becomes more and more promiment with time. They are so important in old SNRs that they essentially modify the dynamics of such SNRs. Theoretical systematization of timescales and the role of different physical processes in the cooling of adiabatic SNR was first reviewed in [4]. In general, the transition to the radiative stage can be studied numerically by following the history of the shocked flow as it is done e. g. by [5–10]. The analytical treatments are of great importance as well, e. g. [11–16].

The physical processes in the radiative blast wave, namely, quick cooling of an incoming flow and formation of the thin dense cold shell which moves due to the pressure of internal gas makes the so called "pressure-driven snowplow" (PDS) model within the "thin-layer" approximation to be adequate for the description of this stage of SNR evolution [15, 17, 18].

The PDS model was introduced by McKee & Ostriker; their analytical solution [12, 15] widely used for the description of evolution of the radiative shell gives a power-law dependence $R \propto t^m$ (where t is age and R is the position of the shock) with the constant m (which equals 2/7 for the uniform medium). However, the numerical studies cited above give somewhat different values of the deceleration parameter m (defined as $m = d\ln R/d\ln t$), namely ≈ 0.33 [5, 10]. We have shown analytically in [16] that the evolution of the radiative shell is given by variable m and that the discrepancy between the analytical and numerical results is only apparent. In fact, the usage of McKee & Ostriker analytical solution assumes that SNR has already reached the asymptotic power-law regime with the constant value of $m = 2/7$. The time needed to reach this asymptotic regime is, however, long in comparison with to the SNR age.

It is common for an approximate theoretical description of SNR evolution to simply switch from the adiabatic solution to the PDS radiative one at some moment of time. However, we stress in this paper the result visible also in previous calculations, namely, the need for an intermediate transition subphase between the adiabatic and radiative stages, with the duration of more than a half of SNR age it has at the time when radiative losses of gas passing through the shock begins to be prominent. Thus the radiative era which begin after the end of the adiabatic one, has to be divided into two phases: the transition subphase, when the radiative losses begin to modify their dynamics and their lead to the formation of the thin radiative shell, and the PDS stage when one can apply the PDS analytic solution. In the present paper the role of nonuniform interstellar medium on the duration of the transition subphase is considered.

II. TRANSITION TO THE RADIATIVE PHASE

A. Definitions of different reference times

Let us consider the spherical shock motion in the medium with the power-law density "o" and "s" refer hereafter to the pre- and post-shock values. The dynamics of the adiabatic shock in such a medium is given by Sedov solutions [19] where the shock velocity $D \propto R^{-(3-\omega)/2}$ and $R \propto t^{2/(5-\omega)}$.

Moving through the medium, the shock decelerates if the ambient density distribution increases or does not quickly decrease ($\omega < 3$). The shock temperature $T_\mathrm{s} \propto D^2$ decreases with time as well. Starting from some age t_low when $T_\mathrm{s} = T_\mathrm{low} \sim 3 \times 10^7\,\mathrm{K}$, which corresponds to the minimum of the cooling function $\Lambda(T)$, the radiative losses of shocked plasma are more and more prominent with the falling of T (Fig. 1). The maximum in the energy losses arises when the shock temperature $T_\mathrm{s} = T_\mathrm{hi} \sim 2 \times 10^5\,\mathrm{K}$, the corresponding Sedov time (i. e. calculated under the assumption that the shock is adiabatic up to this time) is t_hi.

There is a number of reference times in between t_low and t_hi [4, 10, 20]. Once a parcel of gas is shocked its temperature changes due to expansion and cooling $\dot{T}_\mathrm{a} = \dot{T}_\mathrm{a,exp} + \dot{T}_\mathrm{a,rad}$, where the dot marks the time derivative. One may define the "dynamics-affected" time t_dyn by the equation

$$\dot{T}_\mathrm{a,exp}(t_\mathrm{dyn}) = \dot{T}_\mathrm{a,rad}(t_\mathrm{dyn}). \tag{1}$$

If a fluid element is shocked after this time, its temperature decreases faster due to radiation than as a consequence of expansion. At another time t_sag, the radiative cooling begins to affect the temperature distribution inside the shock. When the rate of change of the shock temperature $\dot{T}_\mathrm{s}$ begins to be less than $\dot{T}_\mathrm{a}$, the temperature downstream of the shock will sag rather than rise. Thus the equation for t_sag is

$$\dot{T}_\mathrm{s}(t_\mathrm{sag}) = \dot{T}_\mathrm{a}(t_\mathrm{sag}). \tag{2}$$

Radiative losses cause a faster, in comparison with the adiabatic phase, deceleration of the forward shock. This faster deceleration begins to be prominent around the "transition age" t_tr when the shock pressure decrease due to the radiative losses coming into effect. Then, the shocked gas radiates away its energy rather quickly, cools till the temperature $T \sim 10^4$ K and forms a dense shell. The formation of shell is completed around the "time of shell formation" t_sf which is larger than t_tr; the latter marks the end of the adiabatic era. After t_sf the thermal energy of all the swept-up gas is rapidly radiated and the thin dense shell expansion is caused by the thermal pressure of the interior.

The time t_low is given by the equation

$$T_\mathrm{s}(t_\mathrm{low}) = T_\mathrm{low}. \tag{3}$$

A similar equation defines the time t_hi

$$T_\mathrm{s}(t_\mathrm{hi}) = T_\mathrm{hi}, \tag{4}$$

which was suggested to be a measure of t_tr [13,14]. However, as we will demonstrate later, the post-shock temperature of plasma at t_tr is of the order $10^6\,\mathrm{K} > T_\mathrm{hi}$ and t_hi is larger than t_tr by about 3.5 times (Sect. III A). Therefore it is not correct to calculate the "highest-losses" of SNR age with the shock motion law valid during the adiabatic era.

A simple approach to locate t_tr bases on the comparison of the radiative losses with the initial thermal energy of the shocked fluid [10]. A shocked fluid element cools during the cooling time $\Delta t_\mathrm{cool} \propto \epsilon(T_\mathrm{s}, \rho_\mathrm{s})/\Lambda(T_\mathrm{s}, \rho_\mathrm{s})$, where $\epsilon = (\gamma - 1)^{-1}\rho_\mathrm{s} k_\mathrm{B} T_\mathrm{s}/\mu m_\mathrm{p}$ is its initial thermal energy density, γ is the adiabatic index, k_B is the Boltzman constant, m_p is the proton mass. During the adiabatic phase the cooling time is larger than SNR age t. The radiative losses may be expected to modify their dynamics when the cooling time $\Delta t_\mathrm{cool} \le t$. In such an approach the transition time is a solution of the equation

$$t_\mathrm{tr} = \Delta t_\mathrm{cool}(t_\mathrm{tr}). \tag{5}$$

Let us assume that the cooling function $\Lambda \propto n^2 T^{-\beta}$ with $\beta > 0$ and n is the hydrogen number density. Then $\Delta t_\mathrm{cool} \propto n_\mathrm{o}^{-1} T_\mathrm{s}^{1+\beta} \propto t^{-6(1+\beta)/5}$ with the use of Sedov solutions for uniform medium. For the shock running in the power-law density distribution, the upstream hydrogen number density and the post-shock temperature at time t is

$$n_\mathrm{o} \propto t^{-2\omega/(5-\omega)}, \qquad T_\mathrm{s} \propto t^{-2(3-\omega)/(5-\omega)}. \tag{6}$$

Therefore $\Delta t_\mathrm{cool} \propto t^{-\eta}$ with $\eta = \big(2(3-\omega)(1+\beta) - 2\omega)/(5-\omega)\big)$ for such a distribution of density. For $\beta = 1/2$ the index η is the same as found in [9].

A way of estimating the time of the shell formation t_sf was suggested in [20, 21]. If an element of gas was shocked at the time t_s then the age of SNR will be $t_\mathrm{c} = t_\mathrm{s} + \Delta t_\mathrm{cool}(t_\mathrm{s})$ when it cools down. The minimum of the function $t_\mathrm{c}(t_\mathrm{s})$ has the meaning of SNR age when the first element of gas cools and is called "SNR cooling time" t_cool. Let t_1 be the time when the shock encountered the fluid element which cools first. If so, $t_\mathrm{c} = t_1(t_\mathrm{s}/t_1) + \Delta t_\mathrm{cool}(t_1)(t_\mathrm{s}/t_1)^{-\eta}$. Setting $dt_\mathrm{c}/dt_\mathrm{s}|_{t_\mathrm{s}=t_1} = 0$ one obtain

$$t_\mathrm{cool} = (1+\eta)\Delta t_\mathrm{cool}(t_1), \tag{7}$$

$$\frac{t_\mathrm{cool}}{t_1} = \frac{1+\eta}{\eta}, \tag{8}$$

The cooling time $t_\mathrm{cool} > t_1$ by definition, therefore, the condition $\eta > 0$ must fulfil. This is the case for

$$\omega < 3(1+\beta)/(2+\beta); \tag{9}$$

that is $\omega < 2$ (9/5) for $\beta = 1$ (1/2). The equation

$$t_1 = \eta \Delta t_{\rm cool}(t_1) \tag{10}$$

is more suitable for practical use than (7). If the medium is uniform then $t_{\rm cool} = 17t_1/12$ for $\beta = 1$ and $t_{\rm cool} = 14t_1/9$ for $\beta = 1/2$.

The "SNR cooling time" $t_{\rm cool} = \min(t_{\rm c})$ was initially suggested to be taken as the time of the shell formation. Numerical experiments for shock in the uniform medium suggest that $t_{\rm sf}$ is a bit higher (of the order of 10%) than $t_{\rm cool}$ [22] and the reason for this could be that the compression of the shell is also effective after cooling the first element that takes additional time.

Another point is that the solution for adiabatic shock used in (6) might not formally be applicable there because $t_1 > t_{\rm tr}$ (see Eq. (39)). We believe, however, that the level of accuracy in the estimation of $t_{\rm tr}$, the small difference between $t_{\rm tr}$ and t_1 (about 30% in the case of uniform medium, Sect. III A) as well as close values of $t_{\rm cool}$ and $t_{\rm sf}$ allow one to use the Sedov solution in (6) and to assume $t_{\rm sf} \approx t_{\rm cool}$.

We would like to note once more that the transition time $t_{\rm tr}$ is an approximate estimation on the end of the adiabatic stage and beginning of the radiative era, while the time of the shell formation $t_{\rm sf}$ marks the time when one can start to use the PDS model where hot gas pushes the cold dense shell[1]. The structure of the flow restructurises and the shell forms during the transition subphase given by the time interval $(t_{\rm tr}, t_{\rm sf})$. We will demonstrate later that the ratio $t_{\rm sf}/t_{\rm tr}$ with $t_{\rm tr}$ given by (5) and $t_{\rm sf}$ by (8) is always larger than unity (see Eq. (38)) and that the transition subphase is not short as it is generally assumed.

One more time, namely the "intersection time" $t_{\rm i} \in (t_{\rm tr}, t_{\rm sf})$ was introduced in [16], as a time when two functions — the adiabatic dependence $R = R(t)$ (valid before $t_{\rm tr}$) and the PDS dependence $R_{\rm sh} = R_{\rm sh}(t)$ (valid after $t_{\rm sf}$) — intersect being extrapolated into the transition subphase. This intersection time could be useful in some tasks when the level of accuracy is such that one may sharply switch from the adiabatic solution to the radiative one without the consideration of the transition subphase.

B. Cooling time

The expression

$$\Delta t_{\rm cool} = \frac{\epsilon(T_{\rm s}, \rho_{\rm s})}{\Lambda(T_{\rm s}, \rho_{\rm s})} \tag{11}$$

used in [10] to calculate the cooling time, equates the energy losses $\Lambda \Delta t_{\rm cool}$ with initial thermal energy density $\epsilon_{\rm s}$ of a fluid element on condition that the density and temperature of this element are constant. A more detailed model should account for the density and temperature history during $\Delta t_{\rm cool}$. In feet, the above equation should be replaced with a differential one:

$$d\epsilon/dt = -\Lambda(T, \rho). \tag{12}$$

The total internal energy $U = \epsilon V$ of gas within the volume V changes as $dU = TdS - PdV$ where S is entropy and P is pressure. The evolution of the thermal energy per unit mass $E = \epsilon/\rho$ is therefore

$$\frac{\partial E}{\partial t} - \frac{P}{\rho^2}\left(\frac{\partial \rho}{\partial t}\right) = T\frac{\partial s}{\partial t} \tag{13}$$

where $s = (3k_{\rm B}/2m_{\rm p}\mu)\ln(P/\rho^\gamma)$ is the entropy per unit mass ($m_{\rm p}$ is the mass of proton, μ is the mean particle weight). So, Eq. (12) becomes

$$T\frac{\partial s}{\partial t} = -\frac{\Lambda(T, \rho)}{\rho}, \tag{14}$$

here the temperature T, density ρ, pressure P, energy E are functions of the Lagrangian coordinate a and time t.

Following from (14) and the definition of s, the time $\Delta t_{\rm cool}$ may be also defined as a time taken for the adiabat P/ρ^γ to fall to zero. Kahn [23] has found an interesting result. Namely, if

$$\beta = \frac{2-\gamma}{\gamma-1} \tag{15}$$

(that is $\beta = 1/2$ for $\gamma = 5/3$), then one can derive $\Delta t_{\rm cool}$ from (14) independently of the density and temperature history:

$$\Delta t_{\rm cool}^{\rm Kahn} = \frac{\epsilon(T_{\rm s}, \rho_{\rm s})}{(\beta+1)\Lambda(T_{\rm s}, \rho_{\rm s})}. \tag{16}$$

It can be checked that the same solution may be obtained from (13)–(14) for any β if one assume that the gas is not doing work during $\Delta t_{\rm cool}$ that is equivalent to putting $\partial\rho/\partial t = 0$ in (13). However, the density of fluid is not expected to be constant. In such a situation one should solve the full set of the hydrodynamic equations which can be performed only numerically, while we are interested in a rather simple analytical estimation of cooling time for β generally. Therefore it is more suitable

[1]The PDS analytical solutions which describe the evolution of SNR after the shell formation time are presented in [13,14,16] for uniform ISM and in [14] for ISM with power-law density variation.

to use estimation (11) for the cooling time which follows just from the comparison of the radiative losses with the initial energy. We shall see later that such an approach describes the shock dynamics rather well (Fig. 2).

C. Equations for the reference times

Let us write equations for $t_{\rm tr}$ and $t_{\rm sf}$ for the shock in a nonuniform medium. We assume hereafter $\beta = 1$. Note that all the remaining formulae can easily be modified if one uses β which coincides with the value given by (15); namely, following from the comparison of (16) and (11), $\mathcal{T}$ in (17) has to be simply divided by $\beta + 1$.

If the cooling function for a fluid is approximately $\Lambda = CT^{-\beta} n_e n_{\rm H}$, where C is a constant, then (11) yields

$$\Delta t_{\rm cool} = \mathcal{T} \frac{T_{\rm s}^{1+\beta}}{n_{\rm o}(R)} \quad \text{where} \quad \mathcal{T} = \frac{{\rm k_B}\mu_{\rm e}}{{\rm C}\mu(\gamma+1)}, \tag{17}$$

μ_e is the mean mass of particle per one electron in terms of the proton mass (i. e., $\rho = \mu_e n_e m_{\rm p} = \mu n m_{\rm p}$). The transition time $t_{\rm tr}$ is a solution of equation (5):

$$t_{\rm tr} = \mathcal{T} \frac{T_s(t_{\rm tr})^{1+\beta}}{n_{\rm o}(R(t_{\rm tr}))}, \tag{18}$$

where the dependencies $T_{\rm s}(t)$, $R(t)$ are those valid on the adiabatic phase. The time t_1 can be estimated from (10):

$$t_1 = \eta \mathcal{T} \frac{T_s\,(t_1)^{1+\beta}}{n_{\rm o}\,(R(t_1))}. \tag{19}$$

Now the SNR cooling time $t_{\rm cool}$ and the time of the shell formation $t_{\rm sf} \approx t_{\rm cool}$ is given by (8). The estimations for the transition and the shell formation times are somewhat different in the literature because of different ways used to find the cooling time $\Delta t_{\rm cool}$ and to approximate the cooling function $\Lambda(T)$.

For the adiabatic shock the rate of change of the shock temperature is

$$\dot{T}_{\rm s} = -\frac{2(3-\omega)}{5-\omega}\frac{T_{\rm s}}{t}. \tag{20}$$

Close to the shock, the fluid temperature in Sedov solution [19] is approximately

$$\frac{T(a)}{T_{\rm s}} \approx \left(\frac{a}{R}\right)^{-\kappa(\gamma,\omega)}, \tag{21}$$

where a is the Lagrangian coordinate. The value of κ is given by

$$\kappa = \left(-\frac{a}{T(a)}\frac{\partial T(a)}{\partial a}\right)_{a=R}. \tag{22}$$

where $T(a)$ is the profile from Sedov solutions. It is $\kappa = 1 - 3\omega/4$ for $\gamma = 5/3$ (see Appendix 1). Now we may find that the temperature in a given fluid element a changes due to expansion as

$$\dot{T}_{\rm a,exp} \approx -\frac{2(3-\omega-\kappa)}{5-\omega}\frac{T(a)}{t}. \tag{23}$$

The rate $\dot{T}_{\rm a,rad}$ due to cooling follows from $dE/dt = -\Lambda/\rho$:

$$\dot{T}_{\rm a,rad} = -\frac{\gamma-1}{\gamma+1}\mathcal{T}^{-1} n_{\rm H}(a) T(a)^{-\beta}. \tag{24}$$

Now we have to compare the above rates at the time $t_{\rm s}$, i. e. at the time when the parcel of fluid was shocked. The coordinate $a = R(t_{\rm s})$ by definition. Thus Eq. (1) is rewritten:

$$t_{\rm dyn} = \frac{2(3-\omega-\kappa)}{5-\omega}\Delta t_{\rm cool}(t_{\rm dyn}). \tag{25}$$

Similarly, the equation for $t_{\rm sag}$ follows from (2):

$$t_{\rm sag} = \frac{2\kappa}{5-\omega}\Delta t_{\rm cool}(t_{\rm sag}). \tag{26}$$

As one can see, the most of reference times are given by the equations of the form

$$t_* = K \Delta t_{\rm cool}(t_*), \tag{27}$$

where t_* is a given reference time and K is the corresponding constant. It may be shown that the solution of such an equation may be found as

$$t_* = K^{1/(1+\eta)} t_{\rm tr}. \tag{28}$$

The Sedov radius of the shock at this time is $R_* = K^{2/((5-\omega)(1+\eta))} R_{\rm tr}$.

D. The cooling function

There are two choices of β in the literature, namely 1 and 1/2. The first case is used for *non-equilibrium* cooling model [24] where the cooling function for plasma with solar abundance may be approximated as [10]

$$\Lambda = 10^{-16} n_e n_{\rm H} T^{-1}\ {\rm erg\ cm^{-3}\ s^{-1}}. \tag{29}$$

This approximation is valid for the range of temperatures $T = (0.2-5)\times 10^6\,{\rm K}$ which important for the description of transition into the radiative phase. Another possibility lies in using the *equilibrium* cooling model as it was done in [9, 20, 22, 23, 25]. In this case the approximate

proportionality $\Lambda \propto T^{-1/2}$ is a reasonable one, e. g. for the results on the cooling of the collisional equilibrium plasma from [26, 27]; the actual approximation

$$\Lambda = 1.3 \times 10^{-19} n_e n_{\rm H} T^{-1/2} \text{ erg cm}^{-3} \text{ s}^{-1} \qquad (30)$$

is written for plasma with almost the same abundance as above and is valid for $T = (0.05 \div 50) \times 10^6$ K [23].

Different cooling functions are compared with their approximations on Fig. 1. At lower temperatures, the nonequilibrium cooling is less effective in energy losses than the equilibrium one (compare lines 2 and 5). This is because the cooling rate for temperatures higher than $\sim 3 \times 10^7$ K is mostly due to free–free emission while below this temperature the cooling is mostly due to the line emission from heavy elements (most heavy elements are completely ionized above $\sim 3 \times 10^7$ K). Under nonequilibrium ionization conditions the ions are underionized because electrons are much cooler than ions and thus there is less emission from ions [9, 28] (see also Fig. 18 in [24]).

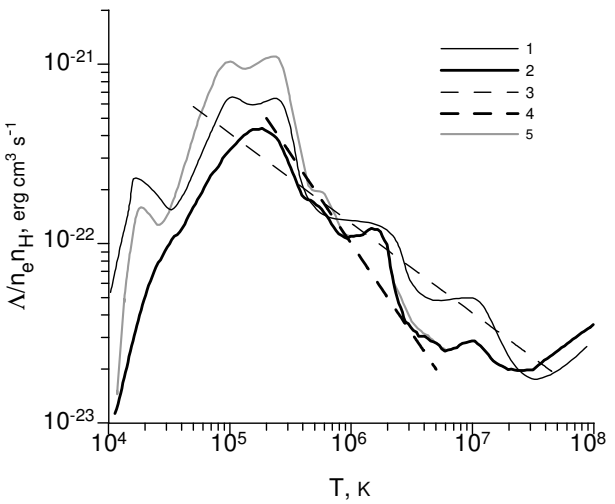

Fig. 1. Equilibrium (line 1) [26] and nonequilibrium (line 2) [24] cooling functions, used in the literature to study the transition of SNRs into the radiative phase, and approximations (30) (line 3) and (29) (line 4). The equilibrium cooling function from [24] is also shown for comparison (line 5).

III. REFERENCE TIMES AND TRANSITION SUBPHASE

A. Shock in a uniform ISM

Let us compare the sequence of different reference times with numerical calculations [10] of transition of the adiabatic shock into the radiative era, for exemplified by the shock motion in the uniform ambient medium. Let us consider the same parameters as in [10], namely $\gamma = 5/3$, $\beta = 1$, the same abundance ($\mu = 0.619$, $\mu_{\rm e} = 1.18$, $\mu_{\rm H} = 1.43$) as well as assume $t_{\rm sf} = t_{\rm cool}$ and use (11) for the calculation of $\Delta t_{\rm cool}$.

If the shock wave moves in the uniform medium, then — with the use of Eq. (18) – the transition time is

$$t_{\rm tr} = 2.84 \times 10^4 E_{51}^{4/17} n_{\rm o}^{-9/17} \text{ yr} \qquad (31)$$

where $E_{51} = E_{\rm SN}/(10^{51}$ erg). The gas element which at first cools (at $t_{\rm cool}$) was then shocked at t_1 which follows from Eq. (19):

$$t_1 = 3.67 \times 10^4 E_{51}^{4/17} n_{\rm o}^{-9/17} \text{ yr.} \qquad (32)$$

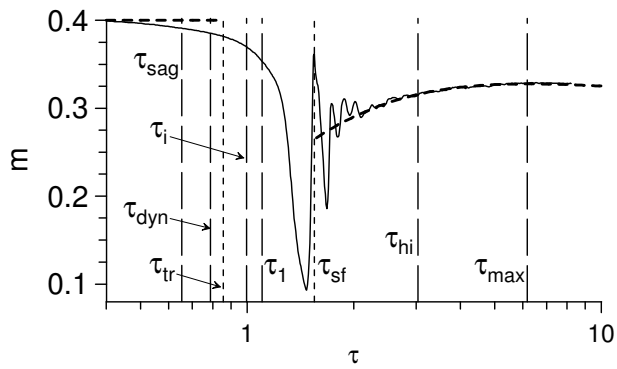

Fig. 2. The evolution of the deceleration parameter m and different reference times for the shock motion in the uniform medium. Solid line — numerical calculations [10], thick dashed lines — Sedov solution (till $\tau_{\rm tr}$) and analytical solution [16] (after $\tau_{\rm sf}$). The dimensionless reference times are $\tau_{\rm sag} = 0.654$, $\tau_{\rm dyn} = 0.802$, $\tau_{\rm tr} = 0.855$, $\tau_{\rm i} = 1.01$, $\tau_1 = 1.10$, $\tau_{\rm sf} = 1.57$, $\tau_{\rm low} = 0.047$, $\tau_{\rm hi} = 3.03$. The function $m(\tau)$ reaches its maximum at radiative phase at $\tau_{\rm max} = 6.18$ [16].

The time of the shell formation is given by Eq. (8):

$$t_{\rm sf} = 5.20 \times 10^4 E_{51}^{4/17} n_{\rm o}^{-9/17} \text{ yr,} \qquad (33)$$

so that $t_{\rm sf}/t_{\rm tr} = 1.83$. The time when the radiative losses of the shocked gas reach their minimum is (3):

$$t_{\rm low} = 1.60 \times 10^3 T_{3e7}^{-5/6} E_{51}^{1/3} n_{\rm o}^{-1/3} \text{ yr} \qquad (34)$$

where $T_{3e7} = T_{\rm low}/(3 \times 10^7$ K). Under the assumption that radiative losses does not change the shock dynamics till $t_{\rm hi}$, with the use of Sedov solutions for the shock motion one has from Eq. (4) that

$$t_{\rm hi} = 1.04 \times 10^5 T_{2e5}^{-5/6} E_{51}^{1/3} n_{\rm o}^{-1/3} \text{ yr} \qquad (35)$$

where $T_{2e5} = T_{\rm hi}/(2 \times 10^5$ K). The fluid temperature drops faster due to cooling than due to expansion from the time

$$t_{\rm dyn} = 2.66 \times 10^4 E_{51}^{4/17} n_{\rm o}^{-9/17}\,{\rm yr}. \qquad (36)$$

The time when one may expect to have the temperature decrease downstream close to the shock is

$$t_{\rm sag} = 2.17 \times 10^4 E_{51}^{4/17} n_{\rm o}^{-9/17}\,{\rm yr}. \qquad (37)$$

The Sedov solutions give at the time $t_{\rm tr}$ the shock radius $R_{\rm tr} = 19 E_{51}^{5/17} n_{\rm o}^{-7/17}$ pc, the shock velocity $D_{\rm tr} = 260\ E_{51}^{1/17} n_{\rm o}^{2/17}$ km/s, the post-shock temperature $T_{\rm tr} = 0.95{\cdot}10^6\ E_{51}^{2/17} n_{\rm o}^{4/17}$ K and the swept up mass $M_{\rm tot}(t_{\rm tr}) = 10^3\ E_{51}^{15/17} n_{\rm o}^{-4/17} M_\odot$.

The above reference times are shown on Fig. 2 together with the evolution of the deceleration parameter $m(\tau)$ calculated numerically [10]. The analytical solutions for the adiabatic [19] and the radiative shock [16] are also shown. Numerical result is found for supernova energy $E_{\rm SN} = 10^{51}$ erg and interstellar hydrogen number density $n_{\rm o} = 0.84\,{\rm cm}^{-3}$. With these values, the times are $t_{\rm sag} = 2.4 \times 10^4$ yr, $t_{\rm dyn} = 2.9 \times 10^4$ yr, $t_{\rm tr} = 3.1 \times 10^4$ yr, $t_1 = 4.0 \times 10^4$ yr, $t_{\rm sf} = 5.7 \times 10^4$ yr, $t_{\rm low} = 1.7 \times 10^3$ yr, $t_{\rm hi} = 1.1 \times 10^5$ yr; the intersection time is $t_{\rm i} = 3.6 \times 10^4$ yr [16]. The function $m(\tau)$ reaches its maximum during the radiative stage at $t_{\rm max} = 2.3 \times 10^5$ yr [16]. The results on Fig. 2 are presented in terms of the dimensionless time $\tau = t/\tilde{t}$ because the analytical solutions allow for scaling (numerical results for various input parameters differ by oscillation transient only; see e. g. Fig. 8 in [10]). The dimensional scale for time determined from fitting analytical and numerical results is $\tilde{t} = 3.6 \times 10^4$ yr [16].

It is apparent from Fig. 2 that the transition time $t_{\rm tr}$ is a reasonable estimation for the end of the adiabatic stage while $t_{\rm sf}$ could be the time when one can start to use the radiative solutions [16] coming from the PDS model of McKee & Ostriker [12]. The duration of the intermediate transition subphase is $(\tau_{\rm sf} - \tau_{\rm tr})/\tau_{\rm tr} = 0.83$ times the age of SNR at the end of the adiabatic stage, i. e., almost the same as duration of the adiabatic stage itself. This means that there is a strong need for a theoretical model which describes the evolution of SNR in this subphase.

For the estimation of reference times, a number of authors [6, 9, 20–22] keep a bit different approach from that used above, namely they use the approximation of the equilibrium cooling function with $\beta = 1/2$ and the Kahn solution for cooling time (16). Let us compare the results of this approach with those obtained above. The evolution of the deceleration parameter in the refereed approach is presented in [6]. There is also the same definition of the time of the shell formation $t_{\rm sf} = t_{\rm cool}$. The estimation is $t_{\rm sf,C} = 4.31 \times 10^4 E_{51}^{3/14} n_{\rm o}^{-4/7}$ yr for their abundance and the cooling function (30). For the parameters used in the numerical calculations $E_{51} = 0.931$ and $n_{\rm o} = 0.1\,{\rm cm}^{-3}$ the time is $t_{\rm sf,C} = 1.58 \times 10^5$ yr while with the use of our Eq. (33) we obtain $t_{\rm sf} = 1.73 \times 10^5$ yr. Both estimations are close. Analytical solutions show that, before $t_{\rm tr}$ and after $t_{\rm sf}$, the evolution of dynamic parameters of the shock can be expressed in a dimensionless form, i. e., independently of E_{51} and $n_{\rm o}$. The behaviour of the shock velocity depends, however, on these parameters during the transition subphase; the difference is in the frequency of oscillations (Fig. 8 in [10]). Nevertheless, as one can see from this figure, the strong deceleration of the shock right after $t_{\rm tr}$ up to the first minimum is almost the same for different parameters, i. e. it can also be scaled. We use this property in order to find the scale factor $\tilde{t}$ for calculations be done in [6]. Namely, the fit of curve $m(\tau)$ from [6] to that of [10] (within the time interval from $t_{\rm tr}$ to the first minimum) gives $\tilde{t}_{\rm C} = 1.05 \times 10^5$ yr.

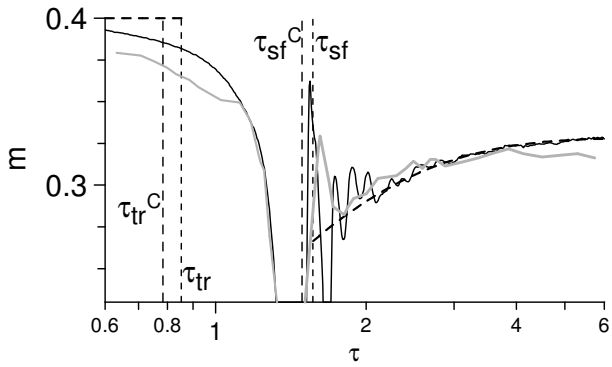

Fig. 3. Numerical calculation of evolution of the deceleration parameter m from [10] (thin black line) and from [6] (thick gray line). The transition and shell formation times from [6] are marked by "C".

Both calculations of the transition to the radiative stage agree rather well as it may be seen on Fig. 3. The dimensionless times for results in [6] are: the shell formation time $\tau_{\rm sf,C} = t_{\rm sf,C}/\tilde{t}_{\rm C} = 1.51$ and the transition time (as it follows from (38)) $\tau_{\rm tr,C} = \tau_{\rm sf,C}/1.92 = 0.785$. Fig. 3 shows that both approaches for the localization of the limits of the transition subphase — with the use of the nonequilibrium-ionization cooling function (29) and the simple estimation for $\Delta t_{\rm cool}$ (11) [10] or with the equilibrium cooling function (30) together with Kahn solution for $\Delta t_{\rm cool}$ (16) [6] give almost the same estimations.

B. The shock in a medium with power-law density variation

Let us now consider the shock motion in the ambient medium with the power-law density variation $\rho^o(R) = AR^{-\omega}$. With the use of (18), (19), (8), (6) and the definition $t_{\rm sf} = t_{\rm cool}$ one can show that the duration of the transition subphase is given by

$$\frac{t_{\rm sf}}{t_{\rm tr}} = \frac{t_{\rm cool}}{t_{\rm tr}} = \frac{1+\eta}{\eta^{\eta/(1+\eta)}}\ . \qquad (38)$$

The shell formation time is always larger than the transition time $t_{\rm tr}$, provided by the fact that $\eta > 0$. The ratio

$$\frac{t_1}{t_{\rm tr}} = \eta^{1/(1+\eta)} \tag{39}$$

is also always larger than unity. Note that these relations do not depend on the abundance and γ. The ratios between all other times may be found from (28).

The consequence of times is $t_{\rm dyn} < t_{\rm tr} < t_1 < t_{\rm sf}$ (Fig. 4) in nonuniform medium with increasing density. The time t_1 may be smaller than $t_{\rm tr}$ and $t_{\rm dyn}$ for the decreasing density medium. The sag time $t_{\rm sag} < t_{\rm tr}$ for $\omega > -6$ only.

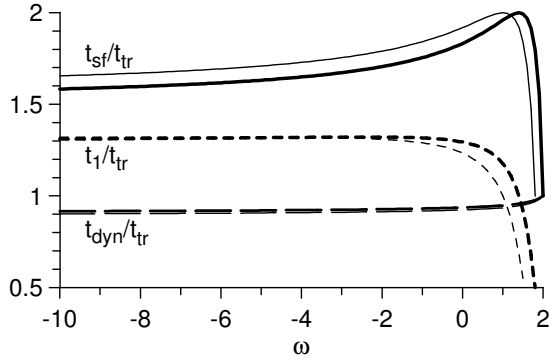

Fig. 4. The ratios of times for $\beta = 1$ (thick lines) and $\beta = 1/2$ (thin lines) as it is obtained from (38) and (39).

Fig. 4 shows the two ratios (38) and (39) as a functions of ω for two values of β. Namely, the ratios $t_1/t_{\rm tr} \approx 1.3$ and $t_{\rm sf}/t_{\rm tr} \approx 1.6 \div 1.8$ are almost the same for shock in the medium with increasing density ($\omega \le 0$). Therefore, in case of a uniform medium and a medium with increasing density, *there is a need of introduction of transition subphase with the duration of more than a half of the SNR age at the beginning of this subphase*, $t_{\rm tr}$. The transition time $t_{\rm tr}$ and therefore the transition subphase $t_{\rm sf} - t_{\rm tr} \propto t_{\rm tr}$ are less for higher density and lower initial energy: $t_{\rm tr} \propto E_{51}^{(2+2\beta+\omega)/\delta} A^{-(7+2\beta)/\delta}$ where $\delta = 11 + 6\beta - \omega(5 + 2\beta)$. Such a dependence on density is also visible in numerical calculations (Fig. 8 in [10]).

1. Medium with decreasing density

It seems that the formulae (38) and (39) suggest for the case of decreasing density that the PDS radiative stage can even begin right after the end of adiabatic stage: $t_{\rm sf}/t_{\rm tr} \to 0$ with $\omega \to 3(1+\beta)/(2+\beta)$. Another result, already stated in [9], also follows: there will be no radiative shell formation for $\omega \ge 3(1+\beta)/(2+\beta)$. In order to understand the reasons of such behavior let us consider more details.

What is the coordinate a_1 of the element which cools first? This element was shocked at $t_1 = \eta^{1/(1+\eta)} t_{\rm tr}$. The Sedov radius at this time is $R(t_1) = a_1 = \eta^{2/((5-\omega)(1+\eta))} R_{\rm tr}$, thus the coordinate $a_1 > R_{\rm tr}$ if $\omega < 1.4$ ($\beta = 1$) as it is shown on Fig. 5. The ratio $a_1/R_{\rm tr}$ is close to unity and is almost the same for such ω, i. e. the fluid we are interested in will be shocked soon after $t_{\rm tr}$. However, if $\omega > 1.4$ then $a_1 \to 0$ quickly with the increasing of ω from 1.4 to 2, i. e. the element which cools first is already inside the shock and may be in a very deep interior. It looks that there could not be any "radiative shell" in a common sense.

It is clear that the trend $t_{\rm sf}/t_{\rm tr} \to 0$ does not mean that radiative processes in the shock develop quickly for $\omega > 1.4$. The transition and the shell formation times correspond to different processes: $t_{\rm tr}$ comes from a comparison of the initial thermal energy density of the shocked fluid with radiative losses though $t_{\rm sf} = t_{\rm cool}$ is a time when the first cooled element appears. The two mentioned processes place in the vicinity of the shock if ambient medium is uniform or with increasing density. Numerical results suggest that they may be used for approximate estimates of the limits of the transition subphase in such media. However, these two processes are separated in space for media with decreasing density. It could be that one (or both) of the times $t_{\rm tr}$ and $t_{\rm sf}$ may not be suitable to mark stages of SNR in medium with decreasing density.

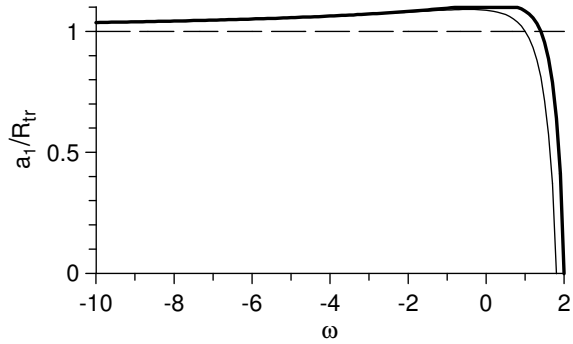

Fig. 5. The ratio $a_1/R_{\rm tr}$ for $\beta = 1$ (thick lines) and $\beta = 1/2$ (thin lines).

The cooling of shock moving in the medium with decreasing density differs from a commonly accepted scenario of the "thin dense shell" formation and should be studied in more details in the future.

IV. CONCLUSIONS

The common approximate scenario of SNR evolution consists of the free expansion stage, the adiabatic phase and the PDS radiative era. It is shown that it is necessary to consider also an additional subphase between the adiabatic and the radiative stages because this subphase lasts more than half of the SNR age it has at the end of the adiabatic stage.

The analytical estimations on the ratios between the reference times which characterize the transition of adiabatic SNR into the PDS radiative stage — $t_{\rm tr}$, $t_{\rm sf}$ and t_1 — do not depend on the initial parameters of SNR and IMS (energy of explosion, number density in the place of explosion, γ etc.) except of the density gradient (i. e.

ω) and assumed β which causes rather small effect. This result is also visible in the numerical calculations for the case of the uniform medium (Fig. 8 in [10]): except for the oscillations (which is indeed different for different $n_{\rm o}$) the durations of the transition subphase in terms of the transition time are almost the same for different values of ISM density.

The ratio $t_{\rm sf}/t_{\rm tr} \approx 1.6$ for shock running in media with constant or increasing densities. The transition time, however, depends on the energy of explosion, the density of the medium and the density gradient: $t_{\rm tr} \propto E_{\rm SN}^{a(\omega)} A^{-b(\omega)}$ with $a > 0$ and $b > 0$ for shock in a medium with $\rho_{\rm o} \propto R^{-\omega}$. This means that the transition subphase is longer for higher explosion energy and smaller density. The dependence of $t_{\rm tr}$ on this parameters are stronger for higher ω because the functions $a(\omega)$ and $b(\omega)$ increase with ω.

The hydrodynamical properties of the shock in media with $\omega > 0$ seem to cause a trend to absence of the radiative phase in a common sense. The cooling of such shocks differs from a commonly accepted scenario of the "thin dense radiative shell" formation and should be studied in more details because it is important for models of non-spherical SNRs which could be only partially radiative.

ACKNOWLEDGEMENTS

I am grateful to B. Hnatyk for valuable discussions.

APPENDIX 1. APPROXIMATION OF THE TEMPERATURE EVOLUTION IN A GIVEN FLUID ELEMENT DOWNSTREAM CLOSE TO THE STRONG ADIABATIC SHOCK

In order to simplify the estimation of $t_{\rm sag}$ and $t_{\rm dyn}$, let us approximate the distribution $\bar{T}(\bar{a}) = T(a,t)/T_{\rm s}(t)$ downstream close to the strong adiabatic shock; here a is Lagrangian coordinate, $\bar{T} = T/T_{\rm s}$ and $\bar{a} = a/R$. Note that hereafter in this Appendix we use the normalized parameters, i. e. divided on their values on the shock front; thus we skip the overlines in the notations. We are interested in the approximation in the form

$$T(a) \approx a^{-\kappa(\gamma,\omega)}. \tag{40}$$

The value of κ is given by

$$\kappa = \left(-\frac{\partial \ln T(a)}{\partial \ln a} \right)_{a=1} \tag{41}$$

where $T(a)$ is the profile from Sedov [19] solutions. The equation of the mass conservation and the equation of the adiabaticity applied for the case of the shock motion in the medium with the power-law density distribution give the distribution of temperature $T(a) = P(a)/\rho(a)$ [29]

$$T(a) = \left(\frac{\gamma - 1}{\gamma + 1} \right)^{\gamma - 1} a^{2\gamma - 5 + \omega} \left(r(a)^2 r_{\rm a}(a) \right)^{-\gamma + 1} \tag{42}$$

where r is Eulier coordinate and $r_{\rm a} = \partial r/\partial a$. Instead of Sedov profiles for $r(a)$ — which is quite complex — we use the approximation

$$r(a) = a^{(\gamma-1)/\gamma} \exp\left(\alpha(a^{\beta} - 1) \right) \tag{43}$$

where α, β are constants; this approximation gives correct values of r and its derivatives in respect to a up to the second order on the shock [29]. Substitution (41) with (42), (43) and with expressions for α, β from [29] yields

$$\kappa = \frac{2\big(8 - (\gamma + \omega)(\gamma + 1)\big)}{(\gamma + 1)^2}. \tag{44}$$

For $\gamma = 5/3$, $\kappa = 1 - 3\omega/4$.

The approximation (40) underestimates Sedov temperature. The smaller a the larger difference. It is about 20% at $a \approx 0.5$ (that corresponds to $r \approx 0.8$).

APPENDIX 2. LIST OF TIMES

$t_{\rm sag}$	"sag" time [4], radiative cooling begins to affect the temperature distribution downstream of the shock;
$t_{\rm dyn}$	"dynamics-affected" time [4], the temperature of a fluid element shocked after this time decreases faster due to radiation than due to expansion;
$t_{\rm tr}$	"transition" time [10], estimation of the time when the deviations from Sedov solutions are prominent; Sedov solution may be approximately used till this time;
$\Delta t_{\rm cool}$	"cooling" time [20, 21, 23], a shocked fluid element cools during this time;
$t_{\rm s}$	"shock" time [20, 21], moment when the shock encountered a given fluid element;
t_1	moment when the shock encountered the fluid element which cools first [20, 21];
$t_{\rm c}$	sum of $t_{\rm s} + \Delta t_{\rm cool}$;
$t_{\rm cool}$	"SNR cooling" time [20–22], the minimum of $t_{\rm s}$, i. e. the age of SNR when the first cooled element appears;
$t_{\rm sf}$	"shell-formation" time [20–22], approximately after this time the shock may be described by the radiative PDS model;
$t_{\rm low}$	moment during the adiabatic stage when the radiative losses of the decelerating shock wave reach their minimum value;
$t_{\rm hi}$	moment when the radiative losses of the decelerating shock wave reach their maximum value [13, 14];
$t_{\rm i}$	"intersection" time [16], moment when two functions — adiabatic $R(t)$ and radiative $R_{\rm sh}(t)$ intersect;
$t_{\rm max}$	moment during the radiative stage when the function $m(\tau)$ reaches its maximum [16];
$\tilde{t}$	timescale;
τ	dimensionless time, $\tau = t/\tilde{t}$.

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ДО ПЕРЕХОДУ АДІЯБАТИЧНИХ ЗАЛИШКІВ НАДНОВИХ ЗІР НА РАДІЯЦІЙНУ СТАДІЮ В НЕОДНОРІДНОМУ МІЖЗОРЯНОМУ СЕРЕДОВИЩІ

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Дано огляд методів оцінки низки часових масштабів, які з'являються в описі переходу сильної адіябатичної ударної хвилі на радіяційну стадію еволюції. Підкреслено потребу виділення додаткової перехідної підфази між кінцем адіябатичної стадії та початком т. зв. радіяційної стадії "тискового снігозгрібача" для ударних хвиль як в однорідному, так і неоднорідному середовищі. Це важливо, зокрема, для вивчення та моделювання взаємодії залишків наднових зір з молекулярними хмарами та розуміння процесів ґенерації космічних променів у таких системах. Тривалість такої підфази — близько 70% віку залишку на момент її початку — слабо залежить від ґрадієнта густини в середовищах зі зростаючою густиною і є більшою для вищої енерґії спалаху зорі та меншої густини в околі вибуху. Показано також, що коли густина зовнішнього середовища спадає, то процеси охолодження відрізняються від загальноприйнятого сценарію формування "тонкої щільної радіяційної оболонки". Вони повинні бути досліджені в майбутньому, оскільки це суттєво для моделювання залишків наднових зір, які можуть бути лише частково радіяційними.