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LIQUID-LIKE PHASES OF $\pi^+\pi^-$ MATTER

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To give a common theoretical description of liquid phases of the charged pion matter in a wide temperature interval, the relativistic quantum φ^6 type model is considered. The liquid states of pion condensate and hot pion matter are investigated.

Key words: pi-meson matter; Bose–Einstein condensate; phase transitions.

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I. INTRODUCTION

The possibility to observe the pion condensate in heavy-ion collisions had served as a subject of theoretical and experimental investigations during the last decades. This condensed state is often imaged like multipion droplets or/and pion liquid. The pion condensate is also a necessary component of neutron stars. The theoretical description of pion condensate is usually given within the framework of phenomenological models leading to the φ^4 type self-interaction between pionic degrees of freedom (see, for example, [1–3]).

Since the pions are the Goldstone bosons for spontaneously broken chiral symmetry, a realistic description of pion subsystem can only be achieved on the basis of a model respecting chiral symmetry. However, if the pion matter is at the temperature much lower than the temperature of the chiral phase transion ($T_\chi \approx 150$ MeV), a chiral perturbation theory is applicable. Using perturbation scheme, we would like to point out the general property of the chiral models, namely, in third-order approximation they result in an attractive two-body interaction (associated with φ^4 term) and three-body repulsive interaction (associated with φ^6 term). To demonstrate it, we appeal to the Skyrme [4] and Weinberg [5] models, where interactions can be presented as $V_{\rm Skyrme}(\nu) \equiv -f_\pi^2 m^2 {\rm Tr}(U + U^\dagger - 2)/4 = f_\pi^2 m^2(\nu^2/2! - \nu^4/4! + \nu^6/6! - \ldots)$ and $V_{\rm Weinberg}(\nu) \equiv f_\pi^2 m^2 \nu^2/(2 + \nu^2/2) = f_\pi^2 m^2(\nu^2/2 - \nu^4/8 + \nu^6/32 - \ldots)$, respectively. Here we use notations for $U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi) \in SU(2)$, $\nu^2 = \vec{\pi}^2/f_\pi^2$; f_π and m are pion decay constant and pion mass, respectively. Note that these interactions are the limited functions of ν. Also note that the theoretical Gasser–Leutwyler interaction [6] of the order of p^4 is attractive, and the experimental data justify the existence of three-pion correlations in heavy-ion collisions [7]. Therefore, if we limit ourselves by consideration of the third-order approximation (leading to the relativistic quantum φ^6 type model), we can expect to observe first-order phase transitions (PTs) in pion subsystem.

Indeed, it is already known from molecular physics that the non-relativistic φ^6 model, in contrast to the φ^4 one, allows us to observe not only second-order PT into condensate state but also its gaseous and liquid phases. It is reached by means of solving the corresponding Gross–Pitaevskii equation [8,9]. Remark that the φ^6 model has been already used in nuclear hydrodynamics and quantum field theory [10]. Here we are trying to obtain similar results (without degrees of freedom of π^0 mesons) in the context of the physics of superdense ions and neutron stars, where pion condensate plays an important role in the softening of nucleonic equation of state [1]. Clearly, in this problem, we should take into account the essentially different nature of interactions in molecular physics and pion subsystem. Moreover, we would like to investigate the conditions of existence of liquid phase at high temperatures (at the temperatures higher than the temperature $T_{\rm cond}$ of the second-order PT into condensate state) with the use of the same model. It is possible that the hot $\pi^+\pi^-$ liquid can be created in relativistic heavy-ion collisions, where the pions play a dominant role at the final stage of the reaction. Note that collective phenomena in particle-nucleus and nucleus-nucleus collisions are well-established and play significant role. An existence of the hot pion liquid has been already predicted in Ref. [11].

Since the liquid condensate of $\pi^+\pi^-$ mesons thermodynamically differs from gaseous condensate, then such a difference should be taken into account, when attempts to register the appearance of a condensate in heavy-ion collisions are performed. On the other hand, the system of the large number of pions at high temperatures, as we shall see below, can also be in a liquid phase which can essentially affect on nucleonic dynamics. We say "liquid", when we deal with a dense phase in thermodynamics. However, the same "liquid" is a state with a high magnitude of pionic field from the field-theoretical point of view. As a consequence, amplification of pionic field can result in the creation of a proton-antiproton pair.

II. THE PHASES OF PION CONDENSATE

The Lagrangian density $\mathcal{L}(\pi^\dagger, \pi)$ of the model is

$$\mathcal{L} = \partial_\mu \pi^\dagger \partial^\mu \pi - m^2 \pi^\dagger \pi + \frac{A}{2!}(\pi^\dagger \pi)^2 - \frac{B}{3!}(\pi^\dagger \pi)^3, \quad (1)$$

where the normal ordering of operator fields is assumed. We deal with the case, when electromagnetic interaction is neglected. Here it is also supposed that $A = m^2/g^2$, $B = 3m^2\lambda/2g^4$, $m = 140$ MeV, g and λ are model parameters which should be fitted.

Constant g plays a role of pion decay constant f_π redefined in the medium. Model parameter λ is introduced to account effectively the higher-order terms of chiral interaction expansion. To analyze the range of λ, let us present chiral interactions (considered above, for example) as $V(\nu) = a_2\nu^2 - a_4\nu^4 + a_6\nu^6\lambda(\nu)$, where $a_n \equiv |V^{(n)}(0)|/n!$ and $\nu^2 = 2\pi^\dagger\pi/g^2$. It turns out that $\lambda(\nu)$ is a smooth function, which is decreased from 1 to 0, when ν runs from 0 to ∞. Thus, if even $|\nu| > 1$, the constant λ introduced instead of function $\lambda(\nu)$ should be less than 1 in order to relate our phenomenological approach with the chiral theory.

Our approach to this model is based on *ansatz* on applicability of the mean-field approximation (MFA) in a wide temperature interval. Moreover, we are limiting ourselves by two cases: i) $T = 0$, when the operator field π can be replaced by a classical complex field ϕ, also called as the order parameter, describing a condensate; ii) $T > T_{\rm cond}$, when there are no anomalous expectation values and the field π coincides with quantum fluctuations χ. Our aim is to investigate the liquid-like states of $\pi^+\pi^-$ matter in these two regimes.

It is appropriate for us to begin from the investigation of different phases of the non-uniform pion condensate at $T = 0$, which appears in the neutron stars and the nuclei with density higher than the saturation one. This situation is modeled by Lagrangian density $\mathcal{L}(\phi^*, \phi)$. In fact the replacement of π by ϕ is analogous to the transition from quantum electrodynamics to the classical description of electromagnetism, when a big number of photons are in approximately the same state. In our case, the presence of a big number of pions in a single state (Bose–Einstein condensate) permits us to introduce the classical function ϕ. In a contrast to Maxwell theory, $\mathcal{L}(\phi^*, \phi)$ contains the quantum constant $\hbar$ explicitly ($\hbar = 1$ in our units). On the other hand, the quantum meaning of classical fields is reviewed in Ref. [12].

From the variational derivative of the corresponding classical action functional, one obtains the following evolution equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi + m^2\phi - A|\phi|^2\phi + \frac{B}{2}|\phi|^4\phi = 0, \qquad (2)$$

which serves as a relativistic generalization of the corresponding Gross–Pitaevskii equation without external trap potential [8].

An applicability of Eq. (2) demands that the following conditions should be satisfied. First, the total number of pions (multiplicity) must be large enough because only in this case we are authorized to use the concept of Bose–Einstein condensation. Second, in order to replace the field operator by the classical field we have to assume both diluteness and the fact that the temperature is low enough. This allows us to ignore both the quantum and thermal depletion of the condensate.

The solution of Eq. (2) essentially depends on physical parameters and boundary conditions. Here we shall deal with exact static quasi one-dimensional solution ("bubble") which can be presented as:

$$\phi(t, z) = \eta_k(z)\exp\left(-imt\sqrt{1 - k^2}\right), \qquad (3)$$

where $\eta_k(z)$ is a real function.

The time dependence is in a phase, corresponding to the non-vanishing charge Q. Such a substitution results in a very large class of solutions $\eta_k(z)$ in terms of elementary and elliptic functions [13, 14]. However, we are interested in stable solutions arising from ordinary charge conservation.

For the boundary conditions

$$\lim_{z\to\pm\infty}\eta_k(z) = 0, \qquad \lim_{z\to\pm\infty}\eta_k'(z) = 0,$$

there is a solitary wave solution of the form [13]:

$$\eta_k(z) = \frac{2gk}{\sqrt{\sqrt{1 - 4\lambda k^2}\cosh(2kmz) + 1}}. \qquad (4)$$

The result obtained describes a narrow layer of the dense phase trapped between dilute phase and leads to a localized energy density. This model solution can describe a region of dense pion matter with respect to the of nucleus-nucleus collision.

One can simply prove that solution (3) is pseudoscalar: it is necessary to replace the coordinate z and the (quasi)momentum k by $-z$ and $-k$, respectively. As it must be, this operation changes a sign of $\phi(t, z)$.

An order parameter in this system is the scalar density $n(z) \equiv |\phi(t, z)|^2$. To determine the phase of pion matter in dense region, we introduce the charge Q (difference between the numbers of particles and antiparticles) and the total energy E per unit area:

$$Q = 8g^2\frac{\sqrt{1 - k^2}}{\sqrt{\lambda}}\,\mathrm{artanh}\sqrt{\frac{1 - \sqrt{1 - 4\lambda k^2}}{1 + \sqrt{1 - 4\lambda k^2}}}, \qquad (5)$$

$$E = m\sqrt{1 - k^2}Q + \frac{g^2mk}{\lambda} - \frac{g^2m}{\lambda^{3/2}}(1 - 4\lambda k^2)\,\mathrm{artanh}\frac{1 - \sqrt{1 - 4\lambda k^2}}{2k\sqrt{\lambda}}. \qquad (6)$$

These quantities are parameterized by k as independent variable. Also, it is not hard to see that, varying k, the behavior of functions Q, E is mainly determined by parameter λ. At this time, the model parameter g influences only the magnitude of these characteristics.

Since the charge Q is conserved in time, there exists a non-vanishing chemical potential $\mu \equiv \partial E/\partial Q$, which is calculated by the formula: $\mu = (\partial E/\partial k)/(\partial Q/\partial k)$. The dependence of μ on k can be explicitly found. However, we do not adduce it here because of its cumbersome form. Note only that $\mu > 0$. It is in accordance with the result of [2], where multipion droplets are considered at low temperature.

As shown in Fig. 1, the central density $\nu \equiv n(0)/g^2$ and the chemical potential μ present backbendings typical for the first-order PT. The transition point, given by the crossing point in $\varepsilon \equiv E/mg^2$ versus $q \equiv Q/g^2$, corresponds to a Maxwell construction in the diagram of μ versus q. However, the system should never explore the backbending part of the diagram because it is a metastable state. It is clear that dense phase is associated with a liquid while dilute phase is a gas. We want to stress that both branches are quantum fluids.

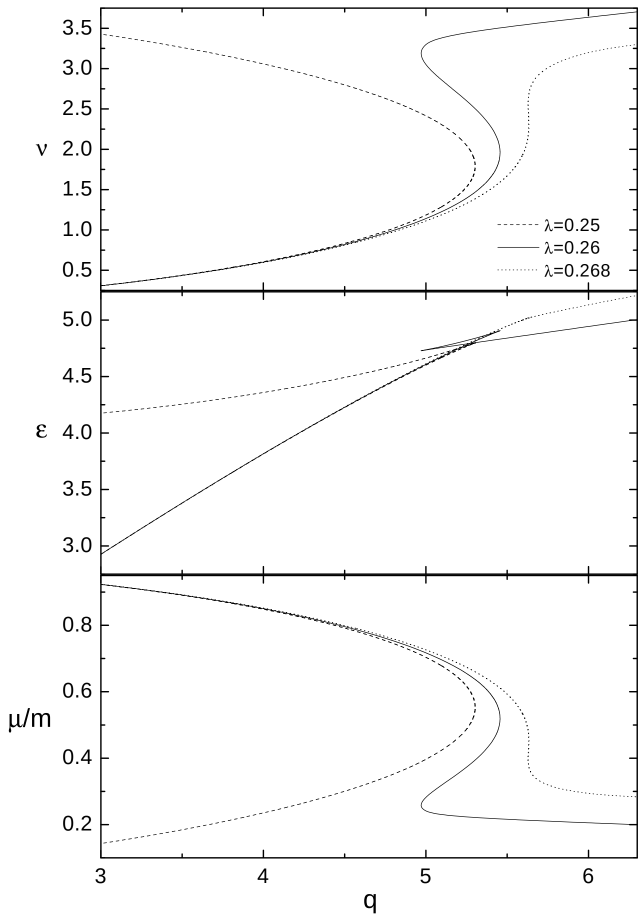

Fig. 1. Central density ν, total energy ε, chemical potential μ per mass m, in dimensionless units, as functions of the reduced charge q.

We see in Fig. 1 that the first-order PT in pion condensate takes place at $0.25 < \lambda < 0.268$ and $\lambda < 1$ as is argued above. Note that, at $\lambda \approx 0.268$, the stable and metastable solutions coincide. It defines a critical point associated with a second-order PT: at this point the derivative of ν as a function of q diverges. For $\lambda = 0.25$ the attractive two-body interaction prevails and the system tends to collapse. In this case the maximum charge is limited by $q \approx 5.3$. Assuming an existence of the first-order phase transition, we put $\lambda = 0.26$ in this paper. The value of constant g will be discussed in the case of hot pion matter.

III. THE HOT PION LIQUID

Now we focus on the description of the $\pi^+\pi^-$ matter at high temperatures in MFA, when the field π is described completely by quantum fluctuations χ. We represent operator field χ in finite volume V as

$$\chi(x) = \sum_{\mathbf{p}} \left[a_{\mathbf{p}} u_{\mathbf{p}}(x) + b^\dagger_{\mathbf{p}} u^*_{\mathbf{p}}(x)\right], \quad (7)$$

$$u_{\mathbf{p}}(x) = \frac{1}{\sqrt{2E_{\mathbf{p}}V}} \exp\left(-iE_{\mathbf{p}}t + i\mathbf{px}\right). \quad (8)$$

The dispersion law is $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_*^2}$, where m_* is effective mass which should be found; $a^\dagger_{\mathbf{p}}$, $b^\dagger_{\mathbf{p}}$ ($a_{\mathbf{p}}$, $b_{\mathbf{p}}$) are creation (annihilation) operators of particles and antiparticles, respectively. The commutation relations between operators are standard: $[a_{\mathbf{p}}, a^\dagger_{\mathbf{q}}] = \delta_{\mathbf{p},\mathbf{q}}$, $[b_{\mathbf{p}}, b^\dagger_{\mathbf{q}}] = \delta_{\mathbf{p},\mathbf{q}}$.

The Hamiltonian operator, $H = \int(\mathcal{H}_0 + \mathcal{W})d^3x$, based on $\mathcal{L}(\chi^\dagger, \chi)$, is determined by the following terms:

$$\mathcal{H}_0 = \partial_t\chi^\dagger\partial_t\chi + \nabla\chi^\dagger\nabla\chi + m_*^2\chi^\dagger\chi, \quad (9)$$

$$\mathcal{W} = (m^2 - m_*^2)\chi^\dagger\chi - \frac{A}{2!}(\chi^\dagger\chi)^2 + \frac{B}{3!}(\chi^\dagger\chi)^3. \quad (10)$$

One has

$$H = \sum_{\mathbf{p}} E_{\mathbf{p}}(n_{\mathbf{p}} + \bar{n}_{\mathbf{p}}) + \int \mathcal{W}\, d^3x, \quad (11)$$

where $n_{\mathbf{p}} \equiv a^\dagger_{\mathbf{p}} a_{\mathbf{p}}$ and $\bar{n}_{\mathbf{p}} \equiv b^\dagger_{\mathbf{p}} b_{\mathbf{p}}$ are the number operators of particles and antiparticles, respectively.

Assuming that the interaction term is the perturbation, thermodynamic potential can be written as

$$\Omega = T\sum_{\gamma,\mathbf{p}} \ln\left[1 - e^{\beta(\gamma\mu - E_{\mathbf{p}})}\right] + V\mathcal{W}_{\mathrm{MFA}}, \quad (12)$$

where μ is the chemical potential, $\beta = 1/T$ is inverse temperature, $\gamma = \pm 1$.

In our approximation, the interaction term is as follows

$$\mathcal{W}_{\mathrm{MFA}} = (m^2 - m_*^2)\sigma + m^2(b\sigma^3 - 2a\sigma^2), \quad (13)$$

where $\sigma = \langle\chi^\dagger\chi\rangle$ is the order parameter, $a = 1/2g^2$, $b = 3\lambda/2g^4$.

We find thermodynamically conjugate variables σ and m_* from the extremizing conditions $\partial\Omega/\partial m_* = 0$, $\partial\Omega/\partial\sigma = 0$, which result in the gap equation,

$$\sigma = \frac{1}{V}\sum_{\gamma,\mathbf{p}} \frac{1}{2E_{\mathbf{p}}} \frac{1}{e^{\beta(E_{\mathbf{p}} - \gamma\mu)} - 1}, \quad (14)$$

and the mass ratio, $s^2 \equiv m_*^2/m^2 = 1 - 4a\sigma + 3b\sigma^2$.

Further investigation of the thermodynamic properties of hot pion matter is carried out in the thermodynamic limit, when the replacement of summation by integration over momentum takes place. Here we neglect a contribution of the surface.

It turns out that, to solve correctly the constraint, it is helpful to operate with dimensionless variable s instead of σ. It leads at once to the splitting of the value interval of σ into two branches: $\sigma = (2g^2/9\lambda)[1 \pm \sqrt{1-(9/2)\lambda(1-s^2)}]$. Such a representation of σ gives us two equations $F_\pm(s,T,\mu)=0$, where

$$F_\pm(s,T,\mu) = 1 \pm \sqrt{1-\frac{9}{2}\lambda(1-s^2)} - \frac{\lambda}{2}\left(\frac{3m}{2\pi g}\right)^2 s^2 \times \sum_\gamma \int_0^\infty \frac{dk}{\sqrt{1+k^2}} \frac{k^2}{e^{\beta\left(ms\sqrt{1+k^2}-\gamma\mu\right)}-1}. \quad (15)$$

The solution of these equations is the function $s(T,\mu)$ which is calculated numerically.

One can say that this splitting reflects the existence of two phases of hot pion matter. If $s=1$, we find that a single phase, corresponding to ideal gas, survives. Switching on an interaction ($s \neq 1$), a new phase, called hot pion liquid, can arise. The appearance of this phase is possible due to the sixth-order term.

In order to compute thermodynamic functions, one needs to know the values of two parameters of our model, namely, g and λ. We have already assumed that $\lambda = 0.26$ to achieve liquid-gas phase transition in condensate. To fit constant g, we try to appeal to experimental data obtained by DLS Collaboration in Berkeley and extract necessary information from dilepton spectra produced in proton–nucleus reaction [15]. If we believe that the main contribution to the spectrum comes from hot $\pi^+\pi^-$ annihilation, one can conclude that the pion annihilation threshold is not equal to twice free pion mass ($2m$) but smaller and approximately is $2ms = 260$ MeV. Our fit to the spectra [15], which depends on the dielectron invariant mass, gives us the temperature of the fireball, formed in this collision, equal to $T = 75$ MeV. Therefore, to find g at chemical equilibrium ($\mu = 0$), we need to substitute these data in our equations. One obtains that $g \approx 8$ MeV from equation $F_+(s,T,0)=0$. Using the equation $F_-(s,T,0)=0$, another value of g is derived. However, the liquid-gas PT is not observed in this case.

Assuming chemical equilibrium, there is the only possibility to observe the transition into liquid phase with changing temperature. Indeed, the dense "liquid" phase appears with increasing temperature (see Fig. 2) that is in contrast to molecular physics, where the number of particles is conserved. This phenomenon has been already pointed out in [11]. The critical temperature of the PT in our model is about 42.3 MeV and less than 136 MeV as it was predicted in [11]. However, we should conclude that, at $T = 75$ MeV of fireball, the pions are in the liquid phase, if the scenario, when $g \approx 8$ MeV, is realized in nature. This outcome demands additional theoretical and experimental verifications.

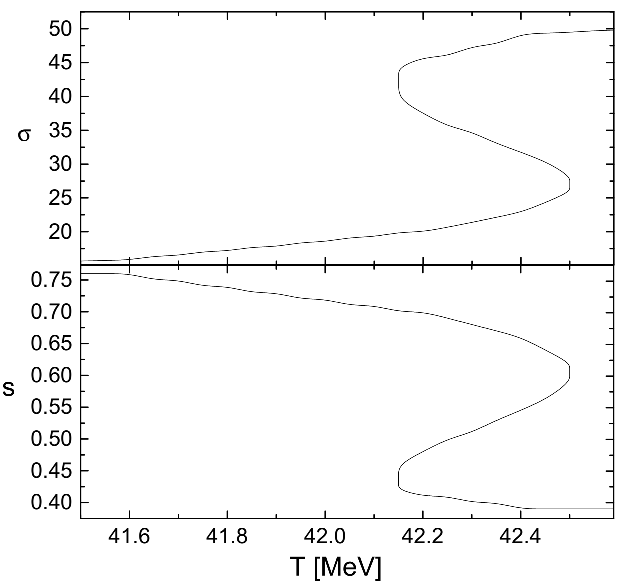

Fig. 2. The dependence of the scalar density σ (in MeV^2) and the reduced effective mass $s \equiv m_*/m$ on temperature.

Note that the calculations with the non-vanishing chemical potential can be carried out and will be published elsewhere.

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РІДИННО-ПОДІБНІ ФАЗИ $\pi^+\pi^-$ МАТЕРІЇ

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Щоб дати у широкому температурному інтервалі єдиний теоретичний опис рідких фаз матерії, що складається із заряджених піонів, розглянуто релятивістську квантову модель типу φ^6. Досліджено рідкі стани піонного конденсату та "гарячої" піонної матерії.