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REGGE TRAJECTORIES IN THE FRAMEWORK OF THE RELATIVISTIC ACTION-AT-A-DISTANCE THEORY

Askold Duviryak
Institute for Condensed Matter Physics of NAS of Ukraine,
1, Svientsitskii Street, Lviv, UA-79011, Ukraine,
e-mail: duviryak@icmp.lviv.ua
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A relativistic quark model of mesons formulated within the formalism of Fokker-type action integrals is proposed, in which an interquark interaction is mediated by a massless vector field and scalar–vector superposition of higher derivative fields. In the non-relativistic limit the model describes a two-particle system with the Cornell potential. In order to analyze the model in the essentially relativistic domain the perturbed circular orbit approximation is applied which permits one to perform the canonical quantization of the model. It is shown that the model reproduces well specific features of the light meson spectroscopy.

Key words: potential models, Cornell potential, Regge trajectories, Fokker action integrals, action-at-a-distance theory.

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I. INTRODUCTION

Mass spectra of heavy mesons (contaning c and b quarks) are described well by means of potential models with the static Cornell potential [1–6]

$$U(r) = -\alpha/r + ar\,. \tag{1.1}$$

The Coulomb part is a non-relativistic limit of the one-gluon exchange interaction while the linear part comes from the Wilson loop. The latter is also related to a string concept of hadrons [7–9]. There are more intricate QCD-motivated potentials such as proposed by Richardson [10], Adler–Piran [11, 12], etc. [2]. But most of them have the short-range Coulomb and the long-range linear asymptotics. Constants α and a vary from one model to another. In particular, $\alpha = \frac{4}{3}\alpha_{\rm s}$ where the *running strong coupling constant* $\alpha_{\rm s}$ grows from $0.2 - 0.25$ for heavy quarks to $0.6 - 0.8$ for light ones [2]. The *string tension* parameter a is frequently used as an adjustable parameter from the range $a = 0.15 \div 0.3\,\mathrm{GeV}^2$ [1–6]. However, in contrast to α, the parameter a is considered as a universal one, i.e. a flavor-independent characteristic of the long-range confining interaction between quarks. The most conventional value $a = 0.18 \div 0.2$ GeV^2 is substantiated recently by QCD simulations on the lattice [13].

Spin effects are taken into account by means of relativistic corrections to the static potential [1–6]. They must be attributed to the scalar or vector type, in accordance with a relativistic field-theoretical nature of the interaction. The corrections to the Coulomb term which originates from the gauge field interaction are of the vector type. The nature of the linear term is less clear; it is specified as a scalar interaction or scalar–vector admixture. The rate ξ of, say, a vector component in the long-range interaction can be used as an adjustable parameter [1–5]. The value of ξ is not well conditioned as yet, since it strongly depends on the data precision and a theoretical interpretation of fine splitting effects. On the other hand, ξ can be considered, similarly to a, as a universal but relativistic characteristic of the confinement. Thus it should be tied to some robust observables of strongly relativistic quark systems, such as light mesons consisting of u, d and s quarks.

Mass spectra of light mesons possess typical features which can be summarized roughly in the following idealized picture [9, 14–16]:

1. As states of a quark-antiquark system, mesons can be classified by the orbital and radial quantum numbers ℓ and n_r.

2. Meson states are clustered in the family of straight-line Regge trajectories in the (M^2, ℓ)–plane where M is a meson mass.

3. Principal ($n_r = 0$) and daughter ($n_r = 1, 2, \dots$) Regge trajectories are parallel; the *slope* index σ is a universal quantity, $\sigma = 1.15 \div 1.2\,\mathrm{GeV}^2$.

4. Spin effects are small, the spectrum is ℓs-degenerated.

5. States of different ℓ and n_r possess an accidental degeneracy which causes a tower structure of the spectrum.

Items 1–4 imply that energy levels of the q-$\bar{\rm q}$ system can be described by the formula:

$$M^2 \approx \sigma(\ell + \varkappa n_r + \zeta), \tag{1.2}$$

where the intercept constant ζ depends on a flavor content of mesons; $\zeta \approx 1/2$ for $(\pi$–$\rho)$-family of mesons, and it grows together with quark masses. Finally, the accidental degeneracy (item 5) constraints the constant coefficient

$\varkappa$ determining the spacing of daughter trajectories to an integer or a rational number [17].[1]

A variety of relativistic models has been invented for the description of light mesons. The most elegant and conceptually important among them are the simple relativistic oscillator (and its variations) [22–24] and string models [7–9]. They lead exactly or asymptotically (at large ℓ) to formula (1.2) with $\varkappa = 2$. Besides, the string models tie the slope index and the string tension together, $\sigma = ka$, with the slope coefficient $k = 2\pi \approx 6.3$, so that the string tension value $a = 0.18$ GeV^2 is suitable.

The subsequent development of potential models is based on various relativistic generalizations of the Schrödinger equation with a confining (Cornell (1.1) or more intricate) potential, such as one- [25–28] and two-particle [15, 16, 29–33] Dirac equations, etc. [34–36]. These models incorporate description of heavy and light hadrons and, in most, reveal asymptotically linear Regge trajectories (1.2) of the slope $\sigma = ka$ with the slope coefficient $k = 4 \div 8$ and with the daughter spacing coefficient $\varkappa = 2$. With this kind of degeneracy, i.e., of $(\ell{+}2n_r)$-type, however, a certain number of mesons falls out the description [9, 15]. The value $\varkappa = 1$ is likely more adequate to experimental data, but is hardly deducible theoretically. For example, to provide for mass formulae derived from the Dirac-type equation model [36] the asymptotical (in the limit $\ell \gg 1$) spacing $\varkappa = 1$, additional selection rules were imposed by hand. In such a way, a proper description of π-, ρ- [37] and K-trajectories [38] was achieved.

In the present paper we propose a relativistic potential model of mesons which reveals asymptotically linear Regge trajectories with the $(\ell{+}n_r)$-degeneracy arising naturally, without applying supplementary selection rules. This model is constructed within the relativistic action-at-a-distance (AaD) theory by the following consideration. The Wheeler–Feynman electrodynamics [39, 40], formulated in terms of the Fokker action integral [41, 42], is the AaD-counterpart of the Coulomb interaction. A classical AaD-prototype of the linear confining interaction was formulated independently by Rivacoba [43] and Weiss [44]. In turn, their Fokker-type action integrals are related, as is shown in [45, 46], to the effective higher-derivative gauge field theory [47–49]. Namely, the interaction between particles is described in terms of a time-symmetric Green function of a fourth-order field equation. Therefore, the model provides both a clear relativization and a field-theoretical interpretation of the Cornell potential.

The Hamiltonian description and the quantization of Fokker-type systems is a rather challenging problem in view of a time-nonlocal character of the interaction [50–53]. The canonical structure in this case can be provided by means of approximated methods [50, 52] which, in most, are not appropriate for strongly coupled systems.

For particular time-symmetric Fokker-type systems one can invent naturally time-asymmetric counterparts in which a time-nonlocality is removed [54]. The Rivacoba–Weiss model is the case. For this but time-asymmetric model an exact Hamiltonian formulation and the corresponding quantum description was elaborated [45, 55]. Despite a suitable degeneracy (i.e., with $\varkappa = 1$), the slope of asymptotic Regge trajectories turned out to be overestimated, with the coefficient $k = 3\sqrt{6} \approx 10.4$. The reason perhaps resides in the fact that the vector character of interaction brought into the model from the underlying gauge theory is not quite suited to an actual nature of a confining interaction in hadrons. Hence, a scalar counterpart of the Rivacoba–Weiss model should be constructed and examined. Unfortunately, a time-nonlocality in this case is unavoidable, and thus the time-asymmetric relativistic Hamiltonian mechanics [54] is unapplicable.

Recently, a quantization method of two-particle Fokker-type (i.e., time-nonlocal) systems in an almost-circular-orbit (ACO) approximation has been proposed by the author [56]. The method is appropriate for strongly coupled systems. Here it is applied to a quantization of the time-symmetric Rivacoba–Weiss model. Moreover, the analogue of this model with scalar confining interaction is built, and the scalar–vector superposition model is considered. There has been studied an asymptotic behavior of the Regge trajectories, from which the slope and daughter spacing coefficients are tied to the string tension a and the scalar–vector mixing parameters ξ.

Finally, the model is supplemented with the short-range Wheeler–Feynman term giving the complete relativistic AaD-counterpart of the Cornell potential model.

[1]In some works the idea has been insisted that a linearity (even approximated) is not a determinant feature of Regge trajectories. For example, from the analysis of experimental data it was stated that a curvature of different meson and baryon trajectories can be either positive or negative [18, 19] though its value turned basically small, especially for light-quark hadron trajectories. From the extended review [19] of potential models leading to nonlinear Regge trajectories one can conclude that the curvature is noticeable mainly for the bottom segments of trajectories. As far as the upper segments are concerned, the majority of relativistic models with a linearly rising long-range potential lead to trajectories which are linear asymptotically for $\ell \gg 1$; see also [20]. Besides, there has been proposed in Ref. [21] a phenomenological fractional-power form of Regge trajectories that is consistent both with dual analytic models as well as with the quark-antiquark pair creation phenomenon limiting a length of trajectories (i.e., a number of bound states). However, the latter, essentially quantum-field effect, is beyond the scope of potential models. On the whole, the arguments in favor of the essential nonlinearity of the Regge trajectories which were offered in different works, still have not been put on a common theoretical ground. In the present paper the author follows a more traditional viewpoint that the Regge trajectories are linear approximately while a minor curvature may be influenced by quark masses, a short-range interaction etc.; see sections VI–VII.

This modification permits us to lower naturally the intercept parameter ζ of Regge trajectories (1.2) in order to fit light meson experimental data.

II. THREE FORMULATIONS OF FOKKER-TYPE ACTION INTEGRAL FOR THE VECTOR CONFINING INTERACTION

We start with the manifestly covariant two-particle action

$$I = I_{\rm free} + I_{\rm int}, \qquad \text{where} \qquad I_{\rm free} = -\sum_{a=1}^{2} m_a \int \mathrm{d}\tau_a \sqrt{\dot{x}_a^2} \tag{2.1}$$

and $I_{\rm int}$ is the Fokker action integral [41, 42] describing an interaction. For the Wheeler–Feynman *electrodynamics* [39, 40] we have:

$$I^{(\rm e)}_{\rm int} = \alpha \iint \mathrm{d}\tau_1\, \mathrm{d}\tau_2\; \dot{x}_1 \cdot \dot{x}_2\, \delta(x_{12}^2). \tag{2.2}$$

In eqs. (2.1) and (2.2) m_a is a rest mass of ath particle (a=1, 2); $x_a^\mu(\tau_a)$ (μ=$\overline{0,3}$) are covariant coordinates of ath world line parameterized by an arbitrary evolution parameter τ_a; $\dot{x}_a^\mu(\tau_a) \equiv dx_a^\mu/d\tau_a$; $x_{12}^\mu \equiv x_1^\mu(\tau_1) - x_2^\mu(\tau_2)$; $x_{12}^2 \equiv \eta_{\mu\nu} x_{12}^\mu x_{12}^\nu$; the coupling constant $\alpha = -q_1 q_2$ is related to particle charges q_a, and $\delta(x_{12}^2)$ is the symmetric Green function of the d'Alembert equation $\Box\delta(x^2) = 4\pi\delta(x)$. We use the Minkowski metrics $\|\,\eta_{\mu\nu}\,\| =$ diag$(+,-,-,-)$, and put the light speed $c = 1$.

One can generalize the integral (2.2) to the arbitrary vector-type interaction:

$$I^{(\rm v)}_{\rm int} = -\iint \mathrm{d}\tau_1\, \mathrm{d}\tau_2\; \dot{x}_1 \cdot \dot{x}_2\, G(x_{12}^2), \tag{2.3}$$

where $G(x_{12}^2)$ is usually proportional to the symmetric Green function of an appropriate field equation, or it may be chosen phenomenologically. Weiss [44] considered the choice $G(x^2) \propto \Theta(x^2)$ in (2.3) (where $\Theta(x)$ is the Heaviside step function) with some coefficient of proportionality which we specify here as follows:

$$G(x^2) = -\tfrac{1}{2} a \Theta(x^2), \qquad a > 0. \tag{2.4}$$

In the non-relativistic limit the Weiss action leads [57] to the interaction potential:

$$U(r) = \int_{-\infty}^{\infty} \mathrm{d}\vartheta\, G(\vartheta^2 - r^2) = -a \int_r^{\infty} \mathrm{d}\vartheta\; = a(r - \infty) \tag{2.5}$$

which corresponds, up to unessential infinite constant, to a linear confinement with the string tension parameter a.

As is shown in [45, 46], the Weiss action principle is related to the higher-derivative theory of the vector field proposed by Kiskis [47] and to its later non-Abelian version [48, 49]. In particular, the function $\Theta(x^2)$ is a symmetric fundamental solution of the fourth-order equation:

$$\Box^2 \Theta(x^2) = 16\pi\delta(x). \tag{2.6}$$

The Fourier transform of this solution $\propto 1/k^4$ coincides with an infrared asymptotics of the gluon propagator [48].

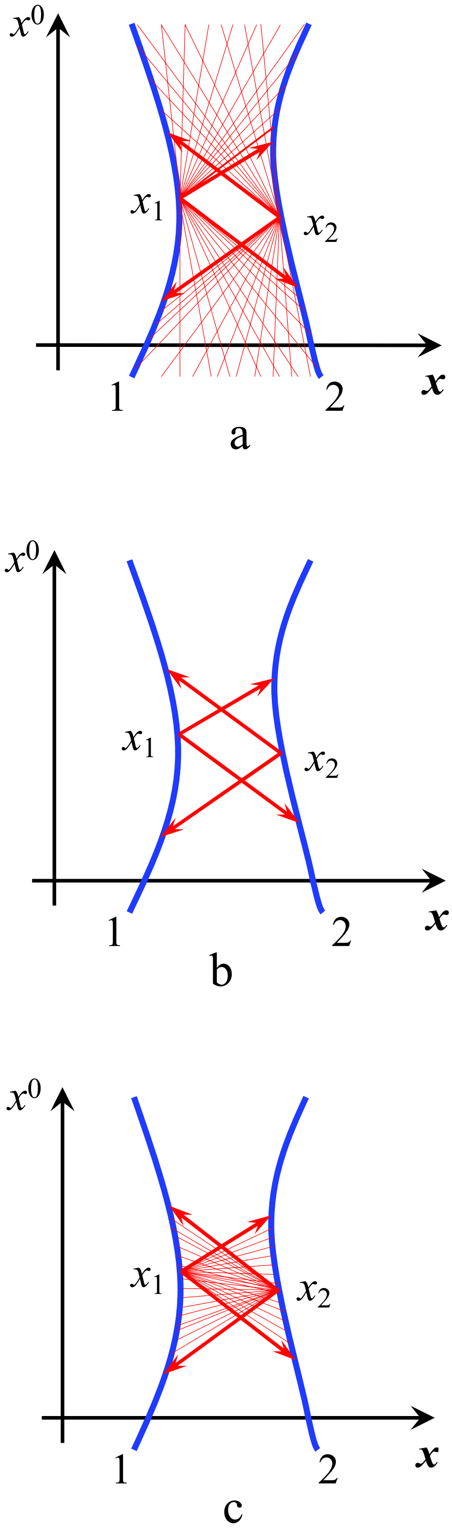

Fig. 1. Interaction causal structure of various Fokker-type action integrals (specified in the text). Solid curves depict world lines of particles 1 and 2. Arrows and thin lines depict generatrices and inwards (a) or outwards (c) of light cones where points of particle world lines are related.

An infinite constant in r.-h.s. of (2.5) indicates that the Fokker action integral (2.3) with the Green function (2.4) is not well posed from the mathematical viewpoint. A formal causal structure of the interaction is that as if each point (say, x_a) of a world line of one particle is related to infinite segments of another word line lying inside the light cone with the center x_a, and the contribution of these segments in the action is infinite; see Fig. 1a. Physically it is not crucial since a variation of the action (2.3)-(2.4) turns $\Theta(x^2)$ into its derivative $\Theta'(x^2) = \delta(x^2)$, and Euler–Lagrange equations relate points of particle world lines along generatrices of light cones only; see Fig. 1b. Nevertheless, integrals of motion, such as the energy and the angular momentum, turn out divergent. In order to avoid this difficulty one can reformulate the Fokker action (2.3), (2.4) via the integration by parts [58]:

$$\begin{aligned} I_{\rm int}^{(\rm v)} &= \frac{a}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\tau_1\,\mathrm{d}\tau_2\ \dot{x}_1\!\cdot\!\dot{x}_2\,\Theta(x_{12}^2) \\ &= -a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\tau_1\,\mathrm{d}\tau_2\ (x_{12}\!\cdot\!\dot{x}_1)(x_{12}\!\cdot\!\dot{x}_2)\,\delta(x_{12}^2) \\ &\quad - \frac{a}{4}\Theta(x_{12}^2)x_{12}^2\Big|_{\tau_1=-\infty}^{\tau_1=\infty}\Big|_{\tau_2=-\infty}^{\tau_2=\infty}. \end{aligned} \tag{2.7}$$

The last divergent term does not contribute in the equations of motion, and we arrive at the equivalent formulation of the problem proposed earlier by Rivacoba [43]. The Fokker-type integral (2.7) itself describes an interaction with the causal structure of Fig. 1b, as in the Wheeler–Feynman electrodynamics (2.2), and leads to finite integrals of motions.

One can propose a third equivalent formulation of the problem which is most convenient for our purpose. Using the equality $\Theta(x^2) = 1 - \Theta(-x^2)$ one obtains:

$$\begin{aligned} I_{\rm int}^{(\rm v)} &= \frac{a}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\tau_1\,\mathrm{d}\tau_2\ \dot{x}_1\!\cdot\!\dot{x}_2\ \left[1-\Theta(-x_{12}^2)\right] \\ &= -\frac{a}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\tau_1\,\mathrm{d}\tau_2\ \dot{x}_1\!\cdot\!\dot{x}_2\,\Theta(-x_{12}^2) \\ &\quad + \frac{a}{2}x_1\!\cdot\!x_2\Big|_{\tau_1=-\infty}^{\tau_1=\infty}\Big|_{\tau_2=-\infty}^{\tau_2=\infty}. \end{aligned} \tag{2.8}$$

The interaction causal structure of integral (2.8) is shown in Fig. 1c. The integrals of motion are the same as in the Rivacoba version of the model. This version (i.e., (2.8)) of the Weiss action (2.3), (2.4) is equally substantiated by the Kiskis field theory [47] since the function $-\Theta(-x^2)$ satisfies the equation (2.6) as well.

III. ACO APPROXIMATION IN THE FOKKER DYNAMICS

The construction of the Hamiltonian description of Fokker-type systems, as a step towards quantization, is a rather difficult task which can be realized within a certain perturbation scheme. A commonly used quasi-relativistic approximation scheme (see, for example, [50]) works well if relativistic effects are weak. But light mesons, as two-quark systems, are essentially relativistic, and they need another approach.

Used here is the almost-circular-orbit (ACO) approximation scheme developed in the previous work of the author [56]. The scheme is based on the fact that all known in the literature two-particle Fokker-type systems with the attractive (in some meaning) interaction possess exact solutions of the shape of concentric planar circular particle orbits of radii $R_a(\Omega)$ dependent on an angular velocity Ω; see [59–61]. In [56] this is proven for a general two-particle Fokker-type system,

$$\begin{aligned} I &= \sum_{a=1}^{2}\int \mathrm{d}t_a\, L_a\left(t_a, \boldsymbol{x}_a(t_a), \dot{\boldsymbol{x}}_a(t_a)\right) \\ &\quad + \iint \mathrm{d}t_1\,\mathrm{d}t_2\,\Phi\left(t_1, t_2, \boldsymbol{x}_1(t_1), \boldsymbol{x}_2(t_2), \dot{\boldsymbol{x}}_1(t_1), \dot{\boldsymbol{x}}_2(t_2)\right), \end{aligned} \tag{3.1}$$

which is invariant under the Aristotle group (including time and space translations and inversions, and space rotations), at least. Manifestly covariant Fokker-type systems [40–46] which by construction are Poincaré-invariant (and the more Aristotle-invariant) as well as possessing reparametrization invariance can be reduced to the form (3.1) by means of the choice of the evolution parameter $\tau_a = t_a \equiv x_a^0$; then the particle positions are $\boldsymbol{x}_a(t_a) = \{x_a^i(t_a), i = 1, 2, 3\}$. The manifestly covariant Rivacoba–Weiss system (2.1), (2.7) (or (2.8)) does possess exact circular orbit solutions even in a far relativistic domain [43]. A set of these solutions can serve as a zero-order approximation in a perturbative treatment of the Fokker-type dynamics.

The invariance of action (3.1) with respect to time translations and space rotations leads to an existence of the energy and the angular momentum integrals of motion [62]:

$$E = \sum_{a=1}^{2}\left\{\dot{\boldsymbol{x}}_a\cdot\frac{\partial}{\partial\dot{\boldsymbol{x}}_a} - 1\right\}(L_a + \Lambda_a) + \iint \mathrm{d}t_1\,\mathrm{d}t_2\,\left\{\frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2}\right\}\Phi, \tag{3.2}$$

$$\begin{aligned} \boldsymbol{J} &= \sum_{a=1}^{2}\boldsymbol{x}_a\times\frac{\partial}{\partial\dot{\boldsymbol{x}}_a}(L_a + \Lambda_a) \\ &\quad - \frac{1}{2}\iint \mathrm{d}t_1\,\mathrm{d}t_2\left\{(\boldsymbol{x}_1+\boldsymbol{x}_2)\times\frac{\partial}{\partial\boldsymbol{x}} + \dot{\boldsymbol{x}}_1\times\frac{\partial}{\partial\dot{\boldsymbol{x}}_1} - \dot{\boldsymbol{x}}_2\times\frac{\partial}{\partial\dot{\boldsymbol{x}}_2}\right\}\Phi, \end{aligned} \tag{3.3}$$

where

$$\Lambda_1 = \int_{-\infty}^{\infty} \mathrm{d}t_2\, \Phi, \quad \Lambda_2 = \int_{-\infty}^{\infty} \mathrm{d}t_1\, \Phi, \quad \bcancel{\iint} \equiv \int_{-\infty}^{t_1}\int_{t_2}^{\infty} - \int_{t_1}^{\infty}\int_{-\infty}^{t_2}.$$

On circular orbits, these integrals are functions of the angular velocity: $E_{(0)}(\Omega)$ and $\boldsymbol{J}_{(0)}(\boldsymbol{\Omega}) \parallel \boldsymbol{\Omega}$ so that we can get the function $J_{(0)}(\Omega)$ where $J_{(0)} = |\boldsymbol{J}_{(0)}|$ and $\Omega = |\boldsymbol{\Omega}|$.

Let us transit to a non-inertial reference frame which is uniformly rotating with the angular velocity $\boldsymbol{\Omega}$. This can be done via the change of variables $\boldsymbol{x}_a(t_a) \to \boldsymbol{z}_a(t_a)$: $\boldsymbol{x}_a(t_a) = \mathsf{S}(t_a)\boldsymbol{z}_a(t_a)$ where $\mathsf{S}(t) = \exp t\Omega \in \mathrm{SO}(3)$ and the skew-symmetric matrics Ω is dual to the vector $\boldsymbol{\Omega}$. Within this reference frame a circular motion of particles is described by static vectors $\boldsymbol{R}_a$ such that $\boldsymbol{R}_2 \uparrow\downarrow \boldsymbol{R}_1$. Then small perturbations of circular orbits are characterized by deviation vectors $\boldsymbol{\rho}_a(t_a) = \boldsymbol{z}_a(t_a) - \boldsymbol{R}_a$.

Expanding the action (3.1) in powers of ρ_a yields in the lowest non-trivial order the quadratic form:

$$I^{(2)} = \tfrac{1}{2}\sum_{kl}\iint \mathrm{d}t\,\mathrm{d}t'\, \rho^k(t) D_{kl}(t-t')\rho^l(t'), \qquad (3.4)$$

where the matrix kernel $\mathsf{D}(t-t') = \|D_{kl}(t-t')\|$ (with multi-indeces $k,l = (a,i),(b,j)$) is invariant under time translations and reversion: $\mathsf{D}^{\mathrm{T}}(t'-t) = \mathsf{D}(t-t')$. The corresponding equations of motion form a time-nonlocal linear homogeneous system:

$$\sum_l \int \mathrm{d}t'\, D_{kl}(t-t')\rho^l(t') = 0, \qquad (3.5)$$

which possesses a certain fundamental set of solutions. Among them the exponential solutions $\rho^k(t) = e^k(\omega)\mathrm{e}^{-\mathrm{i}\,\omega t}$ are of interest. Substituting them into the system (3.5) yields the set of algebraic equations:

$$\sum_l D_{kl}(\omega) e^l(\omega) = 0, \qquad (3.6)$$

which amounts the eigenvector–eigenvalue problem for the polarization vector $e^k(\omega)$ and the frequency ω. The latter is determined by means the secular equation $\det \mathsf{D}(\omega) = 0$ in terms of the dynamical matrix $\mathsf{D}(\omega) = \int \mathrm{d}t\, \mathsf{D}(t)\mathrm{e}^{\mathrm{i}\,\omega t}$. In view of a time-nonlocality of the problem (3.5), the matrix entries $D_{kl}(\omega)$ are, in general, non-polynomial functions of ω, and the set of solutions of the secular equations may appear infinite. Due to symmetric properties of the dynamical matrix this set consists of duplets $\{\pm\omega_\alpha,\ \alpha = 1,2,\dots\}$ if $\omega_\alpha \in \mathbb{R}$ or quadruplets $\{\pm\omega_\alpha, \pm\omega_\alpha^*\}$ if $\mathrm{Im}\ \omega_\alpha \neq 0$. In the latter case the corresponding solution is unbounded and cannot be described correctly within the ACO approximation (where ρ^k must be small). Thus among all eigenfrequencies we select real ones only and arrive at the following solutions of the system (3.5):

$$\rho^k(t) = \sum_\alpha \left\{ A_\alpha e_\alpha^k(\omega_\alpha)\ \mathrm{e}^{-\mathrm{i}\,\omega_\alpha t} + A_\alpha^* \overset{*}{e}{}_\alpha^k(\omega_\alpha)\ \mathrm{e}^{\mathrm{i}\,\omega_\alpha t} \right\}, \qquad (3.7)$$

where complex amplitudes A_α of oscillation modes parameterize the phase space of the system. Only one mode A_r corresponding to mutual radial particle oscillations with the frequency ω_r is physically meaningful. Other modes are either kinematic ones which can be reduced via redefinition of zero-order circular orbits, or non-physical ones which reveal physically unacceptable behavior of particles and arise as a mathematical artefact of the theory. All such modes should be discarded. After this is done and the polarization vectors $e_\alpha^k(\omega_\alpha)$ in (3.7) are appropriately normalized, the angular momentum (3.3) and the energy (3.2) take the form:

$$J = J_{(0)}(\Omega), \qquad (3.8)$$

$$E = E_{(0)}(\Omega) + E_{(2)}(\Omega, A_r) \qquad (3.9)$$

$$\text{where} \qquad E_{(2)}(\Omega, A_r) = \omega_r(\Omega)|A_r|^2. \qquad (3.10)$$

Other integrals of motion following from the Poincaré-invariance of the system vanish; they are the total momentum, $\boldsymbol{P} = 0$ and the center-of-mass integral, $\boldsymbol{K} = 0$. Thus the ACO approximation brings the system into the center-of-mass reference frame.

In order to construct the center-of-mass canonical description of the system one should, first of all, to invert the relation (3.8) with respect to $\Omega = \Omega(J)$. This permits us to obtain the center-of-mass Hamiltonian which is nothing but the total mass of the system:

$$\begin{aligned} M &= M_{(0)}(J) + M_{(2)}(J, |A_r|) \\ &\equiv \left\{E_{(0)}(\Omega) + \omega_r(\Omega)|A_r|^2\right\}_{\Omega=\Omega(J)}. \end{aligned} \qquad (3.11)$$

It is understood as a function of $J = |\boldsymbol{J}|$ where components J_i $(i = 1,2,3)$ of the intrinsic angular momentum $\boldsymbol{J}$ of the system satisfy the Poisson bracket relations (PBR): $\{J_i, J_j\} = \varepsilon_{ij}{}^k J_k$, and of the amplitude of interparticle radial oscillations A_r satisfying the PBR: $\{A_r, A_r^*\} = -\mathrm{i}\,, \quad \{A_r, A_r\} = \{A_r^*, A_r^*\} = 0$.

In order to transit to an arbitrary reference frame one must introduce canonical variables characterizing the state of the system as a whole, for example, the total momentum $\boldsymbol{P}$ and the canonically conjugated CM position variable $\boldsymbol{Q}$. Then a complete Hamiltonian description of the system, i.e., ten canonical generators of the Poincaré group, are determined in terms of M, $\boldsymbol{J}$, $\boldsymbol{P}$ and $\boldsymbol{Q}$ via the Bakamjian–Thomas (BT) model or equivalent constructions [63, 64]. The quantization of BT model is well elaborated [65, 66].

In the present work we are interested mainly in the spectrum of the mass operator $\hat{M}$. It can be obtained directly from (3.11) by means of the following substitution:

$$\boldsymbol{J} \to \hat{\boldsymbol{J}}; \qquad A_r \to \hat{A}_r, \quad A_r^* \to \hat{A}_r^\dagger;$$

$$J \to \sqrt{\hat{\boldsymbol{J}}^2} \to \sqrt{\ell(\ell+1)} \approx \ell + \tfrac{1}{2}, \quad \ell = 0, 1, \ldots \tag{3.12}$$

$$|A_r|^2 \to \tfrac{1}{2}(\hat{A}_r\hat{A}_r^\dagger + \hat{A}_r^\dagger\hat{A}_r) \to n_r + \tfrac{1}{2}, \quad n_r = 0, 1, \ldots \tag{3.13}$$

Here the condition $n_r \ll \ell$ is implied, due to a perturbation procedure.

IV. RIVACOBA–WEISS MODEL IN ACO APPROXIMATION

Let us consider a circular-orbit solution of the Rivacoba–Weiss model. Using the action (2.1), (2.8) for a system of two particles of equal masses $m_a \equiv m$ ($a = 1, 2$) and following the general methodology proposed in [61] or [56], one states a relation between the angular velocity Ω of circulating particles and the radius R of their orbits. It is convenient, instead of R, to work with particle velocities $v_a \equiv v = R\Omega$. Then the relation between Ω and v can be determined implicitly, or parametrically, via an auxiliary angle ϕ [43, 44, 59–61]. It is related with the velocity v by means of the equality:

$$f(\phi) \equiv \phi^2 - 4v^2\cos^2(\phi/2) = 0 \quad (0 \le v < 1) \quad \Longrightarrow$$
$$\Longrightarrow v^2(\phi) = \frac{\phi^2}{2(1+\cos\phi)} \quad \text{or} \quad v(\phi) = \frac{\phi/2}{\cos(\phi/2)}. \tag{4.1}$$

In turn, we have for Ω

$$\frac{m}{a}\Omega = \frac{\phi}{\Gamma v^2}\left[1 - \frac{(1-v^2)\phi}{f'(\phi)}\right] \equiv f_\Omega^{(\mathrm{v})}(\phi) \tag{4.2}$$

where $f'(\phi) \equiv \partial f(\phi)/\partial\phi = 2(\phi + v^2\sin\phi)$, $\Gamma \equiv (1 - v^2)^{-1/2}$, and the superscript "(v)" refers to the vector interaction. Let us note that $v \in (0, 1)$, $R \in (0, \infty)$ and $\Omega \in (\infty, 0)$ if $\phi \in (0, \phi_1)$ where the angle $\phi_1/2 \equiv \chi_1 = 0.235\,\pi$ is a positive solution of the transcendental equation $\chi = \cos\chi$.

The integrals of circular motion $M_{(0)}$ and $J = J_{(0)}$ are expedient to write as:

$$\frac{\Omega M_{(0)}}{a} = \frac{2\phi}{v^2}\left[1 + v^2 - \frac{\phi(1 + v^4\cos\phi)}{f'(\phi)}\right] \equiv f_M^{(\mathrm{v})}(\phi), \tag{4.3}$$

$$\frac{\Omega^2 J}{a} = \tfrac{1}{2}f'(\phi) \equiv f_J^{(\mathrm{v})}(\phi). \tag{4.4}$$

They grow as $M_{(0)} \in (2m, \infty)$ and $J \in (0, \infty)$ if $\phi \in (0, 2\chi_1)$.

In order to study the system in ACO approximation we need to construct the reduced 2×2 dynamical matrix $\mathcal{D}^\perp$ [56] and then to calculate the frequency ω_r of radial oscillations or, equivalently, the fraction $\lambda = \omega_r/\Omega$, as a function of either Ω, J, v or (which is most convenient) ϕ. This is done in Appendix A.

Here we are interested in an asymptotic expression for the total mass (3.11) squared at $J \to \infty$. Within the perturbation procedure the inequality $M_{(2)} \ll M_{(0)}$ is implied. Taking this into account one obtains:

$$M^2 \approx M_{(0)}^2 + 2M_{(0)}M_{(2)} = M_{(0)}^2 + 2M_{(0)}\omega_r|A_r|^2 \tag{4.5a}$$
$$= \frac{M_{(0)}^2}{J}\left\{J + 2\frac{J\Omega}{M_{(0)}}\lambda|A_r|^2\right\}. \tag{4.5b}$$

If the following limits exist and are finite:

$$k = \lim_{J\to\infty}\frac{M_{(0)}^2}{J} = \lim_{\phi\to\phi_1}\frac{f_M^2(\phi)}{f_J(\phi)}, \tag{4.6}$$

$$\varkappa = 2\lim_{J\to\infty}\frac{J\Omega}{M_{(0)}}\lambda = 2\lim_{\phi\to\phi_1}\frac{f_J(\phi)}{f_M(\phi)}\lambda(\phi), \tag{4.7}$$

the asymptotic value for the total mass squared (4.5) takes the form

$$M^2 \sim ka\{J + \varkappa|A_r|^2\} \qquad \text{at} \quad J \to \infty \tag{4.8}$$

and, upon quantization (3.12), (3.13), recovers the Regge trajectories (1.2) with $\sigma = ka$. In the present case of the vector confinement model

$$k^{(\mathrm{v})} = 8\chi_1(1 + \sin\chi_1) \approx 9.896, \qquad \varkappa^{(\mathrm{v})} = 1. \tag{4.9}$$

An asymptotic value of the daughter spacing coefficient $\varkappa^{(\mathrm{v})} = 1$ matches well for a description of the tower structure of meson spectra (see item 5 in Section I). But the slope coefficient $k^{(\mathrm{v})} \approx 9.896$ together with the string tension value $a = 0.18$ GeV^2 yield an overestimated slope of the Regge trajectories: $\sigma = k^{(\mathrm{v})}a = 1.78$ GeV^2 $>$ 1.15 GeV^2. A plausible reason of this disagreement in that the purely vector nature of the interaction in the model does not meet the actual relativistic structure of the confinement interaction which is commonly attributed to the scalar [2, 15, 32] or scalar–vector [4, 25–29] type.

In order to confirm or challenge this assumption we construct in the next section the scalar analogue of the Rivacoba–Weiss confinement model.

V. FOKKER-TYPE ACTION FOR THE SCALAR CONFINEMENT

Let the two-particle action to consist of the free-particle terms (2.1) and the Fokker-action integral of a scalar type [46]:

$$I_{\mathrm{int}}^{(\mathrm{s})} = -\iint \mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\sqrt{\dot{x}_1^2}\sqrt{\dot{x}_2^2}\,G(x_{12}^2). \tag{5.1}$$

If the function $G(x^2)$ is chosen in the form (2.4) it is expected that the action (2.1), (5.1) describes the scalar confinement interaction. Indeed, the action (5.1) can be derived from the higher-derivative theory of scalar field [67].

In this case however one encounters even more significant divergences as in the vector-type model since not only the action itself and integrals of motion but also the equations of motion are ill-posed. Fortunately, the remedy to set the scalar model properly is the same: one replaces the function (2.4) by

$$G(x^2) = \tfrac{1}{2}a\Theta(-x^2), \qquad a > 0, \tag{5.2}$$

which is analogous to the transition from the action (2.3), (2.4) to (2.8) in the case of Weiss model. The replacement of the function (2.4) by (5.2) in the action (5.1) may also be treated as a renormalization of particle rest masses:

$$m_{0\,a} \to m_a = m_{0\,a} - \frac{a}{4}\int_{-\infty}^{\infty} \mathrm{d}\tau_{\bar{a}} \sqrt{\dot{x}_{\bar{a}}^2}, \;\; a = 1, 2, \;\bar{a} = 3 - a,$$

where $m_{0\,a}$ is an infinite bar mass of ath particle while m_a is finite.

A subsequent consideration of the scalar model is similar to one in the vector case. The system of equal rest masses is considered. Dynamical characteristics of circular orbit solution are parameterized by the angle ϕ. In particular, for the angular velocity Ω one can obtain:

$$\frac{m}{a}\Omega = \frac{\phi}{\Gamma}\left[\frac{(1-v^2)\phi}{v^2 f'(\phi)} - 1\right] \equiv f_\Omega^{(\mathrm{s})}(\phi) \qquad (5.3)$$

where $f'(\phi)$, v and Γ as functions of ϕ are defined in section IV. In contrast to the vector case, here $f_\Omega^{(\mathrm{s})}(\phi) \to 0$ if $\phi \to \phi_0 \neq \phi_1$ where $\phi_0/2 \equiv \chi_0$ is a positive solution of the transcendental equation:

$$3\chi^2 \cos\chi + 2\chi^3 \sin\chi - \cos^3\chi = 0, \qquad \chi \in [0, \pi/4].$$

The latter by means of the substitution $\chi = \pi/2 - 3\psi$ can be reduced to the form:

$$\psi = \frac{\pi}{6} - \frac{\sin 3\psi}{6\cos\psi}, \qquad \psi \in [\pi/12, \pi/6]$$

which is suitable to iterate the numerical solution: $\chi_0 = 0.151\pi < \chi_1$. It is surprisingly that particle velocity $v \to 0.535 < 1$ at $\chi \to \chi_0$ while orbit radus $R \to \infty$. This distinquishes the scalar model from the vector one in which $v \to 1$ at $R \to \infty$. For the integrals of (circular) motion we have:

$$\frac{\Omega M_{(0)}}{a} = \frac{2\phi^2(1-v^2)^2}{v^2 f'(\phi)} \equiv f_M^{(\mathrm{s})}(\phi), \qquad (5.4)$$

$$\frac{\Omega^2 J}{a} = (1-v^2)\phi \equiv f_J^{(\mathrm{s})}(\phi). \qquad (5.5)$$

It is easy to verify that $M_{(0)} \in (2m, \infty)$ and $J \in (0, \infty)$ if $\phi \in (0, 2\chi_0)$.

Using the functions (5.4), (5.5) in eqs. (4.6), (4.7) and taking limits at $\phi \to \phi_0$ (instead of $\phi \to \phi_1$) yields the slope and daughter spacing coefficients:

$$k^{(\mathrm{s})} = 2.716, \qquad \varkappa^{(\mathrm{s})} = 1.902. \qquad (5.6)$$

The latter is close to 2, as in the oscillator-like and some string relativistic models of mesons [9, 22–24]. The accidental degeneracy and thus the tower structure of the mass spectrum is recovered approximately. Again, the slope coefficient is not appropriate (similarly to the vector model), but it is considerably less than the conventional values 4÷8.

The difference between the vector and scalar models suggests that general features of the light meson spectroscopy may be recovered (at least, asymptotically) within the Fokker-type model with a scalar–vector confining interaction.

VI. FOKKER-TYPE ACTION INTEGRAL FOR THE SCALAR–VECTOR CONFINING SUPERPOSITION

The Fokker-type system of two particles bound via superposition of scalar and vector confining interactions is naturally defined by means of action (2.1) with

$$I_{\mathrm{int}}^{(\xi)} = (1-\xi) I_{\mathrm{int}}^{(\mathrm{s})} + \xi I_{\mathrm{int}}^{(\mathrm{v})}, \qquad (6.1)$$

where $I_{\mathrm{int}}^{(\mathrm{s})}$ and $I_{\mathrm{int}}^{(\mathrm{v})}$ are defined in eqs. (5.1), (5.2) and (2.8), respectively, while $\xi \in [0, 1]$ is a mixing parameter.

All the functions $f_\Omega^{(\xi)}(\phi)$, $f_M^{(\xi)}(\phi)$ and $f_J^{(\xi)}(\phi)$ determining the dynamics and integrals of circular motion of this model are superpositions of the functions (4.2)-(4.4) and (5.3)-(5.5):

$$f^{(\xi)}(\phi) = (1-\xi) f^{(\mathrm{s})}(\phi) + \xi f^{(\mathrm{v})}(\phi). \qquad (6.2)$$

Then the function $[M_{(0)}^{(\xi)}(J)]^2$ which is a classical analogue of the principal Regge trajectory, can be presented in the parametric form:

$$\frac{M_{(0)}^{(\xi)}}{m} = \frac{f_M^{(\xi)}(\phi)}{f_\Omega^{(\xi)}(\phi)}, \qquad (6.3)$$

$$\frac{aJ}{m^2} = \frac{f_J^{(\xi)}(\phi)}{\left[f_\Omega^{(\xi)}(\phi)\right]^2}, \qquad \phi \in [0, \phi_\xi], \; \xi \in [0, 1]; \qquad (6.4)$$

it is shown in Fig. 2. The maximal angle $\phi_\xi/2 \equiv \chi_\xi$ is the smallest positive root of the equation $f_\Omega^{(\xi)}(2\chi) = 0$. It grows monotonically over the segment $\chi_\xi \in [\chi_0, \chi_1]$ if $\xi \in [0, 1/2]$, and $\chi_\xi = \chi_1$ if $\xi \in [1/2, 1]$. Similarly, the maximal speed of particles (at $R \to \infty$ when $M_{(0)} \to \infty$ and $J \to \infty$) grows monotonically, $v \in [0.535, 1]$ if $\xi \in [0, 1/2]$, and $v = 1$ if $\xi \in [1/2, 1]$.

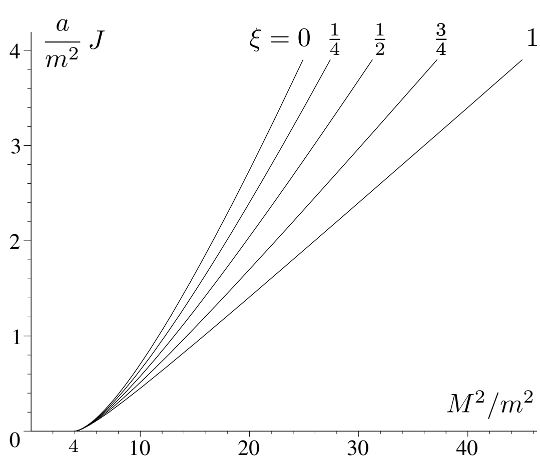

Fig. 2. Classical Regge trajectories for different values of the mixing parameter ξ in the scalar–vector confinement model.

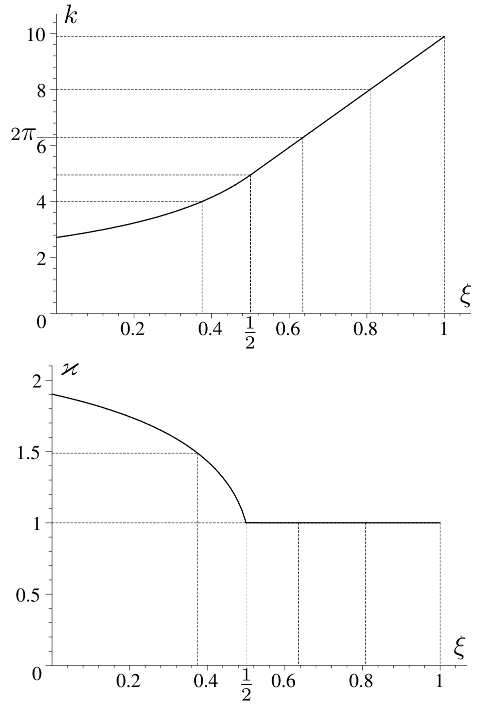

Fig. 3. Slope k and daughter spacing $\varkappa$ coefficients vs mixing parameter ξ in the scalar–vector confinement model. Grid lines connect values ξ with k and $\varkappa$ at $\xi = 1/2, 1$ and $k = 4, 2\pi, 8$.

The slope and daughter spacing coefficients can be calculated similarly to the previous cases, i.e., using eqs. (4.6) and (4.7) with the limiting angle ϕ_ξ (instead of ϕ_1). One can prove that the following equality holds:

$$\lim_{\phi\to\phi_\xi} \frac{f_J^{(\xi)}(\phi)}{f_M^{(\xi)}(\phi)} = \frac{1}{2}, \qquad \xi \in [0,1].$$

Thus formula (4.7) for the daughter spacing coefficient simplifies:

$$\varkappa^{(\xi)} = \lim_{\phi\to\phi_\xi} \lambda^{(\xi)}(\phi), \qquad \xi \in [0,1].$$

The function $\lambda^{(\xi)}(\phi)$ is defined numerically from the secular equation $\det \mathcal{D}^{(\xi)}(\lambda) = 0$ for the matrix (A.17); see Appendix where the graph of $\lambda^{(\xi)}(\phi)$ is shown in Fig. 6.

Both the slope and daughter spacing coefficients are functions of the mixing parameter. In particular,

$$k^{(\xi)} = \xi k^{(\mathrm{v})}, \qquad \varkappa^{(\xi)} = 1, \qquad \xi \in [1/2, 1]. \tag{6.5}$$

The behavior of these functions on the whole segment $\xi \in [0,1]$ is presented in Fig. 3. Grid lines on the graphs take the values of ξ, k and $\varkappa$ into a mutual accordance for particular cases $\xi = 1/2, 1$ and $k = 4, 2\pi, 8$. It is seen from these graphs that the slope coefficient $k^{(\xi)}$ is a monotonically increasing function of the mixing parameter ξ: $k^{(\xi)} \in [k^{(\mathrm{s})}, k^{(\mathrm{v})}] = [2.716, 9.896]$ if $\xi \in [0,1]$. This segment includes conventional values of $k = 4 \div 8$ which occur in relativistic potential models.

Degeneracy properties of the system with scalar-dominating confinement interaction (i.e., at $\xi < 1/2$) differ crucially from those of vector-dominating case: for arbitrary $\xi \in [1/2, 1]$ there is the accidental degeneracy of the $(\ell{+}n_r)$-type. Besides, one can provide in this case an arbitrary value for k from the segment $k^{(\xi)} \in [\frac{1}{2}k^{(\mathrm{v})}, k^{(\mathrm{v})}] = [4.948, 9.896]$, in particular, $k = 2\pi$ and $k = 8$ which are typical for the string models [8,9] and the simple relativistic oscillator models [22,23].

For the scalar-dominating model the lower conventional bound $k = 4$ for the slope [15] is achieved at the mixing $\xi \approx 0.37$ which, in turn, leads to the daughter spacing $\varkappa \approx 3/2$. The accidental degeneracy is present but somewhat hidden.

Upon quantization the mass squared spectrum is calculated by means of the quantization rules (3.12)-(3.13) used in the classical expression (4.5). Practically, one substitutes $J = \ell + \frac{1}{2}$ in l.-h.s. of (6.4) and solves this equation for angles ϕ_ℓ ($\ell = 0, 1...$) which, in turn, are used as arguments of the functions (6.2), (6.3), $f_\Omega(\phi)$ and $\lambda(\phi)$ in r.-h.s. of (4.5).

Let us note that the classical Regge trajectory (6.3), (6.4) (and Fig. 2) starts from $J = 0$ corresponding to $\phi = 0$ and giving $M_{(0)}(0) = 2m$. In the quantum case the bottom value for the dimensionless quantity $j \equiv Ja/m^2$ (in l.-h.s. of (6.4)) corresponding to s-states (i.e., $\ell = 0$) is $j_0 \equiv \frac{1}{2}a/m^2 > 0$, hence $\phi_0 > 0$. For example, taking $a = 0.18$ GeV^2 and $m = 0.15$ GeV (the constituent mass of light quarks) yields $j_0 \approx 4$. In this case a notably curved bottom segment of the principal Regge trajectory which is present in the classical case (see Fig. 2) disappears from the quantum trajectory which thus is closed to a straight line.

Instead, an erroneous curvature acquires in the bottom of daughter trajectories, where $n_r \gtrsim \ell$. The reason in that their classical counterpart (i.e., eq. (4.5) with $A_r \neq 0$) is singular in the slow-motion limit $J \to 0$. In order to correct this drawback, let us consider a nonrelativistic energy of two particles interacting via the linear potential [2,56]:

$$\mathcal{E} \equiv M - 2m = \mathcal{E}_{(0)} + \omega_r |A_r|^2 + o(|A_r|^2) \equiv \frac{3}{2}\left(\frac{2a^2J^2}{m}\right)^{\frac{1}{3}} \left\{1 + \frac{2|A_r|^2}{\sqrt{3}J}\right\} + o(J^{-\frac{1}{3}}).$$

With the same accuracy for $J \gg 0$ one can put:

$$\mathcal{E} \approx \frac{3}{2}\left(\frac{2a^2J^2}{m}\right)^{\frac{1}{3}} \left\{1 + \nu\frac{2|A_r|^2}{\sqrt{3}J}\right\}^{\frac{1}{\nu}} = \frac{3}{2}\left(\frac{2a^2}{m}J^{2-\frac{3}{\nu}}\right)^{\frac{1}{3}} \left\{J + \nu\frac{2}{\sqrt{3}}|A_r|^2\right\}^{\frac{1}{\nu}}. \tag{6.6}$$

This expression is regular in the limit $J \to 0$ provided $\nu \geq 3/2$. Moreover, upon quantization (3.12), (3.13) the expression (6.6) with $\nu = 3/2$ leads to the spectral formula $\mathcal{E}_{\ell\, n_r} \approx \frac{3}{2}\left(2a^2/m\right)^{\frac{1}{3}}\left\{\ell + \frac{1}{2} + \sqrt{3}(n_r + \frac{1}{2})\right\}^{\frac{2}{3}}$ which provides (even for $n_r \gtrsim \ell$) a 3%-accuracy; compare with the quasiclassical s-state (i.e., $\ell = 0$) spectrum: $\mathcal{E}_{0\, n_r} \approx \left(a^2/m\right)^{\frac{1}{3}}\left\{\frac{3\pi}{2}(n_r + \frac{3}{4})\right\}^{\frac{2}{3}}$. Thus one can put for the total mass (3.11):

$$
\begin{aligned}
M \approx& M_{(0)} + \omega_r |A_r|^2 \qquad (6.7)\\
\approx& 2m + \left(M_{(0)} - 2m\right)^{1-\frac{1}{\nu}}\left\{M_{(0)} - 2m + \nu\omega_r|A_r|^2\right\}^{\frac{1}{\nu}} .
\end{aligned}
$$

With $\nu = 3/2$ eq. (6.7) is appropriate in the nonrelativistic domain $\mathcal{E} \ll 2m$. With $\nu = 2$ one reproduces exactly r.h.s of eq. (4.5a) provided $M \gg 2m$, i.e., in the highly relativistic domain. Thus, ν can be used as an adjustable parameter which varies from $\nu = 3/2$ for a heavy quark system to $\nu = 2$ for the light quark one. The last case matches to the description of light mesons. It leads to almost linear Regge trajectories described by the spectral formula (1.2) with the parameters $\sigma = ka$ and $\varkappa$ defined by eq. (6.5) and Fig. 3. The intercept parameter ζ following from the present model can be estimated as follows: $\zeta \geq (1 + \varkappa)/2 + 4m^2/\sigma$. It exceeds the actual value $\zeta \approx 1/2$ for a family of lightest mesons even in the massless limit $m = 0$. Thus meson masses squared are overestimated by the quantity $\Delta M^2 \approx \sigma\varkappa/2 + 4m^2$ at least. This drawback can be removed upon accounting a short-range interaction considered in the next section.

VII. ACCOUNTING A ONE-GLUON EXCHANGE INTERACTION

The classical chromodynamics of pointlike sources admits Abelian solutions of the Yang–Mills equations [68,69] which, on the one hand, are identical to the electrodynamical Liénard–Wiechert potentials, on the other hand, they provide a classical groundwork for a one-gluon exchange interaction of quarks.

Thus, as a relativistic AaD-counterpart of the short-range interquark interaction, the Wheeler–Feynman electrodynamics [39, 40] is appropriate. The corresponding two-body problem is described by the Fokker action (2.1), (2.2), and is a lifelong subject of study [59,60,70–74]. In particular, a circular orbit dynamics is defined by the following equations and integrals of motion [59,60]:

$$
\frac{m}{\alpha\Omega} = 8\,\frac{1+\cos\phi}{[\Gamma f'(\phi)]^3} + 4\,\frac{\phi + \sin\phi}{\Gamma[f'(\phi)]^2} \equiv f_\Omega^{(\mathrm{e})}(\phi), \qquad (7.1)
$$

$$
\frac{M_{(0)}}{m} = \frac{2}{\Gamma}, \qquad \frac{J}{\alpha} = 2\,\frac{1 + v^2\cos\phi}{f'(\phi)} \equiv f_J^{(\mathrm{e})}(\phi). \qquad (7.2)
$$

These definitions will be used in what follows.

Let us superpose the Fokker integral (2.2) with other terms in action (6.1):

$$
I_{\mathrm{int}} = I_{\mathrm{int}}^{(\mathrm{e})} + I_{\mathrm{int}}^{(\xi)}. \qquad (7.3)
$$

Now α in (2.2) is related to the strong coupling constant, $\alpha = \frac{4}{3}\alpha_{\mathrm{s}}$, and the superscript "(e)" refers to 1-gluon *exchange* interaction (rather than the *electromagnetic* one). The action (2.1), (7.3) leads to the Cornell potential (1.1) in the nonrelativistic limit. A relativistic ACO-dynamics of this model is complicated, and here mainly circular orbits will be studied.

The equations of circular motion reduce to the quadratic equation for Ω:

$$
m\Omega - a f_\Omega^{(\xi)}(\phi) - \alpha\Omega^2 f_\Omega^{(\mathrm{e})}(\phi) = 0. \qquad (7.4)
$$

The solution of this equation, as function of ϕ,

$$
\frac{m}{a}\Omega = \frac{2 f_\Omega^{(\xi)}(\phi)}{1 \pm \sqrt{1 - 4\bar{\alpha} f_\Omega^{(\xi)}(\phi) f_\Omega^{(\mathrm{e})}(\phi)}} \equiv f_\Omega^{(\pm)}(\phi) \qquad (7.5)
$$

$$
\text{with} \quad \bar{\alpha} = \frac{\alpha a}{m^2},
$$

is represented by two branches of the following properties:

- $f_\Omega^{(+)}(\phi_{\min}) = f_\Omega^{(-)}(\phi_{\min})$, where $\phi_{\min} > 0$ and $4\bar{\alpha} f_\Omega^{(\xi)}(\phi_{\min}) f_\Omega^{(\mathrm{e})}(\phi_{\min}) = 1$;
- $f_\Omega^{(+)}(\phi) < f_\Omega^{(-)}(\phi), \quad \phi > \phi_{\min}$;
- $f_\Omega^{(+)}(\phi) \underset{\bar{\alpha}\ll 1}{\longrightarrow} f_\Omega^{(\xi)}(\phi), \quad \phi \in (\phi_{\min}, \phi_\xi); \quad f_\Omega^{(-)}(\phi) \underset{\bar{\alpha}\ll 1}{\longrightarrow} [\bar{\alpha} f_\Omega^{(\mathrm{e})}(\phi)]^{-1}, \quad \phi \in (\phi_{\min}, \phi_1)$.

It is seen that the circular motion cannot be as slow as desired. The point $\phi_{\min}$ corresponds to the slowest circular orbit of the radius $R_0 = v_{\min}/\Omega_0$, where $v_{\min} = v(\phi_{\min})$ by (4.1), and $\Omega_0 \equiv \Omega(\phi_{\min})$ by (7.5). For other circular orbits we have the radii:

$$
R^{(\pm)} = \frac{m}{a} v(\phi)/f_\Omega^{(\pm)}(\phi), \; R^{(-)} < R_0 < R^{(+)}, \quad \phi > \phi_{\min}.
$$

Thus functions labeled by the superscript "(–)" or "(+)" describe a short- or long-range dynamics of the system.

Substituting solution (7.5) into the integrals of motion:

$$M_{(0)} = \frac{a}{\Omega} f_M^{(\xi)}(\phi) + 2\alpha \frac{\Omega}{\Gamma} f_\Omega^{(\mathrm{e})}(\phi), \qquad J = \frac{a}{\Omega^2} f_J^{(\xi)}(\phi) + \alpha f_J^{(\mathrm{e})}(\phi) \tag{7.6}$$

yields a classical analogue of the principal Regge trajectories shown in Fig. 4. Comparing Figures 4 and 2 reveals two effects caused by 1-gluon exchange interaction. The first is a deformation of a bottom segment of the trajectory, namely, the curvature for $\bar{\alpha} \lesssim 1$, and straightening for $\bar{\alpha} \gtrsim 1$. The curvature is not important since the bottom segment is cut off upon a quantization while the straightening approximates the trajectory to the linear one. The second effect is a shift of the trajectory as a whole to the left that yields a degression of the intercept constant ζ. Both effects are important in the light meson spectroscopy.

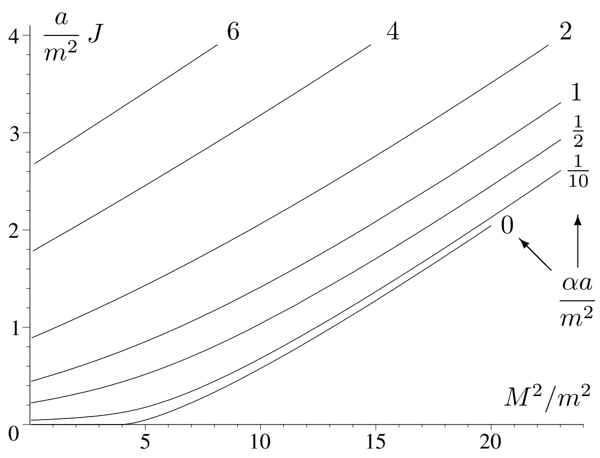

Fig. 4. Classical Regge trajectories for $\xi = 1/2$ and different values of α in the complete model.

Let us consider for a simplicity the limiting case of massless quarks: $m \to 0$ ($\bar{\alpha} \to \infty$). Then the real solution of the equation (7.4) exists if $f_\Omega^{(\xi)}(\phi) < 0$. This is possible for $\phi_\xi \le \phi \le \phi_1$ if $0 \le \xi < 1/2$. More important is the case $1/2 \le \xi \le 1$ where $\phi_\xi = \phi_1$, all coefficients in equation (7.4) vanish, and thus the frequency Ω is undetermined (i.e., arbitrary). In this case the family of circular orbits is parameterized by the angular velocity Ω of quarks rather than their orbital velocity v which is fixed to the light speed: $v(\phi_1) = 1$. Substituting $\phi = \phi_1$ (e.g., $f_M^{(\xi)}(\phi_1) = 0$) into, and eliminating Ω from equalities (7.6) yields the *exact* circular orbit contribution to the total mass squared:

$$M_{(0)}^2(J) = k^{(\xi)} a \{J - \alpha j_0\}, \tag{7.7}$$

where $j_0 \equiv f_J^{(\mathrm{e})}(\phi_1) = 0.4416$, $k^{(\xi)} = \xi k^{(\mathrm{v})}$ and $k^{(\mathrm{v})} = 9.896$ (see (6.5) and (4.9)).

It is seen from (7.7) that the role of the short-range one-gluon exchange interaction becomes asymptotically (at $J \gg 0$) small compared to that of the long-range confining interaction. The same is plausible for the contribution to M^2 caused by radial interparticle oscillations: the short-range term is small and thus negligible as compared to the long-range term $ka\varkappa|A_r|^2$ of the asymptotic formula (4.8). Hence, taking into account that $\varkappa = \varkappa^{(\xi)} = 1$ for $\frac{1}{2} \le \xi \le 1$ and using the quantization rules (3.12)-(3.13) suggest the equation for the quantum Regge trajectories (at least, in the asymptotic regime $\ell \gg n_r$):

$$M^2 \sim k^{(\xi)} a \Big\{ \ell + n_r + 1 - \alpha j_0 \Big\}. \tag{7.8}$$

It coincides with the light meson spectroscopic formula (1.2) provided:

$$\sigma = \xi k^{(\mathrm{v})} a, \qquad \varkappa = 1, \qquad \zeta = 1 - \alpha j_0. \tag{7.9}$$

This result is illustrated in Fig. 5 where the values of input parameters are chosen as follows. The value of the string tension $a = 0.18$ GeV^2 is conventional. Then the mixing parameter $\xi = 0.65$ provides the conventional slope $\sigma = 2\pi a = 1.15$ GeV^2 and the daughter spacing $\varkappa = 1$. Finally, the value $\alpha \equiv \frac{4}{3}\alpha_s = 1$ is chosen in order to provide the best fit of the model Regge trajectories to the experimental data on the family of light unflavored $I = 1$ vector mesons; see Fig. 5 where reliable meson data (marked in boldface) are complemented with data on some questionable meson states [75]. It is obvious the tower structure of spectrum, due to the degeneracy of (ℓ+n_r)-type.

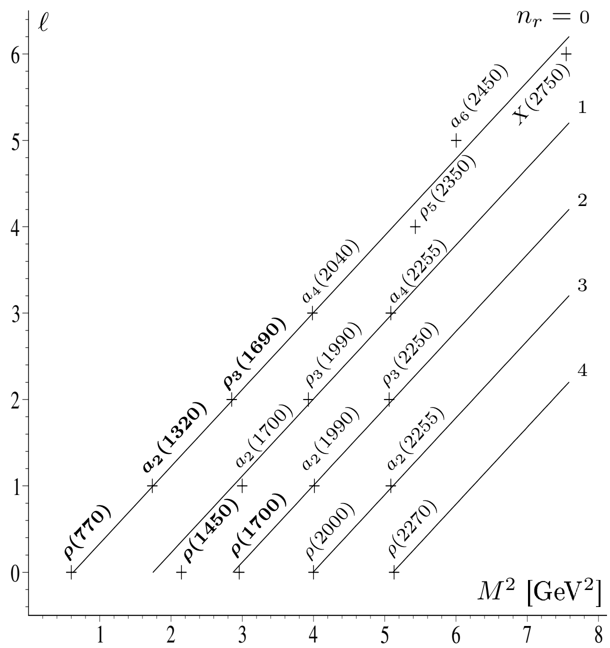

Fig. 5. Quantum Regge trajectories in the complete model vs light unflavored I=1 vector meson spectrum. Reliable meson data marked boldface. Input parameters: $a = 0.18$ GeV^2, $\xi = 0.65$, $\alpha = 1$, $m = 0$; output: $\sigma = 2\pi a = 1.15$ GeV^2, $\varkappa = 1$, $\zeta = 0.56$.

Noteworthy, the choice $\alpha_s(m_q) = 3/4$ of the running strong coupling constant corresponds to the constituent light quark mass $m_q \approx 0.15$ GeV. This follows from the well-known formula: $\alpha_s(m) = \alpha_0\{1 + \frac{7}{2\pi}\alpha_0 \ln \frac{m}{\Lambda}\}^{-1}$, where for the scale $\Lambda = m_Z = 91.19$ GeV of the intermediate Z-boson $\alpha_0 \equiv \alpha_s(m_Z) = 0.117$ [75]. The contradiction between values $m_q \approx 0.15$ GeV and $m = 0$ adopted in eq. (7.8) may lead to the discrepancy in the meson mass squared of order $\Delta M^2 \sim 4m_q^2 \approx 0.1$ GeV^2 which is negligible in the scale of Fig. 5.

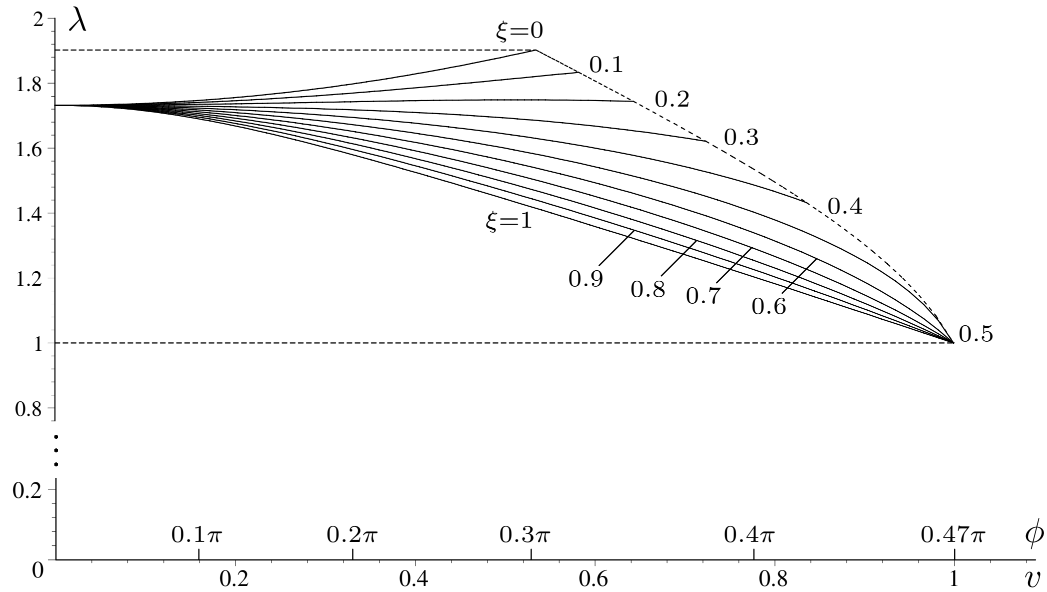

Fig. 6. The relative frequency $\lambda = \omega_r/\Omega$ as a function of the angle ϕ and the velocity v of particle circular motion for different values of the mixing parameter ξ.

For a rigorous proof of eq. (7.8) it is necessary to analyze the ACO dynamics of the complete model (2.1), (7.3), in particular, to determine the corresponding radial frequency ω_r. This is a complicated task which could be a subject of a future study.

VIII. DISCUSSION

In the present paper the ACO-quantization method [56] has been applied to the Rivacoba–Weiss model [43, 44]. This model represents a Fokker-type system of two particles which interaction can be interpreted in terms of the classical higher-derivative theory of a vector gauge field [45–49]. The Green function $\propto 1/k^4$ of this field behaves as the infrared asymptotics of the gluon propagator [48] and leads in the nonrelativistic limit to the linear interaction potential $U = ar$. In the ultrarelativistic limit the model reproduces asymptotically linear Regge trajectories whith the slope $\sigma \approx 9.9a$ related rigidly to the string tension parameter a. The energy spectrum reveals the accidental degeneracy of $(\ell{+}n_r)$-type which provides a tower structure of spectrum. Thus the quantized Rivacoba–Weiss model may serve as a good base for a description of light meson spectra.

In a variety of non-, quasi- and relativistic potential models of heavy and light mesons the linear potential $U = ar$ appears as a scalar or scalar–vector long-range part of inter-quark interaction. If one believes that the string tension a is a universal (i.e., flavor-free) parameter with conventional values in the range $a = 0.15 \div 0.3$ GeV^2 then the Rivacoba–Weiss model overestimates the slope parameter σ. Since the interaction in this model is purely vector, a counterpart model based on the higher-derivative scalar field theory [67] has been constructed. The scalar model, however, underestimates the slope of Regge trajectories. Thus the family of scalar–vector superposition models is studied. It turned out that the slope parameter $\sigma = 1.15 \div 1.2\,$GeV^2 and the string tension parameter $a = 0.15 \div 0.3$ GeV^2 can be mutually agreed if the rate of the vector interaction ranges $\xi = 0.37 \div 0.8$. In particular, the rate $\xi = 0.65$ ties the most conventional values $\sigma = 1.15$ GeV^2 and $a = 0.18$ GeV^2. Besides, a value of the mixing parameter ξ determines the daughter spacing parameter $\varkappa$. In particular, $\varkappa = 3/2$ at $\xi = 0.37$ and $\varkappa = 1$ if $\xi \geq 1/2$, so that the tower structure is also provided.

It is worth noting that within non- and quasi-relativistic potential models the linear interaction is meant mostly as a scalar one. But in many relativistic models, especially those based on the Dirac equation, the scalar–vector structure of a long-range interaction is preferable [4, 26–29]. In particular, the mixture $\xi = 1/2$, as in [25, 29], or contiguous values $\xi = 0.48 \div 0.65$, as in [26], enables to reduce a spin-orbital splitting in accordance to observable values. The present relativistic model assures the scalar–vector structure of confining interaction in the rate $\xi = 0.65$ from another viewpoint.

In order to be appropriate for the description of both light and heavy mesons the vector short-range interaction due to one-gluon exchange is introduced. It is done naturally via complementing action (2.1), (6.1) by the Wheeler–Feynman term (2.2), where $\alpha = \frac{4}{3}\alpha_{\rm s}$ is related to a strong coupling constant $\alpha_{\rm s}$. Then the model reproduces, in the non-relativistic limit, the Cornell potential (1.1). This modification affects some characteristics of the model in a relativistic regime. In particular, it changes bottom segments of Regge trajectories and decreases their intercept ζ (see (1.2)). In turn, a small intercept is appropriate for a description of lightest mesons [23]. Although a complete study of the model complemented with the Wheeler–Feynman term is beyond the scope of this work, some preliminary result has been derived and applied to the meson spectroscopy. In particular, Regge trajectories built within the massless limit of the model fit well the family of light unflavored $I = 1$ vector mesons. It is shown that the tower structure of spectrum reveals the degeneracy of the $(\ell{+}n_r)$-type.

Another extension of the model appropriate for a comprehensive study of the meson spectroscopy is the insertion of a particle spin. One can exploit, as a guideline, a description of spinning particles in terms of anticommuting variables used in the Wheeler–Feynman electrodynamics [76]. A quantization method should be modified appropriately.

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APPENDIX. CALCULATION OF $\mathcal{D}^{\perp}$ AND $\lambda = \omega_r/\Omega$

It is convenient to define a dimensionless 2×2 reduced dynamical matrix

$$\bar{\mathcal{D}} \equiv \frac{1}{a\Omega}\mathcal{D}^{\perp} = \frac{m}{a}\Omega\mathcal{C} + \mathcal{K} - \varXi = f_\Omega(\phi)\mathcal{C} + \mathcal{K} - \varXi \tag{A.1}$$

where:

$$\mathcal{C} = \begin{bmatrix} \Gamma^3 + \lambda^2\Gamma & -{\rm i}\,\lambda\Gamma^3 v^2 \\ {\rm i}\,\lambda\Gamma^3 v^2 & \Gamma + \lambda^2\Gamma^3 \end{bmatrix}, \tag{A.2}$$

$$\mathcal{K} = \int\limits_0^{\phi} {\rm d}\varphi\,\mathcal{K}_0 - \frac{1}{f'(\varphi)}\mathcal{K}_1\bigg|_{\varphi=\phi} + \frac{1}{f'(\varphi)}\frac{\rm d}{{\rm d}\varphi}\frac{1}{f'(\varphi)}\mathcal{K}_2\bigg|_{\varphi=\phi}, \tag{A.3}$$

$$\varXi = \int\limits_0^{\phi} {\rm d}\varphi\,\varXi_0 - \frac{1}{f'(\varphi)}\varXi_1\bigg|_{\varphi=\phi} + \frac{1}{f'(\varphi)}\frac{\rm d}{{\rm d}\varphi}\frac{1}{f'(\varphi)}\varXi_2\bigg|_{\varphi=\phi}. \tag{A.4}$$

The matrix $\mathcal{C}$ comes from the free-particle term of action (2.1). The function $f_\Omega(\phi)$ and components of other matrices $\mathcal{K}$ and $\varXi$ depend on the interaction model.

For the vector (Rivacoba–Weiss) model the function $f^{(\rm v)}_\Omega(\phi)$ is defined in (4.2), and matrices in r.h.s. of (A.3) and (A.4) have the form:

$$\mathcal{K}^{(\rm v)}_0 = 0, \tag{A.5}$$

$$\mathcal{K}^{(\rm v)}_1 = 2\begin{bmatrix} 1{+}v^2{\rm c}(3{+}2{\rm c}) & 0 \\ 0 & 2v^2{\rm s}^2 \end{bmatrix} - 2{\rm i}\,\lambda v^2(1+{\rm c})\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{A.6}$$

$$\mathcal{K}^{(\rm v)}_2 = -4v^2(1+v^2{\rm c})\begin{bmatrix} (1{+}{\rm c})^2 & 0 \\ 0 & {\rm s}^2 \end{bmatrix}, \tag{A.7}$$

$$\varXi^{(\rm v)}_0 = (1+\lambda^2)\begin{bmatrix} {\rm cC} & {\rm sS} \\ -{\rm sS} & {\rm cC} \end{bmatrix} - 2{\rm i}\,\lambda\begin{bmatrix} {\rm sS} & -{\rm cC} \\ {\rm cC} & {\rm sS} \end{bmatrix}, \tag{A.8}$$

$$\begin{aligned}\varXi^{(\rm v)}_1 = {} & -2\begin{bmatrix} {\rm c}(1{+}{\rm v}^2(2{+}3{\rm c})){\rm C} & {\rm s}(1{+}{\rm v}^2(1{+}3{\rm c})){\rm S} \\ -{\rm s}(1{+}{\rm v}^2(1{+}3{\rm c})){\rm S} & (1{+}v^2({\rm c}^2{-}2{\rm s}^2)){\rm C} \end{bmatrix} \\ & - 2{\rm i}\,\lambda v^2\begin{bmatrix} 2{\rm s}(1{+}{\rm c}){\rm S} & ({\rm s}^2{-}{\rm c}(1{+}{\rm c})){\rm C} \\ ({\rm c}(1{+}{\rm c}){-}{\rm s}^2){\rm C} & 2{\rm scS} \end{bmatrix},\end{aligned} \tag{A.9}$$

$$\varXi^{(\rm v)}_2 = 4v^2(1+v^2{\rm c})\begin{bmatrix} (1{+}{\rm c})^2{\rm C} & {\rm s}(1{+}{\rm c}){\rm S} \\ -{\rm s}(1{+}{\rm c}){\rm S} & -{\rm s}^2{\rm C} \end{bmatrix}, \tag{A.10}$$

where $\mathrm{s} \equiv \sin\varphi$, $\mathrm{c} \equiv \cos\varphi$, $\mathrm{S} \equiv \sin(\lambda\varphi)$, $\mathrm{C} \equiv \cos(\lambda\varphi)$.

For the scalar confining interaction the function $f_\Omega^{(\mathrm{s})}(\phi)$ is defined in (5.3), and matrices in r.h.s. of (A.3) and (A.4) have the form:

$$\mathcal{K}_0^{(\mathrm{s})} = \begin{bmatrix} \Gamma^2 & 0 \\ 0 & 1 \end{bmatrix} - \mathrm{i}\,\lambda(\Gamma^2+1)\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \lambda^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{A.11}$$

$$\mathcal{K}_1^{(\mathrm{s})} = 2\begin{bmatrix} 1-v^2(3+2\mathrm{c}) & 0 \\ 0 & 1-v^2 \end{bmatrix} - 2\mathrm{i}\,\lambda v^2(1+\mathrm{c})\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{A.12}$$

$$\mathcal{K}_2^{(\mathrm{s})} = -4v^2(1-v^2)\begin{bmatrix} (1+\mathrm{c})^2 & 0 \\ 0 & \mathrm{s}^2 \end{bmatrix}, \tag{A.13}$$

$$\Xi_0^{(\mathrm{s})} = \Gamma^2 v^2 \mathrm{C}\begin{bmatrix} 1 & -\mathrm{i}\,\lambda \\ \mathrm{i}\,\lambda & \lambda^2 \end{bmatrix} \tag{A.14}$$

$$\begin{aligned} \Xi_1^{(\mathrm{s})} &= -2\begin{bmatrix} (\mathrm{c}(1-3v^2)-2v^2)\mathrm{C} & \mathrm{i}\,(1-2v^2)\mathrm{sS} \\ -\mathrm{i}\,(1-2v^2)\mathrm{sS} & (1-v^2)\mathrm{cC} \end{bmatrix} \\ &\quad - 2\mathrm{i}\,\lambda v^2\begin{bmatrix} 0 & (1+\mathrm{c})\mathrm{C} \\ -(1+\mathrm{c})\mathrm{C} & -2\mathrm{sS} \end{bmatrix}, \end{aligned} \tag{A.15}$$

$$\Xi_2^{(\mathrm{s})} = 4v^2(1-v^2)\begin{bmatrix} (1+\mathrm{c})^2\mathrm{C} & \mathrm{s}(1+\mathrm{c})\mathrm{S} \\ -\mathrm{s}(1+\mathrm{c})\mathrm{S} & -\mathrm{s}^2\mathrm{C} \end{bmatrix}. \tag{A.16}$$

For the scalar–vector superposition the dimensionless dynamical matrix is constructed as follows:

$$\bar{\mathcal{D}}^{(\xi)} = (1-\xi)\bar{\mathcal{D}}^{(\mathrm{s})} + \xi\bar{\mathcal{D}}^{(\mathrm{v})}, \tag{A.17}$$

where ξ is the mixing parameter. The relative frequency λ is then calculated as a real positive root of the reduced secular equation $\det\bar{\mathcal{D}}(\lambda) = 0$. In general, this can be done numerically.

In Fig. 6 the relative frequency $\lambda = \omega_r/\Omega$ as a function of the velocity v of the particle circular motion is shown for various values of the mixing parameter ξ. Let us note that $\lim\limits_{v\to 0}\omega_r/\Omega = \sqrt{3}$ as it must be for the nonrelativistic problem with the linear potential $U = ar$ [56].

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ТРАЄКТОРІЇ РЕДЖЕ В РЕЛЯТИВІСТСЬКІЙ ТЕОРІЇ ПРО ДІЮ НА ВІДСТАНІ

Аскольд Дувіряк
Інститут фізики конденсованих систем НАН України,
вул. Свєнціцького, 1, Львів, UA-79011, Україна

Запропоновано релятивістську кваркову модель мезонів, сформульовану в межах формалізму інтеґралів дії типу Фоккера. Міжкваркова взаємодія в ній переноситься безмасовим векторним полем та скалярно-векторною суперпозицією полів, що описуються рівняннями з вищими похідними. У нерелятивістській границі модель описує двочастинкову систему з потенціалом Корнеля. Для аналізу моделі в істотно релятивістській області застосовано наближення збурених колових орбіт, що дає змогу здійснити канонічне квантування моделі. Показано, що модель добре відтворює особливості спектроскопії легких мезонів.