DETERMINING THE POSITION OF A RADIATION SOURCE USING THE CONICAL DIFFRACTION METHOD

P. P. Vankevych[1], B. D. Drobenko[1], N. Y. Ftomyn[2], Ya. M. Chornodolskyy[2], V. V. Dehtiarenko[3], A. V. Sliusarenko[3], A. D. Chernenko[3], P. A. Bolkot[3]
[1]*Pidstryhach Institute for Applied Problems of Mechanics and Mathematics N.A.S. of Ukraine, 3-b, Naukova St., UA–79060, Lviv, Ukraine,*
[2]*Ivan Franko National University of Lviv, 19, Drahomanov St., Lviv, UA–79005, Ukraine,*
[3]*Hetman Petro Sahaidachnyi National Army Academy, 32, Heroes of Maidan St., Lviv, UA–79026, Ukraine,*
e-mail: nazar.ftomyn@lnu.edu.ua

(Received 18 October 2022; accepted 10 November 2022; published online 19 December 2022)

A diffraction grating based on polyaniline fibers was used to study the effects that occur during the oblique incidence of light rays. Diffractograms of conical diffraction were experimentally obtained for an optical system containing a diffraction grating and a laser. The set of experimental data was approximated using the least squares method. The coefficients of the second-order curves were calculated and the spatial position of the diffraction grating was determined.

Key words: conical diffraction, diffractogram, optical system.

DOI: https://doi.org/10.30970/jps.26.4403

I. INTRODUCTION

A conical diffraction (CD) of light (or of-plane diffraction) causes the distortion of the diffraction pattern (diffracted rays spread along to the surface of a cone) when the laser beam is obliquely incident on the diffraction grating (DG) [1–7]. In this case the position of the individual diffraction maxima (or a band formed by intersection of the diffraction maxima) will be in general described by a second order curve on the screen projection of the resulting diffractogram. As a result the shape of this curve (i.e. ellipse, parabola or hyperbola) significantly depends on the angular position of the laser (or on the orientation of the grating in space if the position of the laser is fixed). This effect is observed for both transmission and reflection DG, in particular, for phase gratings with a spatially modulated refractive index (see e.g., [4]). Furthermore, the problem of rays diffraction for an arbitrary space orientation of DG is important not only for visible radiation, but, for example, in the design of X-ray spectrometers [8], etc.

As a matter of fact, an important and promising application of the CD effect is the construction of technologically simple and low-priced sensors, which can be used to determine the spatial position of the source of optical radiation [9]. The polyaniline fibers DG (refractive index $n \simeq 1.5$) can be easily integrated into various materials. The CD effects will be observed when the laser radiation is incident obliquely on such a system and, as a result, the geometric parameters (i.e., curve shape, curvature, etc.) of the diffraction curves can be easily determined using regression methods (see e.g., [10, 12, 13]).

The purpose of this article is to determine the spatial position of the polyaniline fibers DG (or laser beam propagation direction), based on the analysis of the shape and geometric parameters of the diffraction curves.

II. MEASURING METHOD

The experiment was carried out according to the scheme depicted in Fig. 1. The He–Ne laser (wavelength $\lambda = 632.8$ nm) was used as the source of light. The laser beam propagated through a grating consisting of two layers of polyaniline fiber (the diameter of fiber was $\simeq 0.16$ mm). The distance between the layers was equal to 1 mm. The DG was placed on a rotary table with the angle measuring scale (the accuracy was no worth than $\pm 1°$).

The diffraction pattern was projected onto a screen and after that recorded using a Canon 80d digital camera (CMOS-matrix with ~24 MP resolution). Diffraction curves were stored for each fixed position of the DG (see Fig. 2, 3). The rotation angle of the grating φ_0 was varied with a step of 5° from 50° to 80°, respectively. The obtained images were digitized using the program GetData Graph Digitizer 2.26 (trial version).

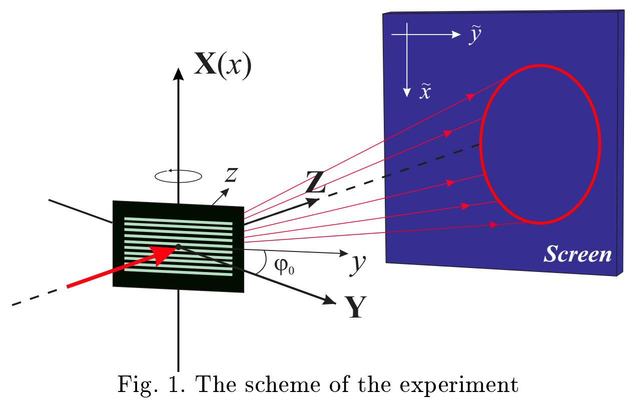

Fig. 1. The scheme of the experiment

The images of the diffraction curves obtained for various grating orientation angles are presented in Figs. 2, 3. As can be seen from these figures, the obtained second-order curves have an elliptical shape and this feature is typical of the given orientation angles of the diffraction element (see analysis [9]).

We applied a general method of the data approximation using a second-order curve:

$$F(\tilde{x}, \tilde{y}) = a\tilde{x}^2 + b\tilde{x}\tilde{y} + c\tilde{y}^2 + d\tilde{x} + e\tilde{y} + f = 0. \quad (1)$$

The a, b, c, d, e, f coefficients were determined using a Python program after applying the functions of the `lsq-ellipse 2.2.1` library [11] (this library was created using the method described in [13]).

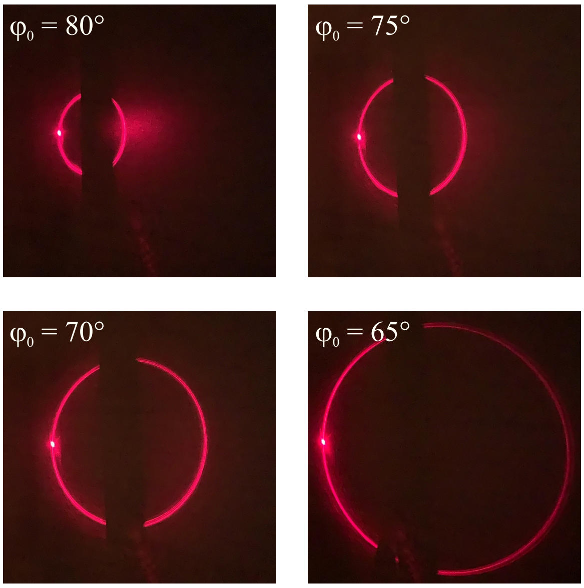

Fig. 2. The diffraction patterns obtained for $\varphi_0 = 80^\circ$, 75°, 70°, 65° orientations of DG

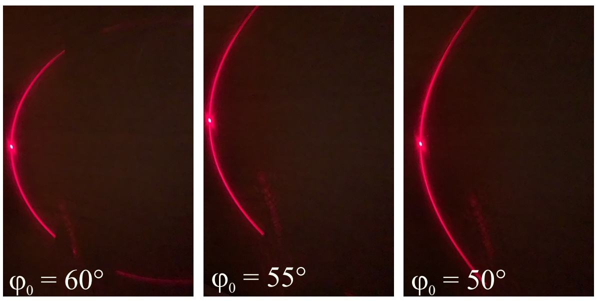

Fig. 3. The diffraction patterns obtained for $\varphi_0 = 60^\circ$, 55°, 50° orientations of the DG

III. RESULTS AND DISCUSSION

It is well known that in the case of the light beam propagation at an angle θ_0 relatively to the normal of the DG (in the plane perpendicular to the grating plane and perpendicular to the direction of the grooves), the position of the diffraction maxima is determined by the following equation [14, 15]:

$$\sin\theta_0 \mp \sin\theta_m = \mp\frac{m\lambda}{d}, \qquad m = 0, \pm 1, \pm 2, \ldots \quad (2)$$

where θ_m denotes the angle at which the m order diffraction maximum is observed, λ is the wavelength of light, d is the grating period. The sign "+" corresponds to the reflection and "−" to the transmission diffraction grating, respectively.

Let us consider the problem of diffraction when a laser beam is obliquely incident on a DG. In this case, the diffractogram will have the form of a second-order curve. As a result, the diffraction equations in the direction cosine approximation for an arbitrary space orientation of the grating are expressed as follows [1–3]:

$$\alpha_i \mp \alpha_m = \mp\frac{m\lambda}{d}\sin\psi, \qquad \beta_i \mp \beta_m = \mp\frac{m\lambda}{d}\cos\psi, \quad (3)$$

where the index i corresponds to the incident beam with the direction cosines which are described by the following equations:

$$\alpha_i = -\sin\theta_0\cos\varphi_0, \quad \beta_i = -\sin\varphi_0. \quad (4)$$

In contrast, both α_m and β_m can be expressed in terms of the coordinates of the mth order diffraction maximum for the diffracted beam:

$$\alpha_m = \frac{x_m}{\sqrt{x_m^2 + y_m^2 + z_m^2}}, \quad \beta_m = \frac{y_m}{\sqrt{x_m^2 + y_m^2 + z_m^2}}. \quad (5)$$

Accordingly, ψ is the angle between the direction of the grooves and the **X** axis, φ_0 corresponds to the angle of DG rotation (the angle between the beam propagation direction, e.i. **Z** axis, and the z axis, see Fig. 1). Furthermore, for our experiment, $\psi = \pi/2$ and $\theta_0 = 0$. Therefore, substituting both (4) and (5) into the equation (3) we obtained:

$$x_m^2 + z_m^2 - y_m^2\operatorname{ctg}^2\varphi_0 = 0. \quad (6)$$

This equation describes a cone with the axis along the y direction (parallel to the grooves of the grating) and with a circular section with radius: $R = y_m \operatorname{ctg}\varphi_0$. Furthermore, the cross-section of the diffraction cone (6) will be described by a second-order curve [9] on the plane of the screen placed perpendicular to the **Z** axis at a distance of l:

$$x_m^2\cos^4\varphi + z_m^2\cos 2\varphi + 2z_m l\sin^3\varphi - l^2\sin^2\varphi = 0, \quad (7)$$

were $\varphi = \pi/2 - \varphi_0$.

As can be seen from equation (7), there is an ellipse curve with the diffraction maxima for angles $\varphi < \pi/4$. In contrast, there are a hyperbola and a parabola observed for $\varphi > \pi/4$ and $\varphi = \pi/4$, respectively. It is necessary to note, that there is a possibility to transform the discrete coordinates of maxima $\{x_m, z_m\}$ into continuous $\{x, z\}$ ones in equation (7) taking into account the diffraction phenomena on individual fibers and unfocusing effects (polymer fiber can be considered as a cylindrical lens). In addition, there is a sufficiently large period of our DG (the linear distance between the diffraction maxima on the screen is inversely proportional to the grating period [1]). Therefore, the diffractograms have the form of solid curves, and there are no separeted diffraction maxima on the screen in our experiment. It is necessary to note that the position of the main diffraction maximum for different orientations of the grating is clearly seen on Fig. 2 and 3 (the brightest point), and there are regions of the geometric shadow from the diffraction element (Fig. 2).

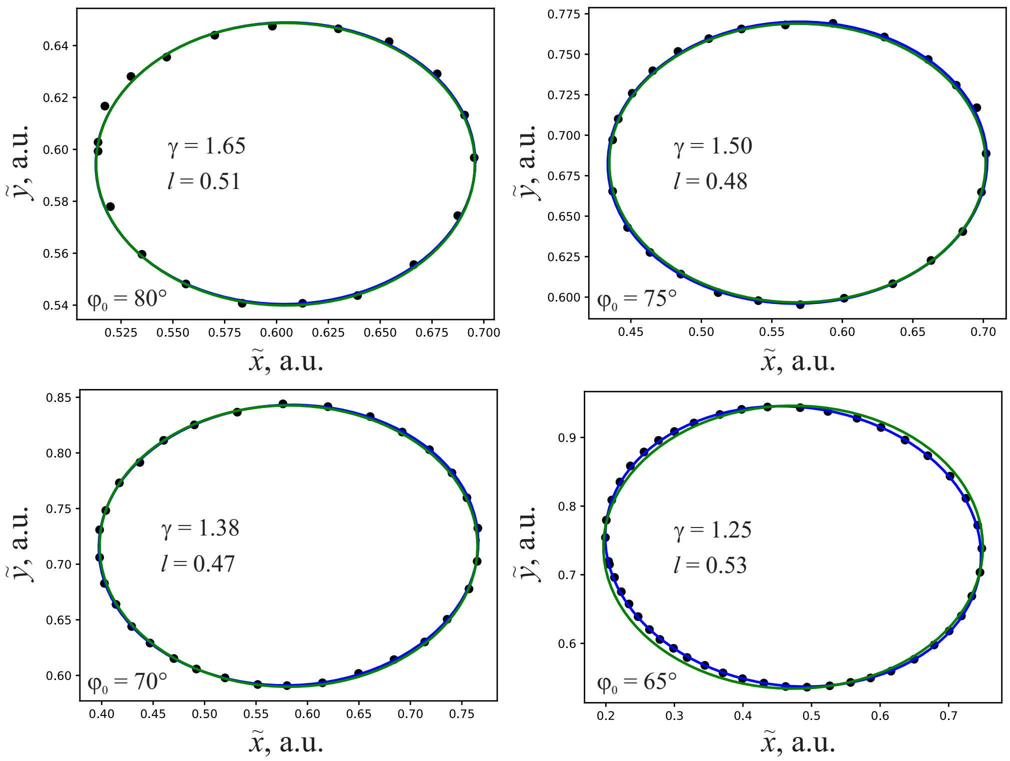

Fig. 4. Diffractograms for $\varphi_0 = 80°, 75°, 70°, 65°$ angles of orientation of the DG. • — digitized values; the curves built using the coefficients expressed in the table (1) are marked in blue color; the curves built using the equation (8) with the coefficients γ and l are marked in green color

φ_0	a	b	c	d	e	f	φ_{0_1}	φ_{0_2}
50	0.7874	0.1012	0.6081	−0.9235	−1.9503	1.1397	57.9	50.0
55	0.9015	−0.0973	0.4216	−1.0627	−1.4849	1.0207	52.7	53.3
60	0.6675	0.0277	0.7441	−0.7142	−1.4457	0.7485	63.2	53.2
65	0.4836	0.0892	0.8707	−0.5238	−1.3331	0.5819	70.2	65.2
70	−0.4237	0.0230	−0.9055	0.4763	1.2853	−0.5850	72.9	68.1
75	0.3872	−0.0127	0.9219	−0.4314	−1.2522	0.5432	74.4	75.7
80	−0.3317	0.0082	−0.9434	0.3960	1.1169	−0.4490	76.6	78.5

Table 1. The calculated a, b, c, d, e, f parameters of elliptic curves. φ_0 is the rotation angle of the DG (in degrees), φ_{0_1}, φ_{0_2} are the calculated values of the rotation angle (in degrees)

The final equation (in the screen ($\tilde{x}$, $\tilde{y}$) coordinate system, see Fig. 1) that can be used for the analysis of the experimental results is expressed as follows (the transformation $x_m \to \tilde{x}$, $z_m \to \tilde{y}/\cos\varphi$ of the coordinate system was applied):

$$\tilde{x}^2 \cos^4\varphi + \tilde{y}^2 \frac{\cos 2\varphi}{\cos^2\varphi} + 2\tilde{y}l\frac{\sin^3\varphi}{\cos\varphi} - l^2\sin^2\varphi = 0. \quad (8)$$

The results of the approximation of the digitized data for the whole range of the change of the φ_0 grating rotation angle are presented in Fig. 4 and 5, and the calculated values of a, b, c, d, e, f parameters of the second-order curves (1) are presented in Table 1. As can be seen from the obtained results, the absolute value of the coefficient c ($|c| < 1$) increases with respect to the rotation angle of φ_0 (except for the value obtained for $\varphi_0 = 55°$). Therefore, the value of the angle φ_0 could be estimated using the equation: $c = \cos 2\varphi/\cos^2\varphi$. By solving this equation with respect to $\cos\varphi = 1/\sqrt{2-c}$, it is possible to calculate φ_0 (see φ_{0_1} in the Table 1).

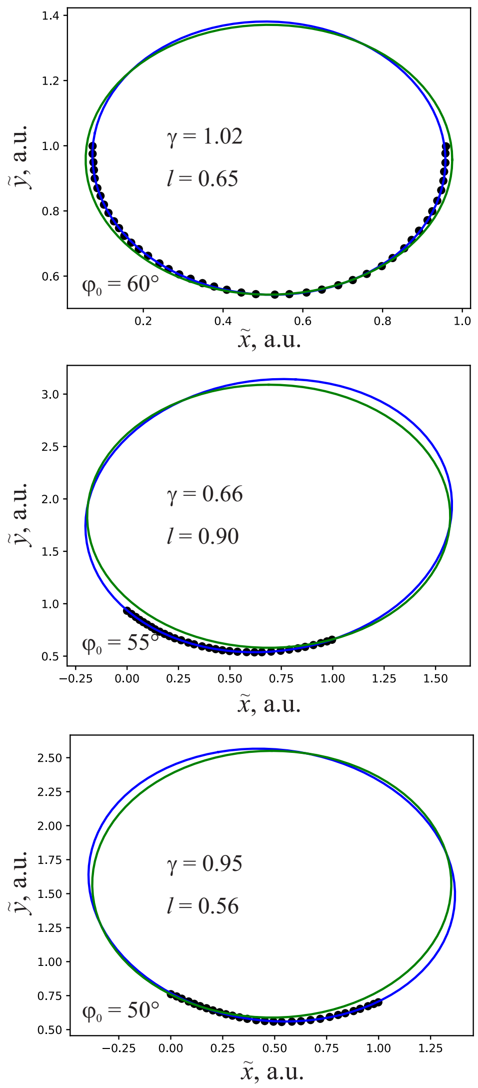

Fig. 5. The diffraction patterns for $\varphi_0 = 60°$, $55°$, $50°$ orientation angles of the diffraction grating. • — digitized values; the curves built using the coefficients of the table (1) are colored in blue, the curves built using the equation (8) with the coefficients γ and l are colored in green

A good agreement between experimental and calculated data was obtained, although for some values of φ_0 quite a significant deviation is observed.

As a matter of fact, the coefficients of equation (1) could be multiplied by an arbitrary parameter (as a result, the same curve is observed) and, consequently, there is some imperfection in the respective method of calculation. Therefore, the normalized coefficients (e.g., c/a, etc) were used for further data analysis. In this case, an additional coefficient γ in equation (8) was entered (by replacing the $\tilde{y} \to \gamma\tilde{y}$) to match the geometric parameters of projected images on the screen and digitized curves. Thus, there are two parameters l and γ, value of whose can be chosen to best agree the curves constructed by equation (8) with the experimental data. Finally, to determine the unknown angle φ, one can use the equation: $c/a = \gamma^2 \cos 2\varphi / \cos^6 \varphi$.

The obtained curves and the values of both l and γ parameters are presented in Figs. 4, 5. The obtained mean values of the parameters are $\bar{\gamma} = 1.20 \pm 0.35$ and $\bar{l} = 0.59 \pm 0.16$ and they may be considered as constants of the measuring system (sensor). The values of the angle φ_0 calculated using to this method are presented as φ_{0_2} in Table 1. As can be seen from the data expressed in Table 1, the values of the coefficients l and γ significantly depend on the angle of rotation of the DG (there is a considerable variation in the l and γ magnitude). These particular results may be attributed to the effect of a small change in the distance between the digital camera and the screen, as well as focusing (to obtain an optimal image of the diffraction pattern).

CONCLUSIONS

The phenomenon of conical diffraction was studied using a diffraction grating based on polyaniline fibers. The main equations which describe the second order elliptical shape curves with the given orientation angles of the diffraction element have been tested.

Based on the obtained experimental diffraction patterns, the values of the coefficients a, b, c, d, e, f of the corresponding curves were calculated. As a result, the values of the grating rotation angles in the range from 50° to 80° were determined, which allows identifying the spatial position of the laser radiation source.

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ВИЗНАЧЕННЯ ПОЛОЖЕННЯ ДЖЕРЕЛА ВИПРОМІНЮВАННЯ МЕТОДОМ КОНІЧНОЇ ДИФРАКЦІЇ

П. П. Ванкевич[1], Б. Д. Дробенко[1], Н. Є. Фтомин[2], Я. М. Чорнодольський[2], В. В. Дегтяренко[3], А. В. Слюсаренко[3], А. Д. Черненко[3], П. А. Болкот[3]
[1]*Інститут прикладних проблем механіки і математики імені Я. С. Підстригача НАН України, вул. Наукова, 3-б, Львів, 79060, Україна*
[2]*Львівський національний університет імені Івана Франка, вул. Драгоманова 19, Львів, 79005, Україна*
[3]*Національна академія сухопутних військ імені гетьмана Петра Сагайдачного, вул. Героїв Майдану 32, Львів, 79026, Україна*

Застосування ефекту конічної дифракції — важливе та перспективне завдання для визначення просторового положення джерел оптичного випромінювання. Цей ефект можна спостерігати, використовуючи сформовану з волокон поліаніліну дифракційну ґратку як складову частину оптичних сенсорних систем. Під час похилого падіння лазерного променя на таку систему можна визначити геометричні параметри дифракційних кривих, застосовуючи аналіз зображень та реґресійні методи обробки даних.

Для двошарової поліанілінової ґратки, орієнтованої під кутами від 50 до 80 градусів, експериментально одержано дифрактограми конічної дифракції, а відповідну оптичну систему побудовано на основі He–Ne-лазера. Отримані криві другого порядку мають еліптичну форму, що типово для заданих кутів орієнтації дифракційного елемента.

У роботі використано метод найменших квадратів, за допомогою якого апроксимовано набір експериментальних даних, одержаних унаслідок оцифрування зображень. Для розв'язання цієї задачі застосовано вільне програмне забезпечення. Розрахункову програму створено мовою програмування Python з використанням функцій та методів бібліотеки `lsq-ellipse 2.2.1`. Наведено результати апроксимації оцифрованих даних для всього діапазону зміни кута φ_0 повороту ґратки та розраховано значення a, b, c, d, e, f параметрів кривих другого порядку. На основі співвідношень між коефіцієнтами еліпса та кутовим положенням дифракційної ґратки розраховано значення кута φ_0. Отримано добре узгодження між експериментальним кутами повороту ґратки та розрахованими значеннями. Визначено величину двох коефіцієнтів вимірювальної системи (сенсора), які зазнають варіацій відносно значень $\gamma \simeq 1.20$ та $l \simeq 0.6$ унаслідок малої зміни відстані від цифрового фотоапарата до екрана.

У разі фіксації дифракційної ґратки таку оптичну систему можна використати, щоб визначити кутове положення джерела лазерного випромінювання.

Ключові слова: конічна дифракція, дифрактограма, оптична система.