ЖУРНАЛ ФІЗИЧНИХ ДОСЛІДЖЕНЬ
т. 11, № 4 (2007) с. 390–395

JOURNAL OF PHYSICAL STUDIES
v. 11, No. 4 (2007) p. 390–395

THE EFFECT OF LATTICE-QCD-BASED GLUON PROPAGATOR ON COHERENCE LENGTH OF QUARK COOPER PAIRS IN TWO-FLAVOR COLOR SUPERCONDUCTOR

Hiyoshi Kiuchi
Department of Nutritional Sciences, Junshin Junior College,
1–1–1 Chikushigaoka, Minami-Ku, Fukuoka City, 815–8510, Japan
(Received April 17, 2007; received in final form September 27, 2007)

The coherence length (ξ) of quark Cooper pairs for two flavors is studied numerically within the framework of a modified QCD-like gauge field theory with the lattice-QCD-based gluon propagator, which is derived from the lattice QCD data. The propagator is considered to include all the nonperturbative effects in the quenched QCD. We find that the coherence length in our model ξ(A) is smaller than that in the QCD-like theory with the tree-level gluon propagator ξ(B). We find that $\xi(\mathrm{A})/d < 1$ at $\mu < 0.65$ GeV, while $\xi(\mathrm{B})/d < 1$ at $\mu < 0.45$ GeV (d: interquark distance, μ: quark chemical potential). Accordingly, Cooper pair in our model is rather bosonic in low to moderate μ region where two-flavor color superconducting phase is possibly realized.

Key words: lattice-QCD-based gluon propagator, QCD-like theory, coherence length, quark Cooper pair, two-flavor quark matter, Bose–Einstein condensation.

PACS number(s): 12.38.−t, 12.38.Mh, 12.39.−x

I. INTRODUCTION

A lot of studies based on the effective models of QCD such as the Nambu–Jona–Lasinio model [1–4], the instanton vacuum model [5,6] and the QCD-like theory [7–10] suggest that, at sufficiently high temperature (T) and/or quark chemical potential (μ), a system with quarks and gluons makes a phase transition from a chiral symmetry broken (CSB) phase to the chirally symmetric one (quark–gluon plasma or color superconductor, which is a condensed state of quark pairs). In the color superconducting phase, color symmetry is spontaneously broken. At $T = 0$, it is believed that a phase transition occurs between a CSB phase and the one of the color superconducting phases [two-flavor color superconducting phase (2SC), color-flavor locking phase (CFL), LOFF state, etc.] [4], [9–13]. This phase transition is considered to occur at moderate μ ($\mu \sim 0.4\,\mathrm{GeV}$) [2,5,6]. In high-$\mu$ region, strangeness is important. As the value of μ decreases, strangeness becomes unimportant. Accordingly, in low to moderate-μ region, especially near the chiral symmetry restorating point, chirally symmetric quark matter may lie in a 2SC phase that is made up of only up and down quarks.

In cold quark matter, if quark-quark interaction is strong enough, two quarks of a Cooper pair (quark–quark pair) may exist close to each other, in consequence, Cooper pairs may be in Bose–Einstein condensation (BEC). In BEC phase, coherence length (ξ), which is the squared mean distance of two paired particles, is smaller than the averaged interparticle distance (d) of relevant particles ($\xi/d < 1$). Because of the asymptotic freedom of QCD, quark–quark coupling strength subsides as μ grows. Accordingly, realizability of quark-BEC increases as μ decreases. Therefore, the vicinity of the chiral symmetry restorating point is the most probable area for the quark BEC. Recently, some studies have been reported concerning ξ and spatial structure of quark Cooper pairs in 2SC based on the Schwinger–Dyson equation (S–D eq.) in the ladder approximation [14,15]. In these studies, the tree-level gluon propagator is used. This simplification may result in neglecting possible nonperturbative effects. It is shown in Ref. (9) that the S–D eq. for the effective mass in the ladder approximation can be derived within the QCD-like theory with the tree-level gluon propagator, which is the usual choice in the theory. We can also show that the S–D eq. for diquark energy gap in the ladder approximation can be derived within the same theory. In the QCD-like theory, the one-loop running coupling ($\bar{g}$) is introduced instead of the coupling constant (g) for the quark–gluon vertex. By this improvement, the asymptotic freedom of QCD is satisfied.

The main goal of the present study is, making use of the lattice-QCD-based gluon propagator instead of the tree-level one, to calculate the coherence length ξ within the framework of the QCD-like gauge field theory in mean-field approximation and to estimate the effect of the lattice-QCD-based gluon propagator on the coherence length. The lattice-QCD-based gluon propagator, which is derived from the lattice QCD data, exhibits infrared vanishing and strong enhancement at the intermediate-energy region $p \sim 1$ GeV (p : transfer momentum) [16,17]. The propagator is considered to include all the nonperturbative effects in the quenched QCD. The intermediate energy region is demonstrated to be the most important region for dynamical chiral symmetry breaking [17]. Assuming that the nonperturbative effects are important in quark–quark (q–q) pairing as well as antiquark–quark ($\overline{q}$–q) pairing, we combine the QCD-like theory and the lattice-QCD-based gluon propagator for the investigation. Hereafter, we refer to the QCD-like theory with the lattice-QCD-based gluon propagator and to the theory with the tree-level one as model A and the model B, respectively. We estimate the effect of the lattice-QCD-based gluon propagator on ξ by comparing values of ξ in model A with those in the model B.

Throughout the paper, we restrict ourselves to $N_f = 2$, corresponding to a system of up and down quarks. The $q-q$ interaction is most attractive in the Lorentz scalar, total spin singlet ($J = 0$), color anti-triplet ($\overline{3}$) and flavor anti-symmetric channel. Consequently, nonzero diquark condensate $\langle qC\gamma_5 q\rangle$ breaks color $SU(3)$ symmetry down to $SU(2)$ symmetry [11].

The outline of the paper is as follows. In the next Section, we combine the lattice-QCD-based gluon propagator and the QCD-like gauge field theory and derive the gap equation for momentum-dependent diquark energy gap $\Delta_{\mathbf{p}}$ in the mean-field approximation. In Section III, we give the equation for coherence length. In Section IV, we solve the gap equation and compute Cooper pair wave function and the coherence length, and present the numerical results. Section V is devoted to conclusions.

II. GAP EQUATION

In this section, we combine the lattice-QCD-based gluon propagator and the QCD-like gauge field theory and derive the gap equation for momentum-dependent diquark energy gap $\Delta_{\mathbf{p}}$ in the mean-field approximation. Let us start with an effective Hamiltonian (H) with gluon exchange interaction:

$$H = H_0 + H_{\rm I}, \tag{1}$$

where

$$H_0 = \int d^3x \overline{\Psi}(x)(i\nabla\!\!\!/ - m)\Psi(x), \tag{2}$$

$$H_{\rm I} = \int d^3x d^3y \frac{g^2}{2}\overline{\Psi}(x)\gamma_\mu \frac{\lambda^A}{2}\Psi(x) \times D(x-y)\overline{\Psi}(y)\gamma^\mu \frac{\lambda^A}{2}\Psi(y), \tag{3}$$

with current quark mass m, the coupling constant g^2 and the color $SU(3)$ matrices λ^A. Here, the gluon propagator $D(x-y)$ is given by

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3}\frac{d(p^2)}{p^2}{\rm e}^{-ip(x-y)}, \tag{4}$$

where $d(p^2)$ is the polarization factor.

In this study, we adopt the polarization factor of the lattice-QCD-based gluon propagator which is derived using the quenched lattice QCD data. The usual choice of the polarization factor in the QCD-like theory is that of the tree-level gluon propagator, i. e., $d(p^2) = 1$.

The polarization factor $d(p^2)$ of the lattice-QCD-based gluon propagator is well described by the following analytic function [16,17]:

$$d(p^2) = Z_g \frac{p^4 + ap^2}{p^4 + \alpha p^2 + \beta}, \tag{5}$$

where $a = 7.887$ GeV^2, $\alpha = 1.254$ GeV^2, $\beta = 0.7175$ GeV^4 and $Z_g = 0.7172$ (Fig. 1).

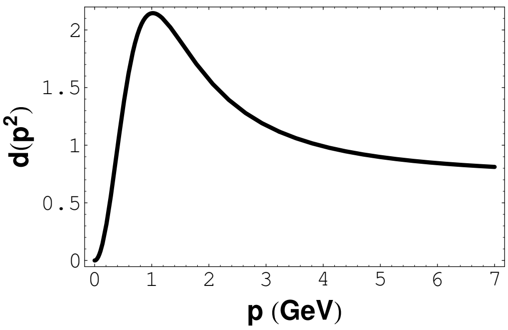

Fig. 1. The polarization factor $d(p^2)$ of the lattice-QCD-based gluon propagator as a function of transfer momentum p exhibits the infrared vanishing and strong enhancement at the intermediate-energy region $p \sim 1$ GeV.

In this study, we concentrate on the Lorentz scalar $qC\gamma_5 q(\overline{q}C^\dagger\gamma_5\overline{q})$ bilinears in two-flavor quark matter. In the $2SC$ phase, a diquark condensate consists of only two of the three colors [11].

Then, the Fierz-rearranged Hamiltonian in 3-momentum space for two flavors is given by

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm I}, \tag{6}$$

where

$$\hat{H}_0 \equiv \sum_{\mathbf{p}} e_{\mathbf{p}} C_{\rm R}^{\dagger\alpha,s}(\mathbf{p})C_{\rm R}^{\alpha,s}(\mathbf{p}) + {\rm R}\to{\rm L}, \tag{7}$$

$$\hat{H}_{\rm I} \equiv -\frac{1}{2}g'^2\sum_{\mathbf{p},\mathbf{p}'}D(\mathbf{p},\mathbf{p}')C_{\rm R}^{\alpha,s\dagger}(\mathbf{p}) \times C_{\rm R}^{\beta,t\dagger}(-\mathbf{p})C_{\rm R}^{\gamma,i}(-\mathbf{p}')C_{\rm R}^{\delta,j}(\mathbf{p}') \times \epsilon_{\alpha\beta 3}\epsilon_{\gamma\delta 3}\epsilon_{st}\epsilon_{ij} + {\rm R}\to{\rm L}, \tag{8}$$

with

$$D(\mathbf{p},\mathbf{p}') = \frac{d(|\mathbf{p}-\mathbf{p}'|^2)}{|\mathbf{p}-\mathbf{p}'|^2} = \frac{Z_g\left(|\mathbf{p}-\mathbf{p}'|^2 + a\right)}{|\mathbf{p}-\mathbf{p}'|^4 + \alpha|\mathbf{p}-\mathbf{p}'|^2 + \beta}. \tag{9}$$

Here $C_{\rm R(L)}^{\dagger\alpha,s}/C_{\rm R(L)}^{\alpha,s}$ denotes the creation/annihilation operator of a right(left)-handed particle with color α and flavor s, $e_{\mathbf{p}} \equiv \sqrt{|\mathbf{p}|^2 + m^2}$, $g'^2 = \frac{1}{6}g^2$, $\alpha, \beta, \gamma, \delta$ denote color indices, i, j, s, t denote flavor indices.

We assume that the Fermi sphere of the quarks bearing the third color is intact, and choose a wave function for the ground state $|\Psi_g\rangle$ of the form,

$$|\Psi_g\rangle = \Psi_{\rm L}^\dagger\Psi_{\rm R}^\dagger|0\rangle, \tag{10}$$

where

$$\Psi_{\rm R}^\dagger = \prod_{\mathbf{p}} [u_{\mathbf{p}} + v_{\mathbf{p}} C_{\rm R}^{\dagger\alpha,s}(\mathbf{p}) C_{\rm R}^{\dagger\beta,t}(-\mathbf{p}) \epsilon_{\alpha\beta3}\epsilon_{st}], \tag{11}$$

$$\Psi_{\rm L}^\dagger = {\rm R} \to {\rm L}. \tag{12}$$

Here, the color indices α and β run from 1 to 2, and the parameters obey the constraint that

$$|u_{\mathbf{p}}|^2 + |v_{\mathbf{p}}|^2 = 1. \tag{13}$$

We find that the diquark condensate is calculated as

$$\langle \Psi_g | C_{\rm R}^{\dagger\alpha,s}(\mathbf{p}) C_{\rm R}^{\dagger\beta,t}(-\mathbf{p}) \epsilon_{\alpha\beta3}\epsilon_{st} | \Psi_g \rangle = (N_c - 1) N_f u_{\mathbf{p}}^* v_{\mathbf{p}}. \tag{14}$$

Following a well-trodden path, we can find the gap equation. Let us rewrite the effective Hamiltonian in terms of quasiparticle creation/annihilation operators. To this end, we perform the inverse Bogoliubov–Valatin transformation. We note that a diquark condensate consists of two quarks bearing the same helicity but different color and flavor. Four kinds of quasiparticles exist, reflecting four kinds of relevant quarks. The transformation is given by

$$\begin{bmatrix} C_{\rm R(L)}^{11}(\mathbf{p}) \\ C_{\rm R(L)}^{22\dagger}(-\mathbf{p}) \end{bmatrix} = \begin{bmatrix} u_{\mathbf{p}}^* & v_{\mathbf{p}} \\ -v_{\mathbf{p}}^* & u_{\mathbf{p}} \end{bmatrix} \begin{bmatrix} a_{\rm R(L)}^{1}(\mathbf{p}) \\ a_{\rm R(L)}^{2\dagger}(-\mathbf{p}) \end{bmatrix}, \tag{15}$$

and

$$\begin{bmatrix} C_{\rm R(L)}^{12}(\mathbf{p}) \\ C_{\rm R(L)}^{21\dagger}(-\mathbf{p}) \end{bmatrix} = \begin{bmatrix} u_{\mathbf{p}}^* & v_{\mathbf{p}} \\ -v_{\mathbf{p}}^* & u_{\mathbf{p}} \end{bmatrix} \begin{bmatrix} a_{\rm R(L)}^{3}(\mathbf{p}) \\ a_{\rm R(L)}^{4\dagger}(-\mathbf{p}) \end{bmatrix}, \tag{16}$$

where $a_{\rm R(L)}^\dagger / a_{\rm R(L)}$ is creation/annihilation operator that creates/annihilates a quasiparticle of right-handed(left-handed) type.

The thermodynamic potential $\hat{\Omega}$ in the quasiparticle basis is given by

$$\hat{\Omega} \equiv \hat{H} - \mu \hat{N} = \sum_p \left[\left(a_{\rm R}^{1\dagger}(\mathbf{p}) \;\; a_{\rm R}^{2}(-\mathbf{p}) \right) \boldsymbol{D} \begin{pmatrix} a_{\rm R}^{1}(\mathbf{p}) \\ a_{\rm R}^{2\dagger}(-\mathbf{p}) \end{pmatrix} + \left(a_{\rm R}^{3\dagger}(\mathbf{p}) \;\; a_{\rm R}^{4}(-\mathbf{p}) \right) \boldsymbol{D} \begin{pmatrix} a_{\rm R}^{3}(\mathbf{p}) \\ a_{\rm R}^{4\dagger}(-\mathbf{p}) \end{pmatrix} \right] + {\rm R} \to {\rm L}, \tag{17}$$

where

$$\hat{N} = \sum_{\mathbf{p}} \left(C_{\rm R}^{\dagger\alpha,s}(\mathbf{p}) C_{\rm R}^{\alpha,s}(\mathbf{p}) + C_{\rm L}^{\dagger\alpha,s}(\mathbf{p}) C_{\rm L}^{\alpha,s}(\mathbf{p}) \right), \tag{18}$$

$$\boldsymbol{D} \equiv \begin{bmatrix} u_{\mathbf{p}} & -v_{\mathbf{p}} \\ v_{\mathbf{p}}^* & u_{\mathbf{p}}^* \end{bmatrix} \begin{bmatrix} e_{\mathbf{p}} - \mu & -\Delta_{\mathbf{p}} \\ -\Delta_{\mathbf{p}}^* & -(e_{\mathbf{p}} - \mu) \end{bmatrix} \times \begin{bmatrix} u_{\mathbf{p}}^* & v_{\mathbf{p}} \\ -v_{\mathbf{p}}^* & u_{\mathbf{p}} \end{bmatrix}. \tag{19}$$

Here, we introduce a gap function $\Delta_{\mathbf{p}}$:

$$\Delta_{\mathbf{p}} \equiv \frac{1}{2} g'^2 \sum_{\mathbf{p}'} D(\mathbf{p}, \mathbf{p}') \times \langle \Psi_g | C_{\rm R}^{\alpha,s}(-\mathbf{p}') C_{\rm R}^{\beta,t}(\mathbf{p}') \epsilon_{\alpha\beta3}\epsilon_{st} | \Psi_g \rangle = (N_c - 1) N_f \frac{g'^2}{2} \sum_{\mathbf{p}'} D(\mathbf{p}, \mathbf{p}') u_{\mathbf{p}'} v_{\mathbf{p}'}^*. \tag{20}$$

The values of the parameters $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are chosen so that $\hat{\Omega}$ has the form of free quasiparticles. After some algebra, we find that the thermodynamic potential $\hat{\Omega}$ is given by

$$\hat{\Omega} = \Omega_g + \sum_{\mathbf{p}} \left[E_{\mathbf{p}} a_{\rm R}^{\nu\dagger}(\mathbf{p}) a_{\rm R}^{\nu}(\mathbf{p}) + {\rm R} \to {\rm L} \right], \tag{21}$$

where the superscript ν runs from 1 to 4, and Ω_g is the groud state thermodynamic potential:

$$\Omega_g = (N_c - 1) N_f \times \sum_{\mathbf{p}} \left[2(e_{\mathbf{p}} - \mu) |v_{\mathbf{p}}|^2 - \Delta_{\mathbf{p}} u_{\mathbf{p}} v_{\mathbf{p}}^* - \Delta_{\mathbf{p}}^* u_{\mathbf{p}}^* v_{\mathbf{p}} \right], \tag{22}$$

with

$$|u_{\mathbf{p}}|^2 = \frac{1}{2} \left(1 + \frac{e_{\mathbf{p}} - \mu}{E_{\mathbf{p}}} \right), \tag{23}$$

$$|v_{\mathbf{p}}|^2 = \frac{1}{2} \left(1 - \frac{e_{\mathbf{p}} - \mu}{E_{\mathbf{p}}} \right), \tag{24}$$

$$E_{\mathbf{p}} = \sqrt{(e_{\mathbf{p}} - \mu)^2 + \Delta_{\mathbf{p}}^2}. \tag{25}$$

In addition, we find that the diquark condensate is given by

$$\langle C_{\rm R}(-\mathbf{p}) C_{\rm R}(\mathbf{p}) \rangle = u_{\mathbf{p}} v_{\mathbf{p}}^* = \frac{\Delta_{\mathbf{p}}}{2E_{\mathbf{p}}} \equiv \phi(\mathbf{p}). \tag{26}$$

Substituting Eq. (26) into Eq. (20), we obtain the following gap equation:

$$\Delta_{\mathbf{p}} = \frac{(N_c - 1) N_f}{12} g^2 \sum_{\mathbf{p}, \mathbf{p}'} D(\mathbf{p}, \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{2E_{\mathbf{p}'}}. \tag{27}$$

In the QCD-like theory, the one-loop running coupling ($\bar{g}$) is introduced instead of the coupling constant (g) for the quark-gluon vertex with an infrared regularization parameter ($p_{\rm R}$)[8–10]:

$$g^2 \to \bar{g}^2(p^2) = \frac{2\pi b}{\log[(p^2 + p_{\rm R}^2)/\Lambda_{\rm QCD}^2]}, \tag{28}$$

where

$$b = \frac{6(N_c^2 - 1)}{2N_c(11 - 2N_f/3)}. \tag{29}$$

It should be noted that the asymptotic freedom in the deep Euclidean region is satisfied by exploiting this running coupling $\bar{g}$.

Finally, we obtain the gap equation we ought to solve:

$$\Delta_{\mathbf{p}} = \frac{(N_c - 1) N_f}{12} \int \frac{d^3 p'}{(2\pi)^3} \bar{g}^2(\mathbf{p}, \mathbf{p}') D(\mathbf{p}, \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{2E_{\mathbf{p}'}} = \int \frac{d^3 p'}{6(2\pi)^3} \cdot \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} \cdot \frac{Z_g \bar{g}^2(\mathbf{p}, \mathbf{p}') \left(|\mathbf{p} - \mathbf{p}'|^2 + a \right)}{|\mathbf{p} - \mathbf{p}'|^4 + \alpha |\mathbf{p} - \mathbf{p}'|^2 + \beta}. \tag{30}$$

III. COHERENCE LENGTH ξ

If $|v_{\mathbf{p}}|^2 \ll 1$, we can show that, with nonrelativistic approximation, the gap equation Eq. (27) reduces to the Bethe–Goldstone equation of the two-body system as

$$\frac{|\mathbf{p}|^2}{m}\phi(\mathbf{p}) + \frac{(N_c-1)N_f g^2}{6}\sum_{\mathbf{p}'} D(\mathbf{p},\mathbf{p}')\phi(\mathbf{p}') \sim 2(\mu - m)\phi(\mathbf{p}), \tag{31}$$

where $m(\neq 2m)$ is the reduced mass of the two-body system.

Accordingly, the two-body wave function of a quark Cooper pair apart from unimportant normalization constant is given by

$$\phi(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3}\phi(\mathbf{p})\mathrm{e}^{-i\mathbf{p}\cdot\mathbf{r}} = \int_0^\infty \frac{p^2dp}{2\pi^2}\phi(\mathbf{p})j_0(pr), \tag{32}$$

where $j_0(pr) = \frac{\sin(pr)}{pr}$ is the zeroth-order spherical Bessel function of the 1st kind with $p = |\mathbf{p}|$ and $r = |\mathbf{r}|$.

The coherence length ξ, which is defined by the squared mean distance of two paired particles, can be calculated as

$$\xi = \left(\frac{\int d^3r|\phi(\mathbf{r})|^2r^2}{\int d^3r|\phi(\mathbf{r})|^2}\right)^{\frac{1}{2}}. \tag{33}$$

The averaged interparticle distance d of relevant quarks is given by

$$d = \frac{\left(\frac{3\pi^2}{4}\right)^{\frac{1}{3}}}{p_{\mathrm{F}}}, \tag{34}$$

where p_{F} stands for the Fermi momentum.

IV. NUMERICAL RESULTS

In this section, after obtaining the momentum-dependent pair wave function by solving the gap equation Eq. (30), we calculate the coherence length (ξ) of quark Cooper pairs. For numerical calculation, we set the values of Λ_{QCD} and the infrared regulator(p_{R}) in the running coupling to 738 MeV and $\mathrm{e}^{0.05}\Lambda_{\mathrm{QCD}}$, respectively [8,9]. In addition, we set the current quark mass to $m = 0$ (the chiral limit). Then, the Fermi momentum equals the quark chemical potential ($p_{\mathrm{F}} = \mu$).

The pair wave functions $|\phi(\mathbf{r})|^2$ at $\mu = 0.3$ GeV and at $\mu = 0.5$ GeV in co-ordinate space are plotted in Fig. 2. The functions have the maximum values at $|\mathbf{r}| = 0$ ($\mathbf{r}$: relative co-ordinate), decrease steeply as $|\mathbf{r}|$ grows and almost vanishes at $|\mathbf{r}| = 0.5$ fm. In addition, we find that the value of $|\phi(\mathbf{r})|$ at $|\mathbf{r}| = 0$ increases as μ increases. This reflects the fact that two quarks of a Cooper pair get close to each other as μ grows. However, as we will see below, the averaged interquark distance d gets shorter more rapidly as μ grows, consequently, the ratio ξ/d increases as μ grows.

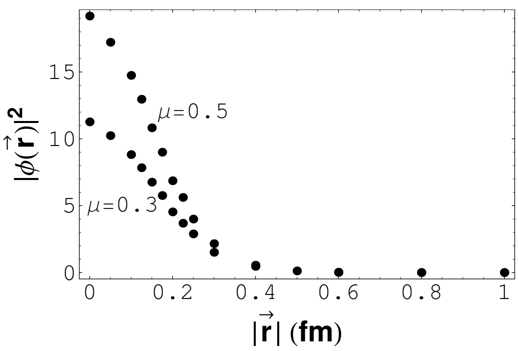

Fig. 2. Cooper-pair wave functions $|\phi(\mathbf{r})|^2$ for different values of μ (0.3 GeV and 0.5 GeV) in co-ordinate space apart from normalization constant.

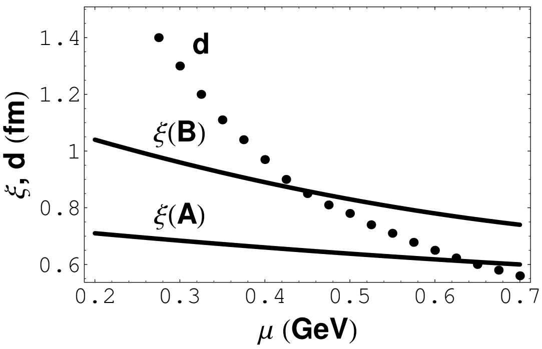

Fig. 3. The coherence length (ξ) and the interquark distance (d) as functions of quark chemical potential (μ). The ξ(A) line, which is the ξ line in the model A, crosses the d line at $\mu \sim 0.65$ GeV. While ξ(B) line, which is the ξ line in the model B, crosses the d line at $\mu \sim 0.45$ GeV.

Figure 3 plots the values of ξ and d as functions of μ. ξ as well as d decreases monotonically as μ grows. The μ dependence of ξ is weaker than that of d. The ξ(A) line, which is the ξ line in the QCD-like theory with the lattice-QCD-based gluon propagatoar (model A), crosses the d line at $\mu \sim 0.65$ GeV. While the ξ(B) line, which is the ξ line in the QCD-like theory with the tree-level gluon propagator (model B), crosses the d line at $\mu \sim 0.45$ GeV.

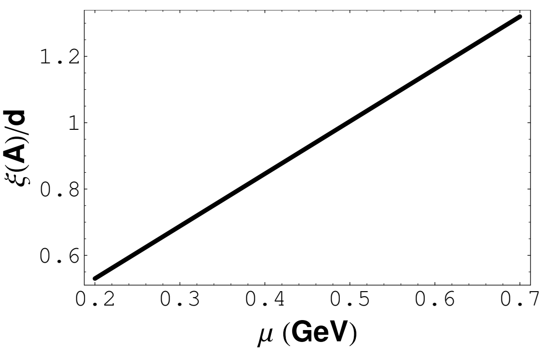

Fig. 4. The ratio $\xi(A)/d$ as a function of μ. $\xi(A)/d$ increases almost linearly as μ grows ($\xi(A)/d \sim 1.42\mu + 0.076$). $\xi(A)/d < 1$ in low to moderate μ region where 2SC phase is possibly realized.

Figure 4 plots the ratio $\xi(\mathrm{A})/d$. The ratio increases as μ grows. We see that $\xi(\mathrm{A})/d < 1$ in moderate-μ region where 2SC phase is possibly realized.

V. CONCLUSIONS

In this paper, coherence length (ξ) of quark Cooper pairs and spatial structure of the pair wave function in two-flavor quark matter have been investigated. We have performed this study within a framework of a modified QCD-like theory in the mean-field approximation. We have employed the lattice-QCD-based gluon propagator, which is extracted from the quenched lattice QCD data. The polarization factor of the lattice-QCD-based gluon propagator is finite at IR region and enhanced at $p \sim 1$ GeV and then decreases as p increases (p: transfer momentum) [16,17]. The intermediate energy region is demonstrated to be the most important region for dynamical chiral symmetry breaking [17]. The usual choice of the gluon propagator in the QCD-like theory is the tree-level one [8–10]. This simplification may result in neglecting possible nonperturbative effects. Meanwhile, the lattice-QCD-based gluon propagator is considered to include all the nonperturbative effects in the quenched QCD. Assuming that the nonperturbative effects are important in quark–quark (q–q) pairing as well as antiquark–quark ($\overline{q}$-q) pairing, we have combined the QCD-like theory and the lattice-QCD-based gluon propagator for the investigation.

So far a few studies were reported about coherence length ξ of quark Cooper pairs for two flavors [14,15]. In Refs. [14] and [15], the one-loop Schwinger–Dyson (S–D) equation in the ladder approximation with infrared safe running coupling is used for obtaining Cooper pair wave function. In one of them (Ref. [15]), they reported the result only in a relatively high density region ($\mu > 0.8\,\mathrm{GeV}$). In the other (Ref. [14]), the result in a moderate density region ($0.3 < \mu < 0.65\,\mathrm{GeV}$) was reported, but the quasiparticle energy in Ref. (14) is that for one-flavor system [18], because it contains $3\Delta_{\mathbf{p}}^2$ $\left(E_{\mathbf{p}} = \sqrt{(e_{\mathbf{p}} - \mu)^2 + 3\Delta_{\mathbf{p}}^2}\right)$ instead of $\Delta_{\mathbf{p}}^2$ $\left(E_{\mathbf{p}} = \sqrt{(e_{\mathbf{p}} - \mu)^2 + \Delta_{\mathbf{p}}^2}\right)$. Fortunately, the coefficient in front of $\Delta_{\mathbf{p}}$ does not affect the value of ξ, although it affects the magnitude of $\Delta_{\mathbf{p}}$. It is shown in Ref. (9), that the one-loop S–D eq. with the ladder approximation can be derived within the QCD-like theory with the tree-level gluon propagator.

At $T = 0$, phase transition between the chiral symmetry is broken and color superconducting phases is considered to occur at moderate-μ ($\mu \sim 0.4\,\mathrm{GeV}$) [2–6]. As suggested in the Introduction, in the moderate μ region, especially near the chiral symmetry restorating point, chirally symmetric quark matter may lie in the 2SC phase. In addition, the region in the vicinity of the chiral symmetry restorating point is the most probable area for the quark-BEC. Hence, we pay special attention to the moderate-μ region. At each value of μ, $\xi(\mathrm{A})$, which is ξ in the QCD-like theory with the lattice-QCD-based gluon propagator (model A), is smaller than $\xi(\mathrm{B})$, which is ξ in the theory with the tree-level gluon propagator (model B). We have found that $\xi(\mathrm{A})/d < 1$ at $\mu < 0.65$ GeV, while $\xi(\mathrm{B})/d < 1$ at $\mu < 0.45\,\mathrm{GeV}$. Thus the Cooper pair in the model A is rather bosonic. This result may imply that the intermediate-energy region in the lattice-QCD-based gluon propagator is important in $q-q$ pairing as well as in $\overline{q}-q$ pairing. The Cooper pair whose ξ is smaller than d suggests that BEC description may be useful as in the analogous example in condensed matter physics [19–21].

It should be noted that, in this study, we have ignored the μ-dependence of the coupling strength. Therefore, if we use a μ-dependent running coupling, we may obtain more or less different result especially in the relatively high μ region.

[1] M. Asakawa, K. Yazaki, Nucl. Phys. **A504**, 668 (1989).
[2] J. Berges, K. Rajagopal, Nucl. Phys. **B538**, 215 (1999).
[3] M. Huang, P. Zhuang, Phys. Rev. D **65**, 076012 (2002).
[4] H. Abuki, M. Kitazawa, T. Kunihiro, Phys. Lett. B **615**, 102 (2005).
[5] G. W. Carter, D. Diakonov, Phys. Rev. D **60**, 016004 (1999).
[6] H. Kiuchi, M. Oka, Prog. Theor. Phys. **114**, 813 (2005).
[7] Y. Taniguchi, Y. Yashida, Phys. Rev. D **55**, 2283 (1997).
[8] O. Kiriyama, M. Maruyama, F. Takagi, Phys. Rev. D **62**, 105008 (2000).
[9] M. Iwasaki, H. Kiuchi, Phys. Rev. D **68**, 014014 (2003).
[10] Y. Hashimoto, Y. Tsue, H. Fujii, Prog. Theor. Phys. **114**, 595 (2005).
[11] M. Alford, K. Rajagopal, F. Wilczek, Phys. Lett. B **422**, 247 (1998).
[12] M. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
[13] M. Alford, J. A. Bowers, K. Rajagopal, Phys. Rev. D **63**, 074016 (2001).
[14] M. Matsuzaki, Phys. Rev. D **62**, 017501 (2000).
[15] H. Abuki, T. Hatsuda, K. Itakura, Phys. Rev. D **65**, 074014 (2002).
[16] P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly, A. G. Williams, Phys. Rev. D **70**, 034509 (2004).
[17] H. Iida, M. Oka,, H. Suganuma, Eur. Phys. J. A **23**, 305 (2005).
[18] M. Iwasaki, T. Iwado, Phys. Lett. B **350**, 163 (1995).
[19] C. A. Regal, C. Ticknor, J. L. Bohn, D. S. Jin, Nature **424**, 47 (2003).
[20] C. A. Regal, M. Greiner, D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
[21] M. W. Zwierlein, C. H. Schunck, C. A. Stan, S. M. F. Raupach, W. Ketterle, Phys. Rev. Lett. **94**, 180401 (2005).

ВПЛИВ ҐРАТКОВО-КХД ҐЛЮОННОГО ПРОПАҐАТОРА НА КОГЕРЕНТНУ ДОВЖИНУ КВАРКОВИХ КУПЕРІВСЬКИХ ПАР У ДВОАРОМАТНОМУ КОЛЬОРОВОМУ НАДПРОВІДНИКУ

Гійоші Кіучі
Відділ харчових наук, Коледж Джуншін, 1-1-1 Чікушіґаока,
Мінамі-ку, Фукуока, 815–8510, Японія

У статті зроблено числові дослідження когерентної довжини (ξ) кваркових куперівських пар для двох ароматів у межах модифікованої КХД-подібної калібрувальної теорії поля із ґратково-КХД ґлюонним пропаґатором, що отримується з ґраткових даних КХД. У пропаґатор включено всі непертурбативні ефекти "замороженої" КХД. Виявлено, що когерентна довжина в нашій моделі ξ(A) менша від отриманої в КХД-подібній теорії з деревоподібним ґлюонним пропаґатором ξ(B). Показано, що $\xi(\mathrm{A})/d < 1$ при $\mu < 0.65$ GeV, тоді як $\xi(\mathrm{B})/d < 1$ при $\mu < 0.45$ GeV (d — міжкваркова відстань, μ — хемічний потенціял кварків). Отже, куперівська пара в нашій моделі є радше бозонною в ділянці малих і середніх μ, де ймовірно реалізується двоароматна кольорова надпровідна фаза.