ISSN 1024-588X. Вісник Львівського університету. Серія фізична. 2015. Випуск 50. С. 72-83
Visnyk of the Lviv University. Series Physics. 2015. Issue 50. P. 72-83

УДК 53.01
PACS 02.40.Gh, 03.65.Ca

PHYSICAL STATES IN DEFORMED SPACE WITH MINIMAL LENGTH

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One-dimensional deformed algebra leading to minimal length is considered. Condition for the wave function to be physical is derived. We obtain that maximally localization states can be presented as linear combination of two eigenstates of position operator and it is unique physical wave function made by two chosen position eigenstates. We prove that any physical function can be presented as linear combination of countable set of maximally localization states and propose simple receipt to do this.

Key words: minimal length, generalized uncertainty principle, maximally localization states

1 Introduction

Recent years there has been a growing interest in quantum mechanic with deformed commutation relations. It is motivated by several independent lines of investigations in theoretical physics (e.g. string theory and quantum gravity), which suggest the existence of minimal length as a finite lower bound to the possible resolution of length [1–3]. Kempf et. al. showed that such an effect can be obtained by modifying usual canonical commutation relations([4–7]). In one dimensional case the simplest deformed algebra leading to minimal length $\Delta X_{min} = \hbar\sqrt{\beta}$ writes

$$[\hat{X}, \hat{P}] = i\hbar(1 + \beta\hat{P}^2). \tag{1}$$

An important feature of quantum theory with minimal length is that eigenstates of the position operator are no longer physical states, since for these states the standard deviation of position is $\Delta X = 0 < \Delta X_{min}$. As a result, we cannot work with the position representation anymore. Generally, the states with $\Delta X < \Delta X_{min}$ do not belong to the domain of physical states while the states with $\Delta X \geq \Delta X_{min}$ do.

One of the possible ways of the spatial description of the system is to consider the set of maximally localized states for which $\Delta X = \Delta X_{min}$ as the generalization of position eigenstates to the case of presence of minimal length. In paper [5] it was proposed to obtain the maximally localization states as particular squeezed states. Hovewer, as it was shown in [8], this result is correct only for few special cases of deformation. It was proposed in [8] more general definition of maximally localized states based on variational principle. The states of maximal localization were considered in case of modification of the commutation relation between position and momentum operators to all orders of the minimum length parameter [9–11], in case of presence of both minimal length and maximal momentum [12] or minimal length and momentum [6], both rotation and translation invariant case of deformation [13] and finally in non commutative quantum theories [14].

Hovewer, the criterion of the state to be physical and connection of particular physical state with the maximally localization states were not considered yet. In present paper we study the properties and the ways of discrete representation of physical states domain in deformed space (1).

The paper is organized as follows. In Section 2 we brief about generalized uncertainty principle, the representations of the deformed algebra (1), position eigenstates and maximally localization states. The requirement for the wave function to be physical and a few simple examples of physical wave functions are presented in Section 3. Next, in Section 4 we present the complete set on the domain of physical states. Finally, Section 5 contains conclusion.

2 A brief on deformed algebra

2.1 Generalized uncertainty principle and representations of the algebra

Uncertainty principle for non-commuting operators $\hat{X}$ and $\hat{P}$ satisfying deformed commutation relation (1) reads

$$\Delta X \geq \frac{\hbar}{2}\left(\frac{1+\beta\langle\hat{P}\rangle^2}{\Delta P}+\beta\Delta P\right). \tag{2}$$

Here we use notation $\Delta X = \sqrt{\langle(\Delta\hat{X})^2\rangle}$ and $\Delta P = \sqrt{\langle(\Delta\hat{P})^2\rangle}$. From the inequality (2) we obtain that standard deviation of the coordinate ΔX has minimum

$$\Delta X_0 = \hbar\sqrt{\beta}\sqrt{1+\beta\langle\hat{P}\rangle^2}, \tag{3}$$

which can be achieved at $\Delta P = \sqrt{\frac{1}{\beta}+\langle\hat{P}\rangle^2}$.

There are different representations of algebra (1). One of them is the momentum representation

$$\hat{X} = i\hbar(1+\beta P^2)\frac{d}{dP}, \quad \hat{P} = P. \tag{4}$$

In the aim of preserving the hermiticity of the operator of coordinate the scalar product has to be modified by the introducing a weight function

$$\langle\Psi|\Phi\rangle = \int_{-\infty}^{+\infty} \frac{dP}{1+\beta P^2}\Psi^*(P)\Phi(P). \tag{5}$$

Another one is the so-called quasi-coordinate representation

$$\hat{X} = i\hbar\frac{d}{dp}, \quad \hat{P} = \frac{1}{\sqrt{\beta}}\tan\sqrt{\beta}p. \tag{6}$$

Here parameter p changes in the region $\left[-\frac{\pi}{2\sqrt{\beta}}, \frac{\pi}{2\sqrt{\beta}}\right]$. The scalar product in this representation has the form

$$< \Psi|\Phi >= \int_{-\frac{\pi}{2\sqrt{\beta}}}^{+\frac{\pi}{2\sqrt{\beta}}} dp\Psi^*(p)\Phi(p). \tag{7}$$

2.2 Eigenvalue problem for position operator

We write eigenvalue equation for position operator in the representation (6)

$$i\hbar\frac{d}{dp}\Psi_\Lambda(p) = \Lambda\Psi_\Lambda(p). \tag{8}$$

Solution of this equation is

$$\Psi_\Lambda(p) = \sqrt{\frac{\sqrt{\beta}}{\pi}}e^{-i\frac{\Lambda}{\hbar}p}. \tag{9}$$

Note that the same result but in representation (4) was obtained in [5]

$$\widetilde{\Psi}_\Lambda(P) = \sqrt{\frac{\sqrt{\beta}}{\pi}}e^{-i\frac{\Lambda}{\hbar\sqrt{\beta}}\arctan\sqrt{\beta}P}. \tag{10}$$

The scalar product of two eigenfunction of position operator

$$\begin{aligned} < \Psi_\Lambda|\Psi_{\Lambda'} > &= \frac{\sqrt{\beta}}{\pi}\int_{-\frac{\pi}{2\sqrt{\beta}}}^{+\frac{\pi}{2\sqrt{\beta}}} dpe^{-i\frac{\Lambda-\Lambda'}{\hbar}p} \\ &= \frac{2\hbar\sqrt{\beta}}{\pi(\Lambda-\Lambda')}\sin\left(\frac{\Lambda-\Lambda'}{2\hbar\sqrt{\beta}}\pi\right). \end{aligned} \tag{11}$$

From (12) we conclude that all amount of eigenstates of the position operator can be divided into sets parameterized by $\lambda \in [-1, 1)$

$$\{\Psi_{(\lambda+2n)\hbar\sqrt{\beta}}(p), n \in Z\}, \tag{12}$$

which are mutually orthogonal

$$< \Psi_{(\lambda+2n)\hbar\sqrt{\beta}}|\Psi_{(\lambda+2m)\hbar\sqrt{\beta}} >= \delta_{m,n}. \tag{13}$$

One can prove that each of these sets is complete. Such proof is equivalent to the proof of the following relation

$$\sum_{n=-\infty}^{n=+\infty} \Psi^*_{(\lambda+2n)\hbar\sqrt{\beta}}(p')\Psi_{(\lambda+2n)\hbar\sqrt{\beta}}(p) = \delta(p-p'), \tag{14}$$

which holds. Each set parameterized by $\lambda \in [-1,1)$ is the set of eigenstates of self-adjoint extension of position operator $\{\hat{X}, D_\lambda(\hat{X})\}$ with

$$D_\lambda(\hat{X}) = \left\{\psi(p), \psi'(p) \in L^2\left(-\frac{\pi}{2\sqrt{\beta}}, \frac{\pi}{2\sqrt{\beta}}\right),\ \psi\left(-\frac{\pi}{2\sqrt{\beta}}\right) = e^{i\lambda\pi}\psi\left(\frac{\pi}{2\sqrt{\beta}}\right)\right\}. \tag{15}$$

The set (12) is presented in fig. (1).

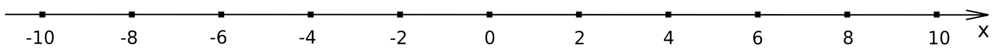

Рис. 1: Complete orthogonal set of position eigenstate ($\lambda = 0$). The eigenstates from the set is denoted by black point. Scale is in units of $\hbar\sqrt{\beta}$.

The considered eigenstates of position operator are nonphysical ones, because they do not fulfill uncertainty relation (2) and mean value of kinetic energy on such states is divergent

$$< \Psi_\Lambda|\hat{P}^2/2m|\Psi_\Lambda >= \infty. \tag{16}$$

But nevertheless we can use complete set (12) for the representation of the wave function.

2.3 Maximally localization states

As we saw in previous section the eigenstates of the position operator are no longer physical states. Consequently, we can use these states for formal representation of some wave function only, but cannot interpret such representation as coordinate one anymore. Thus, in order to recover some information on the spatial distribution of the system, we are forced to consider the maximally localization states for which $\Delta X = \Delta X_{min}$. These states can be considered as the generalization of the position eigenstates on the deformed case.

The states with maximally allowed localization are defined as [5]:

$$\begin{aligned} \langle\Psi^{ml}_\xi|\hat{X}|\Psi^{ml}_\xi\rangle = \xi, \\ \langle\Psi^{ml}_\xi|(\hat{X}-\xi)^2|\Psi^{ml}_\xi\rangle = \Delta X_0. \end{aligned} \tag{17}$$

These states are the states which convert modified Heisenberg uncertainty relation (2) into equality. Maximally localization states can be obtained from the equation:

$$\left[\hat{X} - \langle\hat{X}\rangle + \frac{\langle[\hat{X},\hat{P}]\rangle}{2\Delta P^2}\left(\hat{P} - \langle\hat{P}\rangle\right)\right]\Psi(P) = 0. \tag{18}$$

In case $\langle P\rangle = 0$, $\Delta P = \frac{1}{\sqrt{\beta}}$ equation (18) can be rewritten in the form

$$\hat{Q}\Psi^{ml}_{\xi} = \xi\Psi^{ml}_{\xi}, \tag{19}$$

with

$$\hat{Q} = \hat{X} + i\hbar\beta\hat{P} = \frac{1}{\sqrt{1+\beta\hat{P}^2}}\hat{X}\sqrt{1+\beta\hat{P}^2}. \tag{20}$$

From latter representation for operator $\hat{Q}$ we conclude that $\hat{Q}$ and $\hat{X}$ are isospectral operators despite the fact that $\hat{Q}$ is not hermitian. The solution of equation (19) in representation (6) is:

$$\Psi^{ml}_{\xi} = \sqrt{\frac{2\sqrt{\beta}}{\pi}}\cos(\sqrt{\beta}p)\exp(-\frac{i\xi p}{\hbar}). \tag{21}$$

Scalar product of two maximally localization space is

$$< \Psi^{ml}_{\xi'}|\Psi^{ml}_{\xi} >= \frac{1}{\pi}\left[\frac{\xi-\xi'}{2\hbar\sqrt{\beta}} - \left(\frac{\xi-\xi'}{2\hbar\sqrt{\beta}}\right)^3\right]^{-1}\sin\frac{(\xi-\xi')\pi}{2\hbar\sqrt{\beta}} \tag{22}$$

From (22) we conclude that maximally localization states can be divided into orthogonal sets $\Psi^{ml}_{(\varepsilon+4n)\hbar\sqrt{\beta}}(P), n \in Z$, parameterized by $\varepsilon \in [-2, 2)$ and satisfying relation

$$\langle\Psi^{ml}_{(\varepsilon+4n)\hbar\sqrt{\beta}}|\Psi^{ml}_{(\varepsilon+4m)\hbar\sqrt{\beta}}\rangle = \delta_{m,n}. \tag{23}$$

One can prove that orthogonal set $\{\Psi^{ml}_{(\varepsilon+4n)\hbar\sqrt{\beta}}(P), n \in Z\}$ does not form complete set.

3 Physical requirements for wave function

From generalized uncertainty principle (2) we point out that mean value of $\hat{P}^2$ can be finite only for states which belong to allowed region $\Delta X \geq \Delta X_{min}$. Thus, the finiteness of the mean value of the operator of kinetic energy is the requirement for wave function to be physical.

Let us consider any normalized wave function $F(P)$ in momentum representation (4) satisfying

$$\int_{-\infty}^{\infty} dP\frac{|F(P)|^2}{1+\beta P^2} = 1. \tag{24}$$

We also assume the wave function to be well-behaved, i.e. continuous, finite everywhere and sufficiently smooth.

The mean value of $\hat{P}^2$ in state described by $F(P)$ writes

$$< F|\hat{P}^2|F >= \int_{-\infty}^{\infty} dP\frac{P^2}{1+\beta P^2}|F(P)|^2 = \frac{1}{\beta}\left(\int_{-\infty}^{\infty} dP|F(P)|^2 - 1\right). \tag{25}$$

Thus, the mean value of $\hat{P}^2$ will be finite iff function $F(P)$ is square integrable due to usual scalar product:

$$\int_{-\infty}^{\infty} dP|F(P)|^2 < \infty, \tag{26}$$

and hence goes to zero while P is going to infinity

$$F(\pm\infty) = 0. \tag{27}$$

However, the latter formula is only necessary condition, not sufficient, since condition (26) demands wave function $F(P) \sim P^{-\frac{1}{2}-\varepsilon}$, $\varepsilon > 0$ for $P \to \infty$.

Condition (27) can be written in the form

$$\sum_{n=-\infty}^{+\infty} (-1)^n C_n = 0, \tag{28}$$

with C_n being the expansion coefficients of $F(P)$ over the complete set of position eigenfunctions parameterized by $\lambda \in [-1, 1)$ in momentum representation:

$$F(P) = \sum_{n=-\infty}^{+\infty} C_n \widetilde{\Psi}_{(\lambda+2n)\hbar\sqrt{\beta}}(P). \tag{29}$$

Note that condition (28) is invariant under the choice of λ.

3.1 Connection of maximally localization states with the position eigenstates

Unlike eigenstates of the operator of position maximally localization states are physical ones:

$$< \Psi_\xi^{ml}|\frac{P^2}{2m}|\Psi_\xi^{ml} >= \frac{1}{2m\beta}, \tag{30}$$

which means that maximally localization states have to satisfy condition (28). To show this we present maximally localization state as the series over complete set of position eigenstates (12):

$$\Psi_\xi^{ml} = \sum_{n=-\infty}^{\infty} C_n \Psi_{(\lambda+2n)\hbar\sqrt{\beta}}, \tag{31}$$

$$C_n = \int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} \Psi^*_{(\lambda+2n)\hbar\sqrt{\beta}}(p)\Psi_\xi^{ml}(p)dp. \tag{32}$$

It is easy to obtain that

$$< \Psi_x|\Psi_\xi^{ml} >= \frac{2\sqrt{2}}{\pi\left(1 - (\frac{x-\xi}{\hbar\sqrt{\beta}})^2\right)} \cos\left(\frac{x-\xi}{2\hbar\sqrt{\beta}}\pi\right). \tag{33}$$

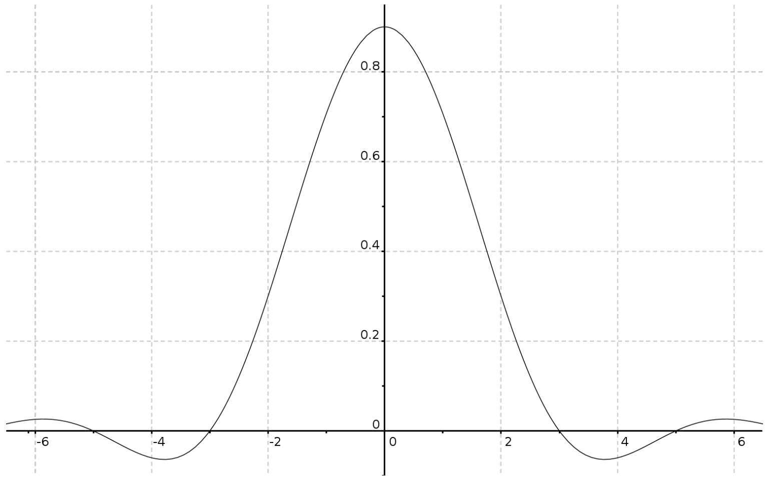

Рис. 2: Plotting of $< \Psi_x|\Psi_\xi^{ml} >$ over $x - \xi$ in units $\hbar\sqrt{\beta}$

This dependence can be plotted (see fig. 2). We see from the fig. 2 that $< \Psi_x|\Psi_\xi^{ml} >$ equals zero while $x = (\xi + \hbar\sqrt{\beta}) + 2n\hbar\sqrt{\beta}$ for $n \in Z,\ n \neq \pm 1$. It means that if we choose $\lambda = \xi + \hbar\sqrt{\beta}$ which denotes complete set of position eigenfunction we obtain the simplest decomposition for Ψ_ξ^{ml}(see fig. 3):

$$\Psi_\xi^{ml} = \frac{1}{\sqrt{2}}\left(\Psi_{\xi-\hbar\sqrt{\beta}} + \Psi_{\xi+\hbar\sqrt{\beta}}\right). \tag{34}$$

This result also can be easily obtained from (21) by presenting the cosine function by exponentials [15]. From (34) we see that maximally localization state satisfy necessary

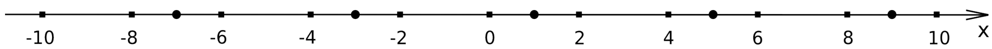

Рис. 3: Orthogonal set of maximally localization states $\Psi^{ml}_{(4n+1)\hbar\sqrt{\beta}}(P), n \in Z$. Maximally localization state is denoted by the black circle placed in a point $(4n+1)\hbar\sqrt{\beta}$. Every maximally localization state can be presented as normalized sum of two adjacent position eigenstates denoted by the black point. Scale is in units of $\hbar\sqrt{\beta}$.

condition of being physical (28) and it is unique physical wave function formed as linear combination of considered two eigenfunctions of position operator.

3.2 Eigenfunctions of $\hat{X}^2$

Another simple example of physical states are the eigenfunctions of $\hat{X}^2$. Let us consider the eigenproblem for $\hat{X}^2$ in representation (6):

$$\hat{X}^2\phi_n(p) = \chi_n\phi_n(p). \tag{35}$$

Due to (27) we assume that eigenfunction vanish at the endpoints of the domain of p. Note, that there exist infinitely many self-adjoint extensions of the operator $\hat{X}^2$ [16], while only the following one

$$\hat{X}^2 = -\hbar^2 \frac{d^2}{dp^2}, D(\hat{X}^2) = \left\{\phi, \phi'' \in L^2\left(-\frac{\pi}{2\sqrt{\beta}}, \frac{\pi}{2\sqrt{\beta}}\right); \phi\left(\pm\frac{\pi}{2\sqrt{\beta}}\right) = 0\right\} \tag{36}$$

has physical eigenfunctions.

Now the considerable problem writes

$$-\hbar^2 \frac{d^2\phi_n(p)}{dp^2} = \chi_n \phi_n(p), \tag{37}$$

with $\phi_n(\pm\frac{\pi}{2\sqrt{\beta}}) = 0$. This problem is rather similar to the problem of particle in a box in undeformed space. The solution of the problem is

$$\chi_n = \hbar^2 \beta n^2, \tag{38}$$

$$\phi_n(p) = \sqrt{\frac{2\sqrt{\beta}}{\pi}} \sin\left(n\sqrt{\beta}(p + \frac{\pi}{2\sqrt{\beta}})\right), \tag{39}$$

with $n = 1, 2, \dots$.

The nonzero matrix elements of $\hat{P}^2$ on eigenfunction of $\hat{X}^2$ are

$$\langle\phi_n|\hat{P}^2|\phi_{n+2k}\rangle = \langle\phi_{n+2k}|\hat{P}^2|\phi_n\rangle = \frac{1}{\beta}\left(2n - \delta_{k,0}\right), \tag{40}$$

with $n, m, k = 1, 2, \dots$.

4 Complete set on the domain of physical states

Let us consider any physical function $F(p)$ in representation (6) and write it in the form

$$F(p) = \sqrt{2}\cos(\sqrt{\beta}p)f(p). \tag{41}$$

The mean value of $\hat{P}^2$ we demand to be convergent

$$\langle\hat{P}^2\rangle = \frac{1}{\beta}\int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} |F(p)|^2 \frac{\sin^2(\sqrt{\beta}p)}{\cos^2(\sqrt{\beta}p)} dp = \frac{2}{\beta}\int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} |f(p)|^2 \sin^2(\sqrt{\beta}p) dp < \infty. \tag{42}$$

Using normalization condition for $F(p)$ we can write:

$$\frac{2}{\beta}\int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} |f(p)|^2 \sin^2(\sqrt{\beta}p) dp = \frac{2}{\beta}\int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} |f(p)|^2 dp - \frac{1}{\beta} < \infty, \tag{43}$$

From (43) we conclude that $f(p)$ has to be square integrable function. Thus, any physical function can be presented by (41) with $f(p)$ being square integrable function.

Mentioned square integrable function can be presented as the series over complete set of position eigenfunctions (12):

$$f(p) = \sum_{n=-\infty}^{\infty} A_n \Psi_{(\lambda+2n)\hbar\sqrt{\beta}}(p), \tag{44}$$

$$A_n = \int_{-\frac{\pi}{2\sqrt{\beta}}}^{\frac{\pi}{2\sqrt{\beta}}} \Psi^*_{(\lambda+2n)\hbar\sqrt{\beta}}(p) f(p) dp, \tag{45}$$

$$\sum_{n=-\infty}^{\infty} |A_n|^2 < \infty. \tag{46}$$

Substituting (44) into (41) we write

$$F(p) = \sum_{n=-\infty}^{\infty} A_n \sqrt{2}\cos(\sqrt{\beta}p)\Psi_{(\lambda+2n)\hbar\sqrt{\beta}}(p) = \sum_{n=-\infty}^{\infty} A_n \Psi^{ml}_{(\lambda+2n)\hbar\sqrt{\beta}}(p). \tag{47}$$

Here we use that $\Psi^{ml}_{(\lambda+2n)\hbar\sqrt{\beta}}(p) = \sqrt{2}\cos(\sqrt{\beta}p)\Psi_{(\lambda+2n)\hbar\sqrt{\beta}}(p)$.

Thus, any physical function can be presented as a series over set of functions (fig. 4)

$$\{\Psi^{ml}_{(\lambda+2n)\hbar\sqrt{\beta}}(P), n \in Z\}, \ \lambda \in [-1, 1), \tag{48}$$

with the expansion coefficients satisfying (46).

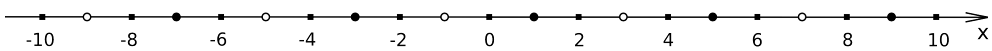

Рис. 4: Two orthogonal sets of maximally localization states $\Psi^{ml}_{(4n+1)\hbar\sqrt{\beta}}(P)$ and $\Psi^{ml}_{(4n-1)\hbar\sqrt{\beta}}(P), n \in Z$ forming complete set on the domain of physical states. Maximally localization states are denoted by the black and white circle placed in a point $(4n+1)\hbar\sqrt{\beta}$ and $(4n-1)\hbar\sqrt{\beta}$ correspondingly. Every maximally localization state can be presented as normalized sum of two adjacent position eigenstates denoted by the black point. Scale is in units of $\hbar\sqrt{\beta}$.

Considerable set (48) is not orthogonal one:

$$< \Psi^{ml}_{(\lambda+1+2m)\hbar\sqrt{\beta}} | \Psi^{ml}_{(\lambda+1+2n)\hbar\sqrt{\beta}} >= \delta_{m,n} + \frac{1}{2}(\delta_{m,n+1} + \delta_{m,n-1}), \tag{49}$$

with $n, m \in Z$.

It is interesting that following matrix elements also have tridiagonal form:

$$\begin{aligned} &< \Psi^{ml}_{2m\hbar\sqrt{\beta}} | P^2 | \Psi^{ml}_{2n\hbar\sqrt{\beta}} >= \frac{1}{\beta}\left(\delta_{m,n} - \frac{1}{2}(\delta_{m,n+1} + \delta_{m,n-1})\right), \\ &< \Psi^{ml}_{2m\hbar\sqrt{\beta}} | X^2 | \Psi^{ml}_{2n\hbar\sqrt{\beta}} >= \hbar^2\beta\left((4n^2+1)\delta_{m,n} + \frac{(2n-1)^2}{2}\delta_{m,n+1} + \frac{(2n+1)^2}{2}\delta_{m,n-1}\right), \\ &< \Psi^{ml}_{2m\hbar\sqrt{\beta}} | P | \Psi^{ml}_{2n\hbar\sqrt{\beta}} >= \frac{i}{2\sqrt{\beta}}\left(\delta_{m,n+1} - \delta_{m,n-1}\right), \\ &< \Psi^{ml}_{2m\hbar\sqrt{\beta}} | X | \Psi^{ml}_{2n\hbar\sqrt{\beta}} >= \hbar\sqrt{\beta}\left(2n\delta_{m,n} + \frac{2n-1}{2}\delta_{m,n+1} + \frac{2n+1}{2}\delta_{m,n-1})\right) \end{aligned} \tag{50}$$

with $n, m \in Z$.

5 Conclusion

We have studied modified commutation relation leading to minimal length proposed by Kempf. Because of the existence of nonzero minimal uncertainty in position, eigenfunctions of the position operator even being square integrable are nonphysical anyway. In general the states with the standard deviation less then minimal length $\Delta X < \Delta X_{min}$ do not belong to the domain of physical states. The physical domain is made by the well-behaved wave function satisfying $\Delta X \geq \Delta X_{min}$. The crucial property of these states is the finiteness of the mean value of kinetic energy.

We have derived the necessary condition of the finiteness of the mean value of kinetic energy. Eigenfunctions of position operator of course do not satisfy this condition, however maximally localization states and eigenstates of $\hat{X}^2$ do.

Note that the eigenproblems for $\hat{X}$ and $\hat{X}^2$ operators in deformed space are rather similar to the eigenproblems for $\hat{P}$ and $\hat{P}^2$ operators for particle in a box in ordinary quantum mechanics. This fact hints that particle in a box problem can be, if one needs, reformulated as free particle in space with minimal uncertainty in momentum.

We have obtained that maximally localization states can be presented as linear combination of two eigenstates of position operator and it is unique physical wave function written by two chosen position eigenstates.

Finally, we propose simple procedure to present chosen physical function as a linear combination of countable set of maximally localization states. This set can be considered as complete set on the physical domain.

6 Acknowledgement

The author thanks Prof. Volodymyr Tkachuk and Dr. Volodymyr Pastukhov for helpful discussion.

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Стаття надійшла до редакції 10.06.2015
прийнята до друку 15.09.2015

ФІЗИЧНІ СТАНИ У ДЕФОРМОВАНОМУ ПРОСТОРІ З МІНІМАЛЬНОЮ ДОВЖИНОЮ

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Розглядається одновимірна деформована алгебра з мінімальною довжиною. Ми вивели умову фізичності хвильової функції. Показано, що максимально локалізовані стани можуть бути представлені як лінійна комбінація двох власних станів оператора координати, і що це єдина фізична хвильова функція утворена з двох обраних координатних власних станів. Ми довели, що будь-яка фізична хвильова функція може бути представлена як лінійна комбінація зліченного набору максимально локалізованих станів та запропонували простий спосіб як це зробити.

Ключові слова: мінімальна довжина, узагальнений принцип невизначеності, максимально локалізовані стани

ФИЗИЧЕСКИЕ СОСТОЯНИЯ В ДЕФОРМИРОВАНОМ ПРОСТРАНСТВЕ С МИНИМАЛЬНОЙ ДЛИНОЙ

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Рассматривается одномерная деформирована алгебра с минимальной длиной. Мы вывели условие физичности волновой функции. Показано, что максимально локализованные состояния могут быть представлены как линейная комбинация двух собственных состояний оператора координаты, и это единственная физическая волновая функция образована из двух выбранных координатных собственных состояний. Мы доказали, что любая физическая волновая функция может быть представлена как линейная комбинация счетно набора максимально локализованих состояний и предложили простой способ как это сделать.

Ключевые слова: минимальная длина, обобщенный принцип неопределенности, максимально локализованные состояния