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GUARANTEED NARMAX MODEL FOR THE PREDICTION OF GEOMAGNETIC $D_{\rm st}$ INDEX

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Theoretical analysis of the guaranteed prediction model of the $D_{\rm st}$ index has been conducted. A new identification approach for the $D_{\rm st}$ index prediction model using experimental data and an evolutionary algorithm has been proposed. All models are based on an observational data set consisting of VB_s, the solar wind parameters as the input and the $D_{\rm st}$ index as the output. This method showed a number of advantages over the earlier proposed methods of space weather forecasting. First, using the dynamic-information approach it is possible to automatically select the most significant regressors from the history of the experimental data. Secondly, it becomes possible to automatically reconstruct a mathematical model and search for optimal model parameters.

Key words: guaranteed prediction, space weather, dynamic-information approach, identification, $D_{\rm st}$ index.

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I. INTRODUCTION

Previous research was concentrated on forecasts of the solar wind parameters (e.g. density, electron and ion temperatures, velocity and magnetic field) at L1 based on magnetogram observations from GONG (Global Oscillation Network Group) in a community-based program to study the solar internal structure and dymamics using helioseismology. It will result in the forecast of potential space weather hazards up to 2 or so days before they arrive at the Earth [1,2]. Scientists try to construct a new set of statistical wave models to describe the plasma wave environment of the inner magnetosphere that will accurately reflect the physics of the dynamics of the radiation belts under the influence of the solar wind. These novel wave models lead to more realistic tensors of diffusion coefficients that are critical for physics based models of the radiation belts. Therefore, the most common way to solve this problem is to use the system science and artificial intelligence techniques [3] that have been applied to space weather since at least the 1990s. Using the state of the art system science methodologies it is necessary to develop new forecasting tools for geomagnetic indices and to assess the prediction efficiency of these new tools alongside those currently available to identify the most reliable techniques to predict the geomagnetic state of the magnetosphere, as expressed by geomagnetic indices, in relation to the solar wind input conditions.

The overall aim of our research is to improve and farther develop the models based on data driven modeling such as ROBUST NARMAX (RN), ROBUST BILINEAR (RB), POLYNOMIAL (PM), and LYPEXP MODEL (LEM). In addition, new models based on the Guaranteed Prediction Approach (GP) of geomagnetic indices have been proposed by Vitaliy Yatsenko [4–6].

SRI (Space Research Institute), models have been implemented for real time operation at SRI and the data and plots will be provided on a SRI web server.

During our research, we used a polynomial discrete model of the $D_{\rm st}$ (disturbance storm time) index [7], which allows to represent the output signal through the system:

$$y(k) = F[y(k-1), \dots, y(k-n_y), u(k-1), \dots, u(k-n_u), \xi(k-1), \dots \xi(k-n_\xi)], \tag{1}$$

where $F[*]$ is polynomial of variables $y(k), u(k)$ and $\xi(k)$ of order l; $u(k)$ is an input signal; $y(k)$ is output signal; $\xi(k)$ is a variable that simulates the possible noise. To select a specific polynomial, we will apply the procedures of structural-parametric identification of the model, whose essence is to add such non-linear terms to improve the predictability and stability. The process of selecting nonlinear terms is usually considered complete when the prediction error satisfies a certain test for the accuracy of the model, which confirms the impossibility of a subsequent error reduction on new observational data.

II. ROUTINE

To simplify the prediction problem of the $D_{\rm st}$ index (nT), we consider the magnetosphere as a system with one input and one output. The description of nonlinear processes can also be improved by introducing free parameters and noise into the model. As shown by numerical calculations, the choice of the parameter VB_s as an input allows us to develop a model that is adequate for the forecast.

Solar wind parameters were measured in the GSM coordinate system. B is the hourly average of the field magnitude with three components B_x, B_y, and B_z. B_z is the southward component of the IMF (interplanetary magnetic field). If $B_z < 0$, $B_s = -B_z$; otherwise $B_s = 0$. V is the solar wind velocity in km/sec.

Obviously, it is more natural to predict not a consequence, i.e. y_{k+n}, the causes of their causes, i.e. $u_{k-(m-l)+1}, u_{k-(m-l)+2}, \ldots y_{k+n-1}, y_{k+n-2}, \ldots$ and further on the basis of the mathematical model of the process (1) determine the predictive estimates $\widetilde{y}_{k+n}$ of the quantities y_{k+n}.

Let us first consider the definition of the forecast estimate $\widetilde{y}_{k+n}$ for $n = 1$. To do this, we need to estimate $\widetilde{u}_{k-(m-l)+1}$ of the value $u_{k-(m-l)+1}$. Consider the class of processes (1) for which there is a priori estimate of the change rate u_k in the form u_k

$$\Delta u_k \in \delta\mathbf{u} = \{\Delta u_k : \Delta u_k = |u_{k-1} - u_k| \le \delta = \text{const}\}. \tag{2}$$

We note that in this case it is not assumed that quantity δ is small. From (2) follows the estimate $\widetilde{u}_{k+1}$ from the values u_{k+1} in the form

$$\widetilde{u}_{k+1} \in \mathbf{u}_{k+1} = \{u_{k+1} : u_k - \delta \le u_{k+1} \le u_k + \delta\}. \tag{3}$$

We rewrite equation (1) for the $(k+1)$th step:

$$y_{k+1} = f\big(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)+1}, u_{k-(m-l)}, \ldots, u_{k-m+1}\big). \tag{4}$$

Substituting in (4) the estimate u_{k+1} (3) for the unknown at the k-th step, we obtain:

$$y_{k+1} = f\big(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)} + \Delta u_k, \ldots, u_{k-m+1}\big). \tag{5}$$

From (5) we find an interval estimate [8] $\widetilde{y}_{k+1}$ of the quantity y_{k+1} in the form:

$$\widetilde{y}_{k+1} \in \widetilde{\mathbf{y}}_{k+1} = \{\widetilde{y}_{k+1} : \underline{y}_{k+1} \le \widetilde{y}_{k+1} \le \overline{y}_{k+1}\}, \tag{6}$$

where

$$\underline{y}_{k+1} = \min_{\Delta u_k \in \delta\mathbf{u}} f(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)} + \Delta u_k, \ldots, u_{k-m+1}) \tag{7}$$

$$\overline{y}_{k+1} = \max_{\Delta u_k \in \delta\mathbf{u}} f(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)} + \Delta u_k, \ldots, u_{k-m+1}) \tag{8}$$

If $f(\ldots, u_{k-(m-l)} + \Delta u_k, \ldots)$ is a monotonic function, then its minimum and maximum belong to the boundaries of the interval $\Delta\mathbf{u}$. If, however, $f(\ldots, u_{k-(m-l)} + \Delta u_k, \ldots)$ is a multiextremal function, then replacing the interval $\{-\delta; \delta\}$ with set of discrete values $\Delta u^{(i)}$, where $i = \overline{1, K}$, we reduce the problem (7), (8) to the combinatorial problem that is solved by a simple listing.

We now find an estimate $\widetilde{y}_{k+2}$ of the quantity y_{k+2}. To do this, we rewrite equation (7) for the step $(k+2)$:

$$y_{k+2} = f(y_{k+1}, y_k, \ldots, y_{k-m+2}, u_{k-(m-l)+2}, \ldots, u_{k-m+2}). \tag{9}$$

The value $u_{k-(m-l)+2}$ is

$$u_{k-(m-l)+2} = u_{k-(m-l)} + \Delta u_{k+1} + \Delta u_k. \tag{10}$$

Since the estimate (2) of the quantity Δu_k is valid for any k, we obtain from (10) and (9) an estimate $\widetilde{u}_{k-(m-l)+2}$ of the quantity $u_{k-(m-l)+2}$:

$$\widetilde{u}_{k-(m-l)+2} \in \widetilde{\mathbf{u}}_{k-(m-l)+2}, \tag{11}$$

where

$$\widetilde{\mathbf{u}}_{k-(m-l)+2} = \{u : \underline{u}_{k-(m-l)+1} - \delta \le u_{k-(m-l)+1} \le \overline{u}_{k-(m-l)+1} + \delta\}. \tag{12}$$

From (7), (11) and (12) follows the estimate $\widetilde{y}_{k+2}$ in the form:

$$\widetilde{y}_{k+2} \in \widetilde{\mathbf{y}}_{k+2}, \tag{13}$$

where $\underline{y}_{k+2} \le \widetilde{y}_{k+2} \le \overline{y}_{k+2}$,

$$\widetilde{y}_{k+1} \in \widetilde{\mathbf{y}}_{k+1} = \{\widetilde{y}_{k+1} : \underline{y}_{k+1} \le \widetilde{y}_{k+1} \le \overline{y}_{k+1}\}, \tag{14}$$

$$\underline{y}_{k+1} = \min_{\Delta u_k \in \delta \mathbf{u}} F^L(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)} + \Delta u_k, \ldots, u_{k-m+1}). \tag{15}$$

From (14), (15) follows that the interval set $\widetilde{\mathbf{y}}_{k+2}$ depends only on three indeterminate quantities y_{k+2}, $u_{k+(m-l)+1}$ and Δu_k for which their corresponding multiple estimates are given. Therefore, the value of the interval $\{\underline{y}_{k+2}; \overline{y}_{k+2}\}$ is greater than the interval $\{\underline{y}_{k+1}; \overline{y}_{k+1}\}$. All that is said about the solution of problems (7), (8) remains valid for problems (14), (15), with the only difference that these optimization problems are solved in a three-dimensional space. The determination of the estimates $\widetilde{y}_{k+n}$ of the value y_{k+n}, where $n \ge 3$, is done in a similar way, and these estimates become rougher with increasing value.

To make predictive estimates of the D_{st} index using the approach described above, we used two models for reconstructing it from experimental data: linear and nonlinear. Using expression (16) and the genetic algorithm for searching for unknown parameters, the following model was obtained (time interval 2000 hours):

$$\overline{y}_{k+1} = \max_{\Delta u_k \in \delta \mathbf{u}} F^L(y_k, y_{k-1}, \ldots, y_{k-m+1}, u_{k-(m-l)} + \Delta u_k, \ldots, u_{k-m+1}), \tag{16}$$

where $\overline{y}_{k+1}$ is the D_{st} index, u is the value of VB_s, k is discrete time, and $\theta_1, \ldots, \theta_{11}$ are unknown parameters. The quality of the simulation was tested using the root-mean-square error and the correlation coefficient between the actual experimental data and the simulation result.

III. NUMERICAL RESULT AND SUMMARY

As the result, the following values of the model parameters were obtained (Fig. 1) for (16): $\theta_1 = 1.255, \theta_2 = -0.355, \theta_3 = 0.053, \theta_4 = 0.013, \theta_5 = -0.416, \theta_6 = -1.741, \theta_7 = 0.951, \theta_8 = 0.437, \theta_9 = -0.202, \theta_{10} = 0.04, \theta_{11} = -0.352$ when $mse = 8.31$ and correlation coefficient is 0.981. VB_s coupling function is adequate for Elman recurrent neural network (NN), which predicts D_{st} index [9]. The correlation coefficient from the use of Elman recurrent NN is smaller [9]. This NARMAX model includes 11 parameters compared with 39 parameters of Elman recurrent NN.

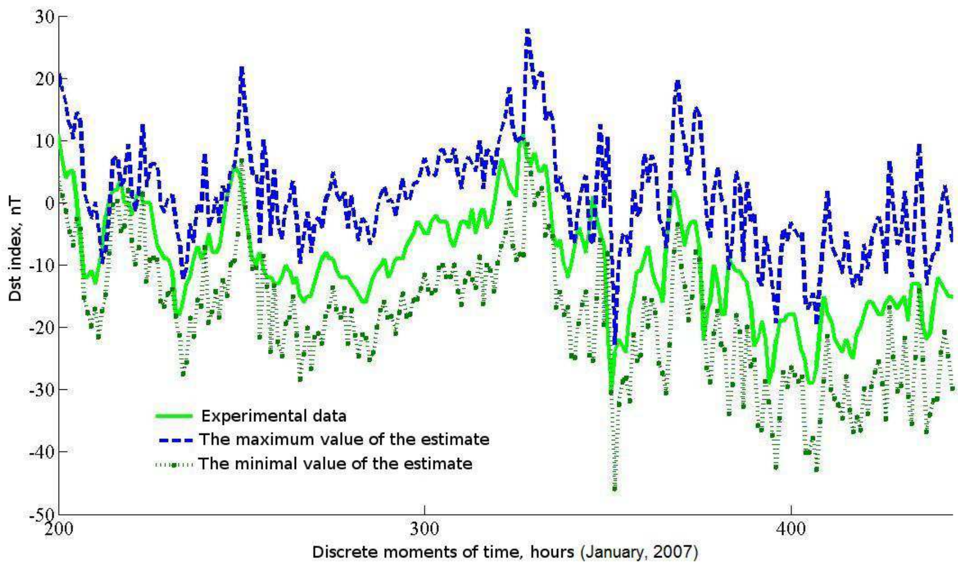

Fig. 1. Results of the calculations of predictive estimates of the D_{st} index value for two hours.

We estimated the forecast horizon of the D_{st} index for a given period of time. Using Wolf's method [10], we calculated the largest localized Lyapunov exponent (Fig. 2). The forecast horizon is $H_f \approx 1/L1$ ($H_f \approx 1 \ / \ 0.09 \approx$ 11.1 hours).

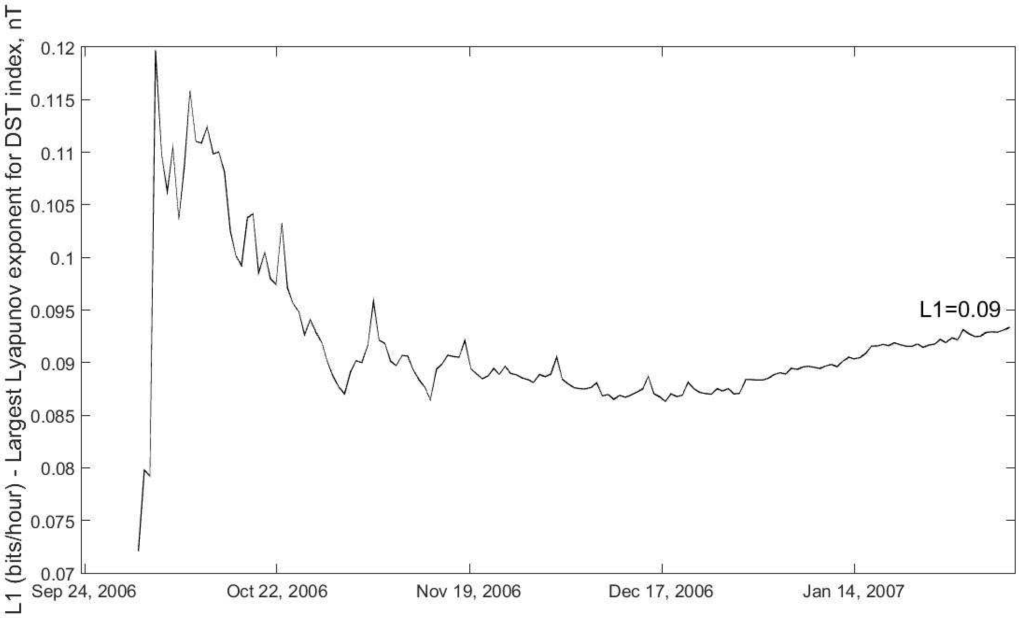

Fig. 2. Largest localized Lyapunov exponent of the D_{st} index

A theoretical analysis of the guaranteed prediction model of the D_{st} index has been completed. A new identification approach for the D_{st} index prediction models using experimental data and an evolutionary algorithm has been proposed. All models are based on an observational data set consisting of VB_s, the solar wind parameter as the input and the D_{st} index as the output. This method showed a number of advantages over the earlier proposed methods of space weather forecasting. First, using the dynamic-information approach, it is possible to automatically select the most significant regressors from the history of the experimental data.

Secondly, it makes it possible to automatically reconstruct a mathematical model and search for optimal model parameters. A numerical algorithm based on the dynamic–information approach, searching for a nonlinear discrete dynamic black box type model to predict the behavior of the D_{st} index has been developed. A software for retrospective forecasting using OMNI2 data as input has been developed. Forecasts of the D_{st} index are then computed based on our models. The results of the forecast (one, two, three, six and nine hours ahead) were analyzed in comparison with the experimental data, which showed that the forecast quality significantly reduced with the increase of time.

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ҐАРАНТОВАНА МОДЕЛЬ NARMAX ДЛЯ ПРОГНОЗУВАННЯ ГЕОМАГНІТНОГО D_{st}-ІНДЕКСУ

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Розглянуто ґарантовану модель прогнозування D_{st}-індексу. Запропоновано новий ідентифікаційний підхід до моделі прогнозування на основі експериментальних даних та еволюційного алґоритму. Усі моделі засновані на даних дистанційних спостережень, що складаються з параметрів сонячного вітру VB_s як вхідних та D_{st}-індексу як вихідного.