ISSN 1024-588X. Вісник Львівського університету. Серія фізична. 2020. Випуск 57. С. 32-46
Visnyk of the Lviv University. Series Physics. 2020. Issue 57. P. 32-46

УДК 53.01, 524.8 https://doi.org/10.30970/vph.57.2020.32
PACS 02.40.Gh, 03.65.Ca, 98.80.-k

Classical dS and AdS cosmologies in the general case of deformed space with minimal length

M. I. Samar

Department for Theoretical Physics, Ivan Franko National University of Lviv,
12 Drahomanov St, Lviv, UA-79005, Ukraine
e-mail: mykolasamar@gmail.com

The effects of the minimal length uncertainty relation on classical de Sitter and Anti-de Sitter cosmological models is studied in the general case of deformed space. We obtain exact solutions for these models in case of some special choices of deformed spaces with minimal length and minimal or maximal momentum. It is shown that minimal length might affect and even change the inflationary nature of the de Sitter cosmology. Anti-de Sitter model with deformation has oscillatory behaviour, but depending on the choice of deformation function the period of oscillations can be larger or smaller in comparison to the undeformed model.

Key words: minimal length, generalized uncertainty principle, classical cosmology, cosmological constant, Hubble parameter

1. Introduction

String theory and quantum gravity independently suggest the existence of minimal length as a finite lower bound to the possible resolution of length [1–3]. Kempf et al. showed that minimal length can be introduced by modifying a canonical commutation relation [4–7]. The deformed commutation relation according to Kempf reads

$$[\hat{X}, \hat{P}] = i\hbar(1 + \beta\hat{P}^2). \quad (1)$$

Deformed algebra (1) can be generalized for a wider class of the deformed commutation relation

$$[\hat{X}, \hat{P}] = i\hbar F(\hat{X}, \hat{P}). \quad (2)$$

where F is a positive function of the position and momentum. Such a modification of the canonical commutation relation can lead to the existence of nonzero minimal uncertainties in position, or in momentum, or both. Function

$$F(\hat{X}, \hat{P}) = 1 + \alpha\hat{X}^2 + \beta\hat{P}^2 \quad (3)$$

is an example of the deformation leading to the minimal length and minimal momentum. In this case $\Delta X_{min} = \hbar\sqrt{\beta/(1-\hbar^2\alpha\beta)}$, $\Delta P_{min} = \hbar\sqrt{\alpha/(1-\hbar^2\alpha\beta)}$.

In case when deformation function depends only on momentum, i.e. $F(\hat{X}, \hat{P}) = f(\hat{P})$, minimal length is [8]

$$\Delta X_{min} = \frac{\pi}{2}\left(\int\limits_0^b \frac{dP}{f(P)}\right)^{-1}, \tag{4}$$

where b denotes limits of $P \in [-b, b]$. Here function of deformation $f(P)$ is assumed to be strictly positive ($f > 0$), even function. This means that minimal length is nonzero if

$$\int\limits_0^b \frac{dP}{f(P)} < \infty. \tag{5}$$

Note, that in case of finite b the maximal momentum occurs in deformed space.

On the other hand, as it was discussed in [9], when the deformation function depends only on the position $F(\hat{X}, \hat{P}) = g(\hat{X})$, there is a connection between unconventional Schrödinger equation based on the use of the deformed canonical commutation relations and the one with a position-dependent effective mass $M(X) = 1/g^2(X)$.

The framework of minimal length hypothesis was applied to different quantum mechanical problems, such as harmonic oscillator [5, 10–13], Dirac oscillator [14, 15], hydrogen atom [16–20], gravitational quantum well [21, 22], a particle in delta potential [23, 24], one-dimensional Coulomb-like problem [23, 25, 26] , particle in the singular inverse square potential [27, 28], the Casimir effect [29], particles scattering [30], et al.

The influence of the quantization of space have been studied at the classical level for the following problems: Keplerian orbits, statistical physics, composite systems et al. [31–37].

The idea of minimal length was also considered in context of classical and quantum cosmology [38–43]. This approach gives a possibility to look at some of problems related to cosmology from different points of view and to search new possibilities to resolve these problems. For example, the effect of the minimal length uncertainty relation on the cosmological constant problem was considered in [38].

In paper [39] the classical dS and AdS cosmological models with minimal length was studied and the solutions in the linear approximation of the deformation parameter were obtained. The other cosmological models were considered in later papers [40–43]. The purpose of this paper is to find exact solutions for classical dS and AdS cosmological models in general case of deformed commutation relation (2). To agree on notation in Section II we review classical dS and AdS cosmological models. Next, we consider dS and Ads cosmological model with deformed Poisson brackets in Section III and IV, correspondingly. Finally, Section V contains the conclusion.

2. A brief review of classical dS and AdS cosmologies

Let us assume that the universe is homogeneous and isotropic. It can be described by the flat Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a(t)(dx^2 + dy^2 + dz^2). \tag{6}$$

Here $a(t)$ is the scale factor of the universe. Corresponding nonzero Christoffel symbols and Ricci tensor components are

$$\Gamma^0_{ij} = a\dot{a}\delta_{ij}, \Gamma^i_{0j} = \frac{\dot{a}}{a}\delta^i_j, \tag{7}$$

$$R_{00} = -\frac{3\ddot{a}}{a}, R_{ij} = (2\dot{a}^2 + a\ddot{a}), \tag{8}$$

where a dot means differentiation with respect to time t. The scalar curvature of Robertson-Walker metric is

$$R = g^{\mu\nu}R_{\mu\nu} = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) \tag{9}$$

To construct the canonical formalism of the theory, let us start with the Einstein-Hilbert action

$$S = \frac{1}{2\kappa}\int (R - 2\Lambda)\sqrt{-g}d^4x, \tag{10}$$

Here $\kappa = \frac{8\pi G}{c^4}$, g is the determinant of the spacetime metric and Λ is the cosmological constant representing the vacuum energy. Substituting (9) into (10) we obtain

$$S = \frac{3V}{c\kappa}\int \left(\ddot{a}a^2 + \dot{a}^2a - \frac{\Lambda c^2 a^3}{3}\right)dt, \tag{11}$$

which can be rewritten as

$$S = -\frac{3V}{c\kappa}\int \left(\dot{a}^2a + \frac{\Lambda c^2 a^3}{3}\right)dt. \tag{12}$$

Here integration over the spatial dimensions gives volume V and term with the total derivative over time is cancelled. Lagrangian of the system is the following

$$L = \dot{a}^2a + \frac{\Lambda c^2 a^3}{3}, \tag{13}$$

which yields the Hamiltonian

$$H = p_a\dot{a} - L = \frac{p_a^2}{4a} - \frac{\Lambda c^2}{3}a^3. \tag{14}$$

Note that in (13) minus is omitted and $\frac{3V}{c\kappa} = 1$. We introduce canonical momentum $p_a = \frac{\partial L}{\partial \dot{a}}$ satisfying

$$\{a, p_a\} = 1. \tag{15}$$

Making canonical transformation

$$u = a^{\frac{3}{2}}, \quad p_u = \frac{2p_a}{3\sqrt{a}}, \tag{16}$$

$$\{u, p_u\} = 1, \tag{17}$$

we rewrite the Hamiltonian into the following form

$$\mathcal{H} = \frac{9p_u^2}{16} - \frac{\Lambda c^2}{3}u^2. \tag{18}$$

In case of $\Lambda > 0$ the Hamiltonian describes the simplest classical inflationary (dS) model and in case of $\Lambda < 0$ the oscillatory (AdS) model. For the dS model the Hamiltonian can be written as

$$\mathcal{H} = \frac{p_u^2}{2m} - \frac{m\Omega^2 u^2}{2}, \tag{19}$$

with $m = \frac{8}{9}$ and $\Omega^2 = \frac{3}{4}\Lambda c^2$. Equations of motion are

$$\dot{u} = \{u, \mathcal{H}\} = \tfrac{p_u}{m}, \tag{20}$$

$$\dot{p}_u = \{p_u, \mathcal{H}\} = m\Omega^2 u, \tag{21}$$

$$\ddot{u} = \Omega^2 u, \tag{22}$$

which yields

$$u(t) = e^{\Omega(t-t_0)}, \tag{23}$$

where we have taken the initial condition $u(t_0) = 1$. The Hubble constant for considered model is

$$H_0 = \frac{\dot{a}}{a} = \frac{2}{3}\frac{\dot{u}}{u} = \frac{2}{3}\Omega \tag{24}$$

For AdS cosmological model the hamiltonian

$$\mathcal{H} = \frac{p_u^2}{2m} + \frac{m\omega^2 u^2}{2}, \tag{25}$$

with $\omega^2 = -\frac{3}{4}\Lambda c^2$.

$$u(t) = \sin(t), \tag{26}$$

where we have taken the initial condition $u(0) = 0$.

3. Classical dS cosmology with deformed Poisson algebra

Let us study the effects of the classical version of commutation relation (2) on the de Sitter cosmological model. In the classical limit quantum mechanical commutators should be replaced by the classical Poisson brackets as $[\hat{X}, \hat{P}] \to i\hbar\{X, P\}$. In the case of canonical variables u and p_u deformed Poisson bracket has the form

$$\{u, p_u\} = F(u, p_u). \tag{27}$$

Note that in case of $F(u, p_u) = f(p_u)$ the existence of minimal length imposes the constraint on the function of deformation $f(p_u)$

$$\int\limits_0^a \frac{dp_u}{f(p_u)} < \infty. \tag{28}$$

The equations of motion are

$$\dot{u} = \{u, H\} = \tfrac{p_u}{m} F(u, p_u), \tag{29}$$

$$\dot{p}_u = \{p_u, H\} = m\Omega^2 u F(u, p_u). \tag{30}$$

From equations (29) and (30) we obtain

$$\frac{\dot{u}}{\dot{p}_u} = \frac{1}{m^2\Omega^2}\frac{p_u}{u}. \tag{31}$$

The last equation yields

$$p_u = \pm m\Omega u, \tag{32}$$

with the constant of integration which equals to zero. Finally, substituting (32) into (29) we obtain

$$\dot{u} = \pm\Omega u F(u, \pm m\Omega u). \tag{33}$$

Integration of the latter equation gives the law of inflation for dS model which includes the effects caused by minimal length

$$\int\limits_1^u \frac{du}{uF(u, \pm m\Omega u)} = \pm\Omega(t - t_0) \tag{34}$$

Hubble parameter then can be presented as

$$H = \frac{\dot{a}}{a} = \frac{2}{3}\frac{\dot{u}}{u} = \frac{2}{3}\Omega F(u, \pm m\Omega u) = H_0 F(u, \pm m\Omega u). \tag{35}$$

In the linear approximation on parameter of deformation formula (35) yields

$$H = H_0 F(e^{\Omega(t-t_0)}, \pm m\Omega e^{\Omega(t-t_0)}). \tag{36}$$

Now let us consider a few special examples of deformation function.

Example 1. Let us consider the deformation function proposed by Kempf $F(u,p_u) = 1+\alpha u^2+\beta p_u^2$. We obtain that the expansion law is

$$u(t) = \frac{e^{\Omega(t-t_0)}}{\sqrt{1+\gamma\left(1-e^{2\Omega(t-t_0)}\right)}}, \tag{37}$$

with

$$\gamma = \alpha + \beta m^2\Omega^2. \tag{38}$$

In the linear approximation on the parameter of deformation γ the expansion law writes

$$u(t) = e^{\Omega(t-t_0)} + \frac{\gamma}{2}\left(e^{3\Omega(t-t_0)} - e^{\Omega(t-t_0)}\right). \tag{39}$$

The last result was obtained in paper [39] in case of $\alpha = 0$. However, from the exact result (37) we obtain that inflation of the universe will last for finite time t_r, thus we have Big Rip scenario

$$t_r = t_0 + \frac{1}{2\Omega}\ln\left(1+\frac{1}{\gamma}\right). \tag{40}$$

For this example of deformation function the Hubble parameter also can be found exactly

$$H = H_0\frac{1+\gamma}{1+\gamma(1-e^{2\Omega(t-t_0)})}. \tag{41}$$

Example 2. The second example of deformation function that possesses the exact solution is the following

$$F(u,p_u) = f(p_u) = (1+\beta p_u^2)^{\frac{3}{2}}. \tag{42}$$

The solution can be presented as

$$\frac{1}{\sqrt{1+\gamma_\beta u^2}} - \frac{1}{\sqrt{1+\gamma_\beta}} + \frac{1}{2}\ln\frac{\sqrt{(1+\gamma_\beta u^2}-1)(\sqrt{1+\gamma_\beta}+1)}{(\sqrt{1+\gamma_\beta u^2}+1)(\sqrt{1+\gamma_\beta}-1)} = \Omega(t-t_0), \tag{43}$$

with

$$\gamma_\beta = \beta m^2\Omega^2. \tag{44}$$

In the linear approximation on parameter of deformation the expansion law writes

$$u(t) = e^{\Omega(t-t_0)} + \frac{3\gamma_\beta}{4}\left(e^{3\Omega(t-t_0)} - e^{\Omega(t-t_0)}\right). \tag{45}$$

Comparing (45) with (39) we see that for this example of deformation the expansion rate is higher than the one for the previous example. Plots of exact expansion laws (43) and (37) are present in Fig.1.

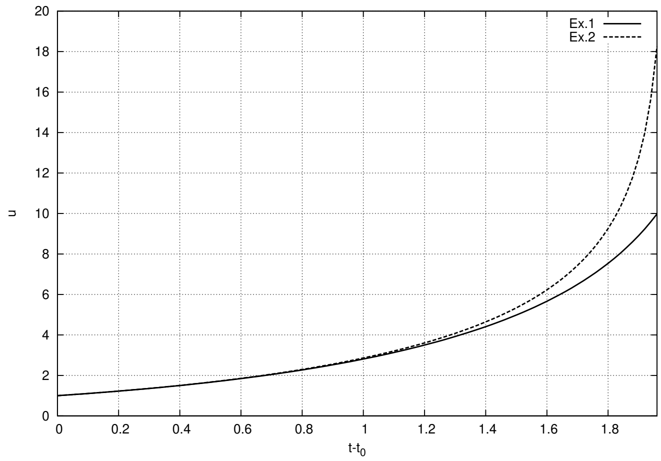

Fig. 1: Expansion laws for Examples 1 and 2 of deformation function with $\Omega = 1$ and $\gamma = \gamma_\beta = 0.01$

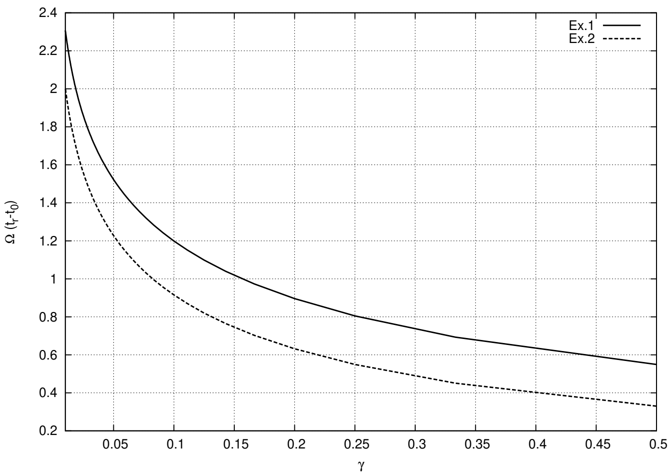

Fig. 2: Dependencies of the lifetime of the universe $t_r - t_0$ on $\gamma = \gamma_\beta$ for Examples 1 and 2 of the deformation function

Similarly to the previous case, the end of the universe will occur in some moment t_r in the future

$$t_r = t_0 - \frac{1}{\Omega\sqrt{1+\gamma_\beta}} + \frac{1}{2\Omega}\ln\frac{\sqrt{1+\gamma_\beta}+1}{\sqrt{1+\gamma_\beta}-1}. \quad (46)$$

The dependencies of $t_r - t_0$ on deformation parameter $\gamma = \gamma_\beta$ are given in Fig.2.

We again arrive at the Big Rip scenario. In fact, this ultimate fate of the universe can be predicted in the general case of deformed function $F(u, p_u) = f(p_u)$ with minimal length and without maximal momentum. Really, using constraint (28) in case of $u_{max} =$

∞ we can write

$$t_r - t_0 = \frac{1}{\Omega}\int\limits_1^\infty \frac{du}{uf(\pm m\Omega u)} < \frac{1}{\Omega}\int\limits_1^\infty \frac{du}{f(\pm m\Omega u)} < \infty. \tag{47}$$

Example 3. The last example of deformation function with maximal momentum is $f(p_u) = \sqrt{1-\beta p_u^2}$, $|p_u| < \frac{1}{\sqrt{\beta}}$. In such a case the expansion phase of the evolution of the universe will proceed until $u_{max} = \frac{1}{\sqrt{\gamma_\beta}}$ by the law

$$u(t) = \frac{1}{\cosh\Omega(t-t_0) - \sqrt{1-\gamma_\beta}\sinh\Omega(t-t_0)}. \tag{48}$$

In the linear approximation on parameter of deformation the expansion law writes

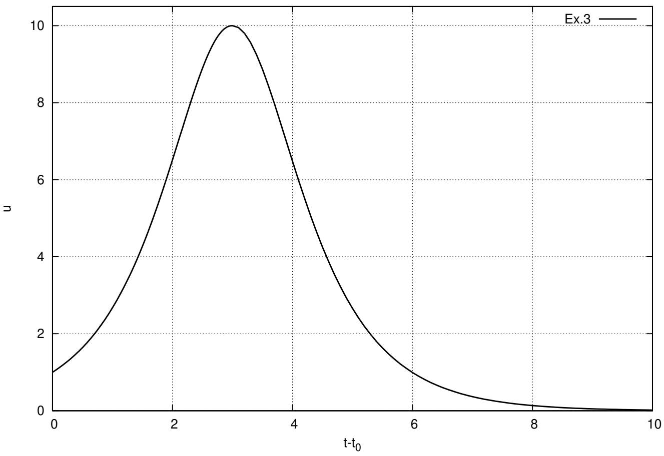

Fig. 3: The effect of minimal length on the scale factor of the universe for Examples 3 of deformation function with $\Omega = 1$ and $\gamma_\beta = 0.01$

$$u(t) = e^{\Omega(t-t_0)} - \frac{\gamma_\beta}{4}\left(e^{3\Omega(t-t_0)} - e^{\Omega(t-t_0)}\right). \tag{49}$$

However, at the moment in time

$$t_c = t_0 + \frac{1}{2\Omega}\ln\left(\frac{1+\sqrt{1-\gamma_\beta}}{1-\sqrt{1-\gamma_\beta}}\right) \tag{50}$$

the expansion of the universe reverses and the universe recollapses by the law

$$u(t) = \frac{1}{\sqrt{\beta}m\Omega\cosh\left[\Omega(t-t_c)\right]}. \tag{51}$$

Thus, the ultimate fate of the universe in this model is the Big Crunch (*see Fig.3*) .

In the general case of deformation function $F(u, p_u) = f(p_u)$ with maximal momentum it can also be proven similarly as in (47) that expansion phase of the evolution of the universe lasts a finite period.

4. Classical AdS cosmology with deformed Poisson algebra

Now let us consider classical AdS cosmology with minimal length scenario. The Hamiltonian corresponding to this problem is

$$\mathcal{H} = \frac{p_u^2}{2m} + \frac{m\omega^2 u^2}{2}, \tag{52}$$

with $\omega^2 = -\frac{3}{4}\Lambda c^2$.

The equations of motion are

$$\dot{u} = \{u, H\} = \tfrac{p_u}{m} F(u, p_u), \tag{53}$$

$$\dot{p}_u = \{p_u, H\} = -m\omega^2 u F(u, p_u). \tag{54}$$

The law of evolution of the universe for AdS model can be written as

$$\int_0^u \frac{du}{\sqrt{u_0^2 - u^2} F(u, m\omega\sqrt{u_0^2 - u^2})} = \omega t, \tag{55}$$

with $u_0 = \sqrt{\frac{2E}{m\omega^2}}$ being the amplitude of the scale factor and $E = \frac{p_u^2}{2m} + \frac{m\omega^2 u^2}{2}$ being the energy of the system.

We obtain exact analytical results for two special examples of deformation function.

Example 1. Let us consider the deformation function proposed be Kempf $F(u, p_u) = 1 + \alpha u^2 + \beta p_u^2$. The evolution of the universe is governed by the oscillatory law

$$u(t) = \sqrt{\frac{2E(1+\varepsilon_\beta)}{m\omega^2}} \frac{\sin[\sqrt{(1+\varepsilon_\alpha)(1+\varepsilon_\beta)}\omega t]}{\sqrt{1 + \varepsilon_\beta \sin^2[\sqrt{(1+\varepsilon_\alpha)(1+\varepsilon_\beta)}\omega t] + \varepsilon_\alpha \cos^2[\sqrt{(1+\varepsilon_\alpha)(1+\varepsilon_\beta)}\omega t]}}, \tag{56}$$

with $\varepsilon_\alpha = \frac{2\alpha E}{m\omega^2}$, $\varepsilon_\beta = 2m\beta E$. The period of oscillations can be calculated exactly by the following formula

$$T = \frac{2\pi}{\omega\sqrt{(1+\varepsilon_\alpha)(1+\varepsilon_\beta)}}. \tag{57}$$

As it turned out this period is smaller than in ordinary AdS model.

Example 2. The last example of deformation function with maximal momentum is $f(p_u) = \sqrt{1 - \beta p_u^2}$, $|p_u| < \frac{1}{\sqrt{\beta}}$. The exact solution can be presented by incomplete elliptic integral of the first kind as

$$\omega t = \frac{1}{\sqrt{1-\varepsilon}} EllipticF\left(\frac{u}{u_0}, \sqrt{\frac{\varepsilon}{\varepsilon - 1}}\right). \tag{58}$$

In the linear approximation on parameter of deformation (58) writes

$$u(t) = u_0 \left(1 - \frac{\varepsilon}{4}\cos^2(\omega t)\right) \sin[(1 - \frac{\varepsilon}{4})\omega t]. \tag{59}$$

Comparison of results (56) and (58) with the undeformed one is presented on Fig. 4. From Fig. 4 we can conclude that depending on the choice of deformation function the period of oscillations can be larger or smaller in comparison to the ordinary model.

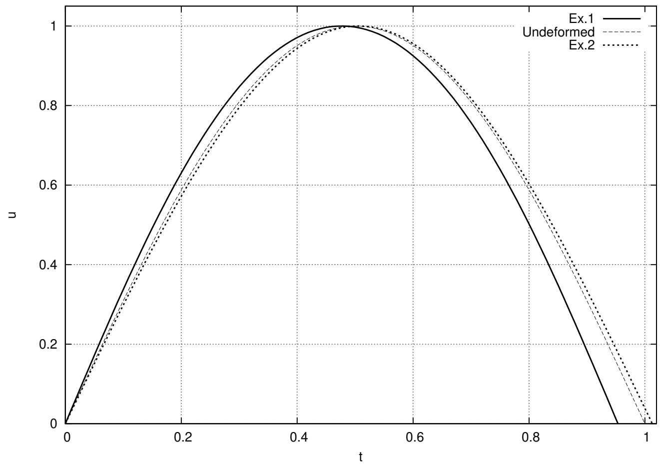

Fig. 4: The scale factor of the AdS universe for Example 1 and 2 of deformation function and undeformed case with $\Omega = 1$ and $\gamma_\beta = 0.01$

5. Conclusion

In this paper, we have studied the effects of minimal length uncertainty relation on classical dS and AdS cosmologies. We have shown in the general case that for the deformation with minimal length and without maximal momentum dS cosmology gives an inflationary universe with Big Rip ultimate fate.

We also obtain an exact solution for the dS model in case of deformed commutation relation with both minimal length and minimal momentum. In this case, Big Rip scenario is also realized.

For the deformation with minimal length and maximal momentum in dS model, we obtain that at some moment of time the expansion of the universe reverses and the universe recollapses. Thus, we arrive at Big-Crunch scenario.

In the classical AdS model, there are some differences between ordinary and deformed models. The model with deformation has oscillatory behaviour but the period of oscillations can be larger or smaller in comparison to the ordinary model. This means that depending on the choice of deformation function in AdS model with minimal length the corresponding Big-Crunch occurs later or earlier than in an ordinary model.

Acknowledgements

This work was supported by the project FF-63Hp (No. 0117U007190) from the Ministry of Education and Science of Ukraine.

1. *Gross D. J. and Mende P. F.* Nucl. Phys. B **303**, 407 (1988).
2. *Maggiore M.* Phys. Lett. B **304**, 65 (1993).
3. *Witten E.* Phys. Today **49**, 24 (1996).
4. *Kempf A.* J. Math. Phys. **35**, 4483 (1994).
5. *Kempf A., Mangano G. and Mann R. B.* Phys. Rev. D **52**, 1108 (1995).
6. *Hinrichsen H. and Kempf A.* J. Math. Phys. **37**, 2121 (1996).
7. *Kempf A.* J. Phys. A: Math. Gen. **30**, 2093 (1997).
8. *Maslowski T., Nowicki A. and Tkachuk V. M.*, J. Phys. A **45**, 075309 (2012).
9. *Quesne C. and Tkachuk V. M.* J. Phys. A **37**, 4267 (2004).
10. *Quesne C. and Tkachuk V. M.* J. Phys. A **36**, 10373 (2003).
11. *Quesne C. and Tkachuk V. M.* J. Phys. A **37**, 10095 (2004).
12. *Chang L. N. , Minic D., Okamura N. and Takeuchi T.*, Phys. Rev. D **65**, 125027 (2002).
13. *Dadic I., Jonke L. and Meljanac S.* Phys. Rev. **67**, 087701 (2003).
14. *Quesne C. and Tkachuk V. M.* J. Phys. A **38**, 1747 (2005).
15. *Quesne C. and Tkachuk V. M.* J. Phys. A **39**, 10909 (2006)
16. *Brau F.* J. Phys. A **32**, 7691 (1999).
17. *Benczik S., Chang L. N., Minic D. and Takeuchi T.* Phys. Rev. A **72**, 012104 (2005).
18. *Stetsko M. M. and Tkachuk V. M.* Phys. Rev. A **74**, 012101 (2006).
19. *Samar M. I. and Tkachuk V. M.* J. Phys. Stud. **14**, 1001 (2010).
20. *Samar M. I.* J. Phys. Stud. **15**, 1007 (2011).
21. *Brau F. and Buisseret F.* Phys. Rev. D **74**, 036002 (2006).
22. *Pedram P., Nozari K., Taheri S.H.* J. High Energy Phys. **1103**, 093 (2011).
23. *Samar M. I. and Tkachuk V. M.* J. Math. Phys. **57**, 042102 (2016).
24. *Ferkous N.* Phys. Rev. A **88**, 064101 (2013).
25. *Samar M. I. and Tkachuk V. M.* J. Math. Phys. **57**, 082108 (2016).
26. *Fityo T. V., Vakarchuk I. O. and Tkachuk V. M.* J. Phys. A **39**, 2143 (2006).
27. *Bouaziz D. and Bawin M.* Phys. Rev. A **76**, 032112 (2007).
28. *Bouaziz D. and Bawin M.* Phys. Rev. A **78**, 032110 (2008).
29. *Frassino A. M. and Panella O.* Phys. Rev. D **85**, 045030 (2012).
30. *Stetsko M. M. and Tkachuk V. M.* Phys. Rev. A **76**, 012707 (2007).
31. *Benczik S., Chang L. N. et al.* Phys. Rev. D **66**, 026003 (2002).
32. *SilagadzeZ. K.* Phys. Lett. A **373**, 2643(2009).
33. *Fityo T.* Phys. Lett. A **372**, 5872 (2008).
34. *Frydryszak A. M. and Tkachuk V. M.* Czech. J. Phys. **53**, 1035 (2003).
35. *Buisseret F.* Phys. Rev. **82**, 062102 (2010).
36. *Quesne C. and V. M. Tkachuk V. M.* Phys. Rev. A **81**,012106 (2010).
37. *Tkachuk V. M.* Phys. Rev. A **86**, 062112 (2012).
38. *Chang L. N., Minic D., Okamura N. and Takeuchi T.* Phys. Rev. D **65**, 125028 (2002).
39. *Vakili B.* Int. J. Mod. Phys. D **18**, 1059 (2009)
40. *Vakili B.* Phys. Rev. D **77** , 044023 (2008).
41. *Battisti M. V. and Montani G.* Int. J. Mod. Phys. A **23**,1257 (2008).
42. *Jalalzadeh S., Rasouli S. M. M. and Moniz P. V.* Phys. Rev. D **90**, 023541 (2014).
43. *Pedram P.* Phys. Rev. D **91**, 063517 (2015).

References

1. D. J. Gross and P. F. Mende, Nucl. Phys. B **303**, 407-454 (1988). doi:10.1016/0550-3213(88)90390-2.
2. M. Maggiore, Phys. Lett. B **304**, 65-69 (1993). doi:10.1016/0370-2693(93)91401-8.
3. E.Witten, Phys. Today **49**, 24-31 (1996). doi:10.1063/1.881493
4. A. Kempf, J. Math. Phys. **35**, 4483-4496 (1994). doi:10.1063/1.530798.
5. A. Kempf , G. Mangano and R. B. Mann, Phys. Rev. D **52**, 1108-1118 (1995). doi:10.1103/PhysRevD.52.1108.
6. H. Hinrichsen and A. Kempf, J. Math. Phys. **37**, 2121-2137 (1996). doi:10.1063/1.531501.
7. A. Kempf, J. Phys. A: Math. Gen. **30**, 2093-2102 (1997). doi: 10.1088/0305-4470/30/6/030.
8. T. Maslowski , A. Nowicki and V. M. Tkachuk, J. Phys. A **45**, 075309 (2012). doi: 10.1088/1751-8113/45/7/075309.
9. C. Quesne and V. M. Tkachuk, J. Phys. A **37**, 4267-4281 (2004). doi: 10.1088/0305-4470/37/14/006.
10. C. Quesne and V. M. Tkachuk, J. Phys. A **36**, 10373-10391 (2003). doi:10.1088/0305-4470/36/41/009.
11. C. Quesne and V. M. Tkachuk, J. Phys. A **37**, 10095-10114 (2004). doi:10.1088/0305-4470/37/43/006.
12. L. N. Chang , D. Minic , N. Okamura and T. Takeuchi, Phys. Rev. D **65**, 125027 (2002). doi:10.1103/PhysRevD.65.125027.
13. I. Dadic , L. Jonke and S. Meljanac, Phys. Rev. D **67**, 087701 (2003). doi:10.1103/PhysRevD.67.087701.
14. C. Quesne and V. M. Tkachuk, J. Phys. A **38**, 1747-1766 (2005). doi:10.1088/0305-4470/38/8/011.
15. C. Quesne and V. M. Tkachuk, J. Phys. A **39**, 10909-10922. (2006). doi:10.1088/0305-4470/39/34/021.
16. F. Brau, J. Phys. A **32**, 7691-7696 (1999). doi:10.1088/0305-4470/32/44/308.
17. S. Benczik , L. N. Chang , D. Minic and T. Takeuchi, Phys. Rev. A **72**, 012104 (2005). doi:10.1103/PhysRevA.72.012104.
18. M. M. Stetsko and V. M. Tkachuk, Phys. Rev. A **74**, 012101 (2006). doi:10.1103/PhysRevA.74.012101.
19. M. I. Samar and V. M. Tkachuk, J. Phys. Stud. **14**, 1001-1-1001-5 (2010). doi:10.30970/jps.14.1001.
20. M. I. Samar, J. Phys. Stud. **15**, 1007-1-1007-7 (2011). doi:10.30970/jps.15.1007.
21. F. Brau and F. Buisseret, Phys. Rev. D **74**, 036002 (2006). doi:10.1103/PhysRevD.74.036002.
22. P. Pedram , K. Nozari and S.H. Taheri , J. High Energy Phys. **1103**, 093 (2011). doi: 10.1007/JHEP03(2011)093.
23. M. I. Samar and V. M. Tkachuk, J. Math. Phys. **57**, 042102 (2016). doi: 10.1063/1.4945313.

24. N. Ferkous, Phys. Rev. A **88**, 064101 (2013). doi: 10.1103/PhysRevA.88.064101.
25. M. I. Samar and V. M. Tkachuk, J. Math. Phys. **57**, 082108 (2016). doi: 10.1063/1.4961320.
26. T. V. Fityo , I. O. Vakarchuk and V. M. Tkachuk, J. Phys. A **39**, 2143-2149 (2006). doi: 10.1088/0305-4470/39/9/010.
27. D. Bouaziz and M. Bawin, Phys. Rev. A **76**, 032112 (2007). doi:10.1103/PhysRevA.76.032112.
28. D. Bouaziz and M. Bawin, Phys. Rev. A **78**, 032110 (2008). doi:10.1103/PhysRevA.78.032110.
29. A. M. Frassino and O. Panella, Phys. Rev. D **85**, 045030 (2012).doi:10.1103/PhysRevD.85.045030.
30. M. M. Stetsko and V. M. Tkachuk, Phys. Rev. A **76**, 012707 (2007). doi: 10.1103/PhysRevA.76.012707.
31. S. Benczik, L. N. Chang, D. Minic, N. Okamura, S. Rayyan and T. Takeuchi, Phys. Rev. D **66**, 026003 (2002). doi:10.1103/PhysRevD.66.026003.
32. Z. K. Silagadze, Phys. Lett. A **373**, 2643-2645 (2009). doi:10.1016/j.physleta.2009.05.053.
33. T. Fityo, Phys. Lett. A **372**, 5872-5877 (2008). doi:j.physleta.2008.07.047.
34. A. M. Frydryszak and V. M. Tkachuk, Czech. J. Phys. **53**, 1035-1040 (2003). doi:10.1023/B:CJOP.0000010529.32268.03.
35. F. Buisseret, Phys. Rev. A **82**, 062102 (2010). doi: 10.1103/PhysRevA.82.062102.
36. C. Quesne and V. M. Tkachuk, Phys. Rev. A **81**,012106 (2010). doi:10.1103/PhysRevA.81.012106.
37. V. M. Tkachuk, Phys. Rev. A **86**, 062112 (2012). doi:10.1103/PhysRevA.86.062112.
38. L. N. Chang , D. Minic , N. Okamura and T. Takeuchi, Phys. Rev. D **65**, 125028 (2002). doi:10.1103/PhysRevD.65.125028.
39. B. Vakili, Int. J. Mod. Phys. D **18**, 1059-1071 (2009). doi:10.1142/S0218271809014935.
40. B. Vakili, Phys. Rev. D **77** , 044023 (2008). doi:10.1103/PhysRevD.77.044023.
41. M. V. Battisti and G. Montani, Int. J. Mod. Phys. A **23**,1257-1265 (2008). doi:10.1142/S0217751X08040184.
42. S. Jalalzadeh , S. M. M. Rasouli and P. V. Moniz, Phys. Rev. D **90**, 023541 (2014). doi:10.1103/PhysRevD.90.023541.
43. P. Pedram, Phys. Rev. D **91**, 063517 (2015). doi: 10.1103/PhysRevD.91.063517.

Статтю отримано: 11.05.2020
Прийнято до друку: 01.10.2020

Класичні dS та AdS космології в загальному випадку деформованого простору з мінімальною довжиною

М. І. Самар

Львівський національний університет імені Івана Франка,
кафедра теоретичної фізики,
Україна, Львів, вул. Драгоманова 12, 79005
e-mail: mykolasamar@gmail.com

Теорія струн і квантова гравітація незалежно припускають наявність мінімальної довжини як скінченної нижньої межі можливої роздільної здатності для довжини. Кемпф показав, що узагальнений принцип невизначеності, який приводить до мінімальної довжини, може бути отриманий шляхом модифікації канонічних комутаційних співвідношень для операторів координати та імпульсу. Деформовану алгебру Кемпфа можна узагальнити до ширшого класу деформованих комутаційних співвідношень з довільною функцією деформації, що крім мінімальної довжини може призвести до мінімального чи максимального імпульсів. У цій роботі досліджено вплив узагальненого співвідношення невизначеності з мінімальною довжиною на класичні космологічні моделі де Сіттера та анти-де Сіттера. В загальному випадку деформованого простору показано, що для деформації з мінімальною довжиною і без максимального імпульсу космологія де Сіттера дає інфляційний Всесвіт з Великим Розривом. Також отримано точні розв'язки для моделі де Сіттера у випадку деформованого комутаційного співвідношення з мінімальною довжиною та мінімальним імпульсом. У цьому випадку також реалізується сценарій Великого Розриву. Для деформації з мінімальною довжиною та максимальним імпульсом у моделі де Сіттера ми отримуємо, що в якийсь момент часу розширення Всесвіту повертається в зворотному напрямку, і Всесвіт стискається. Так, спостерігатиметься сценарій Великого Стиснення. У класичній моделі анти-де Сіттера є деякі відмінності між звичайними та деформованими моделями. Модель з деформацією, як і недеформована, має коливальну поведінку, але період коливань може бути більшим або меншим порівняно зі звичайною моделлю. Це означає, що залежно від вибору функції деформації в моделі анти-де Сіттера з мінімальною довжиною Велике Стисненння відбувається раніше або пізніше, ніж у звичайній моделі.

Ключові слова: мінімальна довжина, узагальнений принцип невизначеності, класична космологія, космологічна стала, параметр Хаббла.