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EFFECT OF MAGNETIC FIELD MODULATION ON BÉNARD–TAYLOR FERROCONVECTION

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This paper is concerned with the influence of rotation on the onset of ferromagnetic fluid convection in the presence of both modulated and unmodulated magnetic fields. The effects of magnetic field modulation and rotation on the onset of ferroconvection are of interest from both practical and theoretical points of view. Modulation of an appropriate parameter may significantly affect the motion and improve the stability of various systems, including charges in an electrostatic field and ferromagnetic resonance. Rotating ferrofluids have potential uses in a variety of fields including rotating turbomachines and chemical processing industry. The resulting eigenvalue problem is solved using isothermal boundary conditions and the regular perturbation method under the assumption of a small modulation amplitude. On the assumption that the principle of exchange of stabilities is valid, the onset criteria are formulated. The magnetic parameter, the Taylor number, the Prandtl number, and the magnetic field modulation frequency are all functions of the thermal Rayleigh number shift. The influence of various physical factors is perceived to be significant at moderate values of the magnetic field modulation frequency. The study shows that, in the presence of both magnetization and rotation, the magnetic field modulation has a destabilizing impact on the system with convection occurring faster than in the unmodulated system.

Key words: magnetic fluid, magnetic field modulation, perturbation method, stability, rotation.

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I. INTRODUCTION

Ferrofluids are a specific kind of colloidal suspension made up of numerous tiny particles of a solid ferromagnetic substance (iron-Fe, cobalt-Co, nickel-Ni etc.) in a liquid carrier (hydrocarbon, ester and fluorocarbon). The ferromagnetic components must be coated with a shell made of a suitable substance in order to forbid swarming in the presence of a magnetic field [1]. According to the coating, the ferrofluids are classified into two main groups: surfacted ferrofluids, if the coating is a surfactant molecule, and ionic ferrofluids, if it is an electric shell. A surfacted ferrofluid is used in this investigation. The liquid's ability to react to a magnetic field is one of its most significant features. This characteristic results from the magnetic body force occurring in a magnetic field. Many researchers and technologists, however, are fascinated by colloidal magnetite (Fe_3O_4), the most thoroughly studied ferrofluid, due to its diverse applications in thermal engineering, bio-medical domain, and aerospace [2–4]. The notion of ferroconvection to thermal expansion in a layer enclosing ferrofluid is comparable to the conventional Bénard convection and has sparked considerable interest due to its potential value as a heat exchanger.

According to Finlayson's study on linear heat transfer in a non-rotating thin layer of a magnetic fluid, the convection of a magnetic fluid exhibiting variable susceptibility to magnetic fields gives rise to a non-uniformity in magnetic body force, which causes thermomagnetic heat transfer [5]. In addition to visualizing the flow pattern, Schwab and coworkers experimentally acknowledged Finlayson's results [6]. But in order to properly align the convection rolls, they had to add an additional magnetic force to the system. In the absence of this longitudinal field, they discovered an erratic convection pattern. The work of Finlayson served as a sufficient source of motivation for several researchers, who explored the ferroconvective instability problem under a range of practical constraints [7–12]. Recent research using the higher order Galerkin method has shown that the effect of magnetic force and second sound boosts the starting point of Brinkman ferroconvection. The commencement of Brinkman ferroconvection is, however, prevented by the porous parameter and MFD viscosity [13].

The motion of a variety of domains, notably magnetic field sensors, ions in an electrode material, modulators, ferromagnetic resonant, and optical switches, may be significantly influenced by the modulation of a suitable parameter, which can also increase the stability of the system. The alteration in the magnetic field with respect to time on the threshold of ferroconvection and the conflict between harmonic and sub-harmonic modes using the Floquet theory, the Chebyshev pseudospectral procedure, and QZ method have been examined in some detail [14–16]. Additionally, it is revealed in an experimental study [17] how the features of the starting point of thermo-magnetic advection of ferromagnetic smart liquid have a substantial impact on the stationary and periodically modulated magnetic fields. According to a theoretical study [18] concerning the magnetic convection system in a nonuniformly rotating electroconductive medium exposed to an external alternating magnetic field, the magnetic modulation effect is significant. It may be used to mitigate heat transfer. On the basics of the Stokes micro-continuum theory, the combined effect of couple stresses signifying non-Newtonian characteristics of the ferrofluid and magnetic field modulation was reported in a theoretical work [19] and it was revealed that the effect of the couple stress setbacks the starting point of ferroconvection. In a theoretical paper published recently [20] utilizing the regular perturbation approach, the effect of a modulated magnetic field in a densely packed anisotropic permeable magnetic fluid layer is discussed.

Engineering applications as well as theoretical considerations drive the study of convection in rotating fluid layers. Food and chemical processing, metal solidification and centrifugal casting, rotating equipment, and biomechanics are a few of the key application fields in engineering. The analysis of ferrofluids with rotation is a noteworthy subject in itself, thus the effects of rotation on the heat transfer rate are important from a scientific and technological standpoint. In order to examine the thermal convective instability in a rotating ferrofluid layer, Das Gupta & Gupta [21] employed the linear stability theory, proving that overstability is impossible if $\mathrm{Pr} > 1$. In a theoretical paper [22], the issue of weakly nonlinear two and three-dimensional oscillatory convection in the form of standing waves for a horizontal fluid layer heated from below and revolving along a vertical axis is covered. The Coriolis effect on centrifugally driven convection in a rotating porous layer using the linear stability theory is reported in a theoretical work [23]. In annother article [24], it is examined how the magnetic field affects the starting point of rotational convection in a ferromagnetic-fluid-filled differentially heated permeable layer positioned in zero gravity and revealed that the magnetic field has a destabilizing effect. The combined effects of gravity modulation and rotation on the commencement of thermal convection in a horizontal fluid layer and a fluid-saturated porous layer have been well explained [25]. Rotation is discovered to have twofold effects, increasing or decreasing the modulating effect in the case of viscous fluid layers and Brinkman porous layers while increasing the destabilizing effect in the case of Darcy porous layers. In a theoretical paper [26], the regular perturbation approach is used to explore the combined impact of centrifugal acceleration and time-varying boundary temperatures on the onset of convective instability in a rotating magnetic fluid layer. It is delineated that, for bottom wall modulation, rotation tends to stabilize the system at low frequencies and the opposite is true for moderate and large frequencies. In a recent theoretical paper, the bio-thermal convection in a rotating layer of a porous media saturated with a Newtonian fluid containing gyrotactic microorganisms was explored [27]. The findings reveal that while increasing the rotation parameter might postpone the commencement of the bioconvection, an increase in the cell strangeness can stimulate the commencement of the bioconvection.

The literature study mentioned above indicates that no research has been done on the effect of rotation (Coriolis force) on the starting point of ferroconvection in the presence of a time-varying magnetic field. The purpose of this work is to use the regular perturbation approach to analyze the topic at hand. The study in this article is based on the assumption that the convective currents are minimal and the magnetic field modulation dimension is extremely small, preventing nonlinear effects. As a result, the onset of ferroconvection in a rotating fluid layer can potentially be sped up or slowed down depending on the frequency of magnetic field modulation. This research might be helpful for magnetic fluid technologies, including magnetic field sensors, modulators, ferromagnetic resonators, and optical switches.

II. FORMULATION OF THE PROBLEM

The system is comprised of a horizontal infinite layer of a ferrofluid separated by the two planes $z = 0$ and $z = d$. The system is working under the effect of the Coriolis force due to rotation, acceleration due to gravity $\mathbf{g} = -g\hat{k}$ and a time-varying magnetic field $\mathbf{H}_0^{\mathrm{ext}}(t) = H_0^{\mathrm{ext}}(t) = H_0(1 + \varepsilon\cos\omega t)\,\hat{k}$ due to the magnetic field modulation as shown in Fig. 1. The origin of the Cartesian coordinate system (x, y, z) is at the bottom of the fluid layer, and the z-axis is directed vertically upward. The upper and lower surfaces retained at different uniform temperatures with a gradient ΔT.

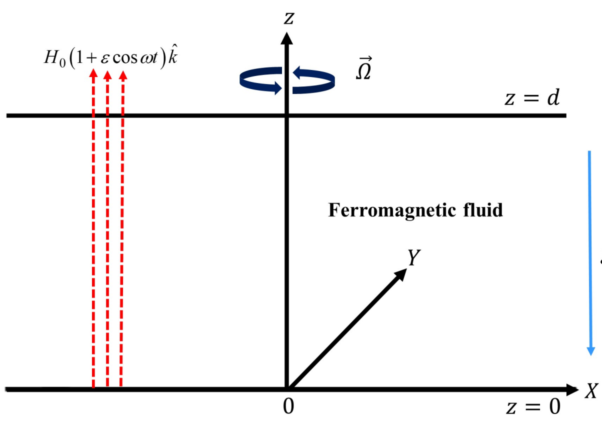

Fig. 1. Schematic Diagram

The mathematical governing equations of the present study under the Boussinesq approximation are [14, 21]

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial\mathbf{q}}{\partial t}+(\mathbf{q}\cdot\nabla)\,\mathbf{q}-2\mathbf{q}\times\mathbf{\Omega}\right] = -\nabla p+\rho\mathbf{g}+\mu\nabla^2\mathbf{q}+\nabla\cdot\ (\mathbf{H}\,\mathbf{B})\,, \tag{2}$$

$$C_1\left[\frac{\partial T}{\partial t}+(\mathbf{q}\cdot\nabla)\,T\right]+\mu_0\,T\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{V,\ H} \times\left[\frac{\partial\mathbf{H}}{\partial t}+(\mathbf{q}\cdot\nabla)\,\mathbf{H}\right]=K_1\nabla^2T, \tag{3}$$

$$\rho=\rho_0\left[1-\alpha\,(T-T_{\mathrm{R}})\right], \tag{4}$$

$$\mathbf{M}=\frac{\mathbf{H}}{H}M\,(H,\ T)\,, \tag{5}$$

$$M=M_0+\chi_{\mathrm{m}}\,(H-H_0)-K_{\mathrm{m}}\,(T-T_{\mathrm{R}})\,. \tag{6}$$

The relevant Maxwell equations are

$$\nabla\cdot\mathbf{B}=0,\quad \nabla\times\mathbf{H}=0,\quad \mathbf{B}=\mu_0\,(\mathbf{H}+\mathbf{M})\,, \tag{7}$$

where $\mathbf{q}$ is the velocity of the fluid, ρ the density, ρ_0 the reference density, μ the dynamic viscosity, μ_0 the magnetic permeability, T the temperature, g is the acceleration due to gravity, $\mathbf{\Omega}$ angular velocity, H_0 is the uniform magnetic field, ε is the small amplitude, ω is the frequency, t is the time, $\mathbf{H}$ the total magnetic field, $\mathbf{M}$ the magnetization, $\mathbf{B}$ the magnetic induction, K_1 the thermal conductivity, α the coefficient of thermal expansion, T_{R} the reference temperature, $C_1=\rho_0C_{V,H}-\mu_0\mathbf{H}\cdot\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{V,H}$, $\rho_0C_{V,H}$ the specific heat at a constant volume and magnetic field, χ_{m} is the differential magnetic susceptibility and K_{m} is the pyromagnetic coefficient. In Eq. (2), the pressure term, $p=p_{\mathrm{f}}-\frac{\rho_0}{2}\nabla|\mathbf{\Omega}\times\mathbf{r}|^2$, accommodates the centrifugal acceleration with p_{f} representing the fluid pressure and the term $2\rho_0(\mathbf{q}\times\mathbf{\Omega})$ represents the Coriolis force. The lower and upper surface temperatures respectively are $T=T_{\mathrm{R}}+\frac{1}{2}\,\Delta T$ at $z=0$ and $T=T_{\mathrm{R}}-\frac{1}{2}\,\Delta T$ at $z=d$.

III. LINEAR STABILITY THEORY

On applying the method of small perturbation and introducing the magnetic potential ϕ, we obtain the following stability equations [20, 21, 26]

$$\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2W+\sqrt{\mathrm{Ta}}\frac{\partial\zeta}{\partial z} =\left[R+RM_1(1+\varepsilon\cos\omega t)^2\right]\nabla_1^2T -RM_1(1+\varepsilon\cos\omega t)^2\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right), \tag{8}$$

$$\left(\frac{\partial T}{\partial t}-W\right)=\nabla^2T, \tag{9}$$

$$\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2\zeta=\sqrt{\mathrm{Ta}}\frac{\partial W}{\partial z}, \tag{10}$$

$$\nabla^2\phi=\frac{\partial T}{\partial z}, \tag{11}$$

where, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ and $\nabla^2=\nabla_1^2+\frac{\partial^2}{\partial z^2}$. The dimensionless parameters are Pr the Prandtl number, R the Rayleigh number, M_1 the buoyancy-magnetization parameter and Ta the Taylor number.

We now confine ourselves to the case when the boundary conditions on velocity are appropriate to a free surface which is constrained to be flat. While this case is somewhat unrealistic, it is mathematically significant because we can come up with an analytical solution whose characteristics would indicate the qualitative aspects of convective instability. Moreover, the existing results in the viscous case indicate that within the limits of a large Taylor number, the Rayleigh number is independent of the surfaces bounding the fluid [21, 28]. Equations (8)–(11) are to be solved using suitable boundary conditions [26]

$$W=\frac{\partial^2W}{\partial z^2}=T=\frac{\partial\phi}{\partial z}=\frac{\partial\zeta}{\partial z}=0\ \text{ at }\ z=0\,,1. \tag{12}$$

The magnetic boundary conditions are due to the continuity of the normal component of the magnetic induction and the tangential component of the magnetic field across the boundary. It is suitable to state the whole problem in terms of the vertical component of the W velocity. Upon combining Eqs. (8)–(11), we obtain the following equation

$$\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}-\nabla^2\right)^2\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^4W+\mathrm{Ta}\left(\frac{\partial}{\partial t}\nabla^2-\nabla^4\right)\frac{\partial^2W}{\partial z^2} =R\nabla^2\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_1^2W+RM_1\left(\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}-\nabla^2\right)(1+\varepsilon f)^2\nabla_1^4W, \tag{13}$$

where $f=\mathrm{Re}\left\{e^{-i\omega t}\right\}=\cos\omega t$.

The boundary conditions in Eq. (12) can also be expressed in terms of W in the form [26]

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = \frac{\partial^8 W}{\partial z^8} = 0 \quad \text{at } z = 0,\ 1. \tag{14}$$

IV. METHOD OF SOLUTION

The eigenfunctions W and the eigenvalues R are associated with the above eigenvalue problem for a modulated magnetic field that is different from the constant magnetic field by a small quantity of order ε. We therefore assume the solution of Eq. (12) in the form [25]

$$\begin{pmatrix} W \\ R \end{pmatrix} = \begin{pmatrix} W_0 \\ R_0 \end{pmatrix} + \varepsilon \begin{pmatrix} W_1 \\ R_1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} W_2 \\ R_2 \end{pmatrix} + \dots. \tag{15}$$

Substituting (15) into (13) and equating the corresponding terms upto $O(\varepsilon^2)$, we obtain the following system of equations:

$$LW_0 = 0, \tag{16}$$

$$LW_1 = R_1 \nabla^2 L_1 \nabla_1^2 W_0 + R_1 M_1 L_1 \nabla_1^4 W_0 + 2f R_0 M_1 L_1 \nabla_1^4 W_0, \tag{17}$$

$$\begin{aligned} LW_2 &= R_1 \nabla^2 L_1 \nabla_1^2 W_1 + R_2 \nabla^2 L_1 \nabla_1^2 W_0 + R_1 M_1 L_1 \nabla_1^4 W_1 \\ &+ R_2 M_1 L_1 \nabla_1^4 W_0 + 2f R_0 M_1 L_1 \nabla_1^4 W_1 + 2f R_1 M_1 L_1 \nabla_1^4 W_0, \end{aligned} \tag{18}$$

where $L = L_1^2 L_2 \nabla^4 - R_0 \nabla^2 L_1 {\nabla_1}^2 - R_0 M_1 L_1 {\nabla_1}^4 + L_2 \nabla^2 \mathrm{Ta} D^2$, $L_1 = \frac{1}{\mathrm{Pr}} \frac{\partial}{\partial t} - \nabla^2$, $L_2 = \frac{\partial}{\partial t} - \nabla^2$, $D = \frac{\partial}{\partial z}$.

Equation (16) which is obtained at $O(\varepsilon^0)$ is the one used in the study of ferroconvection in a rotating fluid layer in the absence of magnetic field modulation. The marginally stable solution for the problem is

$$W_0 = \sin(\pi z). \tag{19}$$

Substituting (19) into Eq. (16), we obtain the expression for the Rayleigh number R_0 for the rotating ferromagnetic fluid layer in the absence of magnetic field modulation

$$R_0 = \left[\frac{\left(\pi^2 + \alpha^2\right)^4 + \left(\pi^2 \,\mathrm{Ta} \left(\pi^2 + \alpha^2\right)\right)}{\alpha^2 \left[\pi^2 + (1 + M_1)\, \alpha^2\right]} \right] \tag{20}$$

with $\alpha^2 = \alpha_x^2 + \alpha_y^2$ being overall horizontal wavenumber, α_x and α_y being wavenumbers in x and y directions, respectively. Equation (20) is the expression for the thermal Rayleigh number as a function of the wavenumber, the buoyancy-magnetization parameter and the Taylor number for the unmodulated Bénard–Taylor ferroconvection problem. In the absence of magnetic force (i.e., when $M_1 = 0$), the expression for R_0 reduces to that of Chandrasekhar [28]. The plot of R_0 versus wave number α is delineated in the results and discussion (Section V).

Since changing the sign of ε amounts to a shift in the time origin by half a period and such a shift does not affect the stability of the problem, it follows that all the odd coefficients $R_1,\ R_3,\ R_5 \dots$ in Eq. (15) must vanish [25, 29]. Following the analysis of Malashetty and Swamy [25], one obtains the following expression for R_2 (the first non-zero correction to R_0)

$$R_2 = \frac{R_0^2 M_1^2 \alpha^6}{\left(\pi^2 + \alpha^2\right) \left[\pi^2 + (1 + M_1)\, \alpha^2\right]} \sum_{n=1}^{\infty} \frac{A_n\, C_n}{D_n}, \tag{21}$$

where

$$A_n = \left(\frac{\omega}{\mathrm{Pr}}\right)^2 + \left(n^2 \pi^2 + \alpha^2\right) \left(\pi^2 + \alpha^2\right),$$

$$C_n = 2B_1,$$

$$D_n = B_1^2 + B_2^2$$

with

$$B_1 = \left(\begin{aligned} - \left(\frac{\omega}{\mathrm{Pr}}\right)^2 (1 + 2\,\mathrm{Pr}) \left(n^2 \pi^2 + \alpha^2\right)^3 + \left(n^2 \pi^2 + \alpha^2\right)^5 + n^2 \pi^2 \,\mathrm{Ta} \left(n^2 \pi^2 + \alpha^2\right)^2 \\ - R_0 \alpha^2 \left(n^2 \pi^2 7 + \alpha^2\right) \left[n^2 \pi^2 + (1 + M_1)\, \alpha^2\right] \end{aligned} \right)$$

and

$$B_2 = \begin{pmatrix} \omega\left(\dfrac{\omega}{\mathrm{Pr}}\right)^2 \left(n^2\pi^2+\alpha^2\right)^2 - \omega\left(1+\dfrac{2}{\mathrm{Pr}}\right)\left(n^2\pi^2+\alpha^2\right)^4 - \omega\, n^2\pi^2\,\mathrm{Ta}\left(n^2\pi^2+\alpha^2\right) \\ +\dfrac{1}{\mathrm{Pr}} R_0\alpha^2\omega\left[n^2\pi^2+\left(1+M_1\right)\alpha^2\right] \end{pmatrix}.$$

The Rayleigh number R at its critical value is calculated up to $O\left(\varepsilon^2\right)$ by computing R_0 and R_2 at $\alpha_0 = \alpha_c$, where α_c is the value at which R_0 is minimum. Supercritical instability occurs provided R_{2c} is positive. On the other hand, subcritical instability is said to occur when R_{2c} turns out to be negative.

V. RESULTS AND DISCUSSION

The impact of magnetic field on the Bénard–Taylor ferroconvective instability is carried out in detail for the following two cases:

(a) unmodulated magnetic field;

(b) modulated magnetic field.

The above-mentioned cases are discussed and analyzed with the help of appropriate graphs (refer to Figs. 2 through 6).

A. Onset of ferroconvection (unmodulated case)

This section analyzes the effects of the buoyancy-magnetization parameter M_1 and the Taylor number Ta on the unmodulated Bénard–Taylor ferroconvection problem [21]. In Fig. 2, the effect of M_1 (the ratio of magnetic force to gravitational force) is depicted and the graph is plotted between the stationary thermal Rayleigh number denoted by R_0 and the wave number denoted by α. The parameter M_1 bears the values as $M_1 = 5$, 25, 50, and the other parameter is supposed to be constant as $\mathrm{Ta} = 500$. It can be seen that as M_1 increases, R_0, decreases and from the definition of M_1 it follows that an increase in the magnetic force has a destabilizing influence on the flow. Further, the critical wave number α, corresponding to R_0, decreases with an increase in M_1.

Fig. 3 illustrates the values of R_0 with respect to α for different values of the Taylor number Ta, fixing the other parameter as $M_1 = 25$. It can be observed that the values of R_0 increase with an increase in Ta, indicating that the effect of rotation is to stabilize the flow in the absence of magnetic field modulation.

B. Onset of ferroconvection (modulated case)

The problem at hand is determining the criteria for the commencement of ferroconvection in a horizontal magnetic smart fluid layer with the Coriolis force and a time-varying magnetic field. The principle behind the stability analysis is based on the condition of the minimal amplitude of magnetic field modulation. The regular perturbation approach is implemented in order to achieve the critical Rayleigh number and wavenumber. The expression for the correction Rayleigh number R_2 is found to be proportional to the modulation frequency ω, the buoyancy-magnetization parameter M_1, the Prandtl number Pr and the Taylor number Ta. The influence of these parameters on the stability of the system is illustrated with the help of figures 4 through 6. The sign of the critical correction thermal Rayleigh number R_{2c} is accountable for the stabilizing or destabilizing impact of the magnetic field fluctuation on the stability of the system. A positive R_{2c} means that the magnetic field modulation effect is stabilizing, whereas a negative R_{2c} indicates that the magnetic field modulation effect is destabilizing.

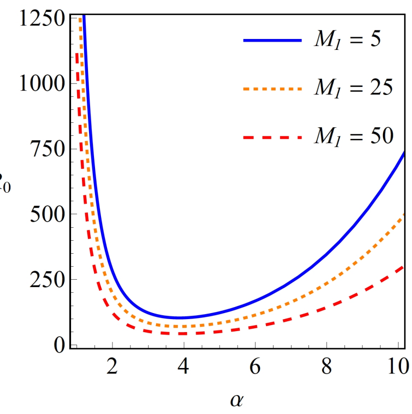

Fig. 2. Effect of buoyancy-magnetization parameter M_1 on R_0 with respect to α.

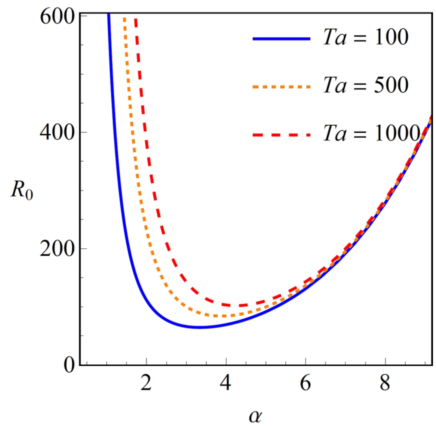

Fig. 3. Effect of Taylor number Ta on R_0 with respect to α.

Figure 4 shows the impact of the buoyancy magnetization parameter M_1 over the critical correction thermal

Rayleigh number R_{2c} and for fixed values of $\mathrm{Pr} = 10$ and $\mathrm{Ta} = 500$. Parameter M_1 is the ratio of magnetic force to gravitational force. The general effect of M_1 is to make the system unstable. It is seen that R_{2c} increases with an increase in the parameter M_1, indicating that magnetic mechanism has a stabilizing effect on the system. This is due to the fact that when M_1 increases, the magnetic force diminishes because of the Centrifugal force and makes the system stable. It is also clear from Fig. 4 that R_{2c} decreases with on increase in ω, then reaches the peak negative value at $\omega = 150$ and increases further increase in ω. This means that the system is stabilized for small, moderate and large values of ω. When ω is sufficiently large, the effect of modulation disappears altogether.

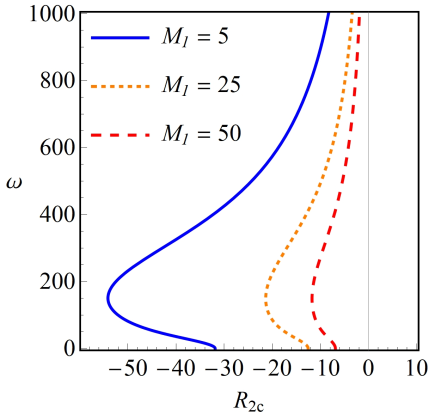

Fig. 4. Variation of R_{2c} with respect to ω and M_1.

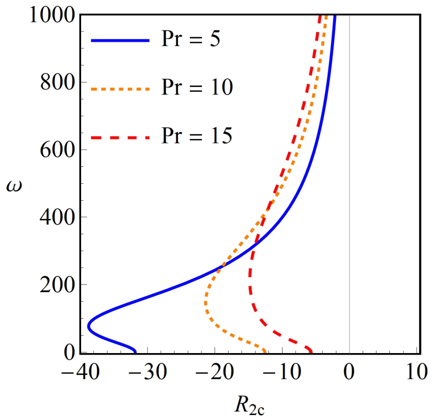

Fig. 5. Variation of R_{2c} with respect to ω and Pr.

The variation of R_{2c} with ω for different values of the Prandtl number Pr and for fixed values of $M_1 = 25$ and $\mathrm{Ta} = 500$ is displayed in Fig. 5. Parameter Pr is the ratio of the speed of momentum propagation to that of heat transport. It should be remarked that the expression for R_0 does not involve the Prandtl number Pr, and the Prandtl number Pr affects only R_{2c}. We observe from Fig. 5 that the value of R_{2c} grows with incrementing the value of Pr, provided ω is small and the trend reverses for moderate and large values of ω. It is also observed that the subcritical instability occurs when increasing the value of Pr, provided ω is small, and the supercritical motion can be seen for moderate and large values of ω.

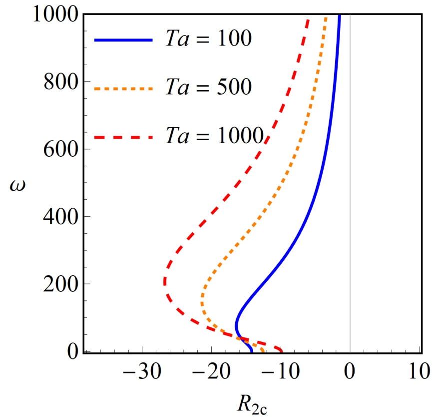

Fig. 6. Variation of R_{2c} with respect to ω and Ta.

Figure 6 shows the deviance of R_{2c} with ω for different values of the Taylor number Ta and for fixed values of $M_1 = 25$ and $\mathrm{Pr} = 10$. The Taylor number characterizes the importance of centrifugal forces due to rotation of a fluid around a vertical axis relative to viscous forces. The stabilizing effect of rotation is obvious from Fig. 3 (unmodulated case). We further note that the value of R_{2c} grows with incrementing the value of Ta, provided ω is very small, and the trend reverses for a moderate and large values of ω. We detect from Fig 6 that the effect of increasing Ta is to reduce the stabilizing effect of modulation on the system for moderate and very large values of ω. This is due to the fact that when Ta increases, either the centrifugal force increases or the viscous force decreases, which shows that increasing Ta decreases the viscous force and makes the system more unstable. Further, the destabilizing effect of modulated magnetic field on the system can be seen for a very small value of ω.

The analysis presented is based on the assumption that the amplitude of the modulating magnetic field is minimal and the convective currents are weak, allowing nonlinear effects to be ignored. Thus, the validity of the results obtained here depends on the value of the modulating frequency of the magnetic field. Hence, the onset of ferroconvection can be advanced or delayed in the presence of a rotating fluid layer. Magnetic field modulation could thus be used to control convective instability in a rotating ferromagnetic fluid layer.

VI. CONCLUSIONS

The effect of rotation on the onset of ferroconvection is carried out for the following cases: unmodulated and modulated magnetic fields. A linear stability analysis and the regular perturbation approach are used to solve the problem. The investigation has led to the following conclusions:

1. Magnetic mechanisms tend to stabilize the system in the presence of magnetic field modulation and destabilize the system in the absence of magnetic field modulation.

2. The effect of Pr is to stabilize the system when the frequency of modulation is small and to destabilize the system with moderate and large values of ω.

3. Rotation tends to destabilize the system with moderate and large values of frequency ω when the magnetic field modulation is present. However, for a very small value of frequency ω, the effect of rotation is to stabilize the system. Further, the effect of rotation is to stabilize the flow in the absence of magnetic field modulation.

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ВПЛИВ МОДУЛЯЦІЇ МАГНІТНОГО ПОЛЯ НА ФЕРОКОНВЕКЦІЮ БЕНАРДА–ТЕЙЛОРА

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Ця стаття стосується впливу обертання на початок конвекції феромагнітної рідини за наявності як модульованих, так і немодульованих магнітних полів. Вплив модуляції та обертання магнітного поля на початок фероконвекції становить інтерес як з практичного, так і з теоретичного погляду. Модуляція відповідного параметра може істотно вплинути на рух і підвищити стабільність різних систем, включно з зарядами в електростатичному полі й феромагнітним резонансом. Обертові ферофлюїди мають потенційне застосування в різних галузях, зокрема в обертових турбомашинах та хімічній промисловості. Отриману проблему власних значень розв'язують з використанням ізотермічних граничних умов і методу реґулярних збурень за припущення малої амплітуди модуляції. У припущенні справедливості принципу обміну стійкостями сформульовано критерії настання конвекції. Магнітний параметр, число Тейлора, число Прандтля та частота модуляції магнітного поля є функціями теплового зсуву числа Релея. За помірних значень частоти модуляції магнітного поля вплив різноманітних фізичних чинників суттєвий. Дослідження показує, що за наявності як намагніченості, так і обертання модуляція магнітного поля має дестабілізувальний вплив на систему, водночас конвекція відбувається швидше, ніж у немодульованій системі.

Ключові слова: магнітна рідина, модуляція магнітного поля, метод збурень, стійкість, обертання.