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Hydrogen atom in noncommutative space

Khrystyna P. Gnatenko

Ivan Franko National University of Lviv, Department for Theoretical Physics,
12 Drahomanov St., Lviv, 79005, Ukraine

We consider the hydrogen atom in noncommutative space which is rotationally invariant. We find the perturbation of the energy levels of the hydrogen atom caused by the noncommutativity of coordinates.

Introduction

Recently, noncommutativity has received considerable interest owing to the development of String Theory and Quantum Gravity. [1,2]

Noncommutative space can be realized as a space where the coordinate operators X_i satisfy the following relation

$$[X_i, X_j] = i\hbar\theta_{ij}, \tag{1}$$

with θ_{ij} being a constant antisymmetric matrix. Different problems have been studied in this space (see, for instance, Ref. 3 and references therein). It is worth mentioning that in noncommutative space (1) there is a problem of rotational symmetry breaking. [4] To preserve the rotational symmetry we consider the generalization of the constant antisymmetric matrix to a tensor which is defined in the following form $\theta_{ij} = \frac{\alpha}{\hbar}(a_ib_j - a_jb_i)$, where α is a dimensionless constant and a_i, b_i are additional coordinates governed by the harmonic oscillator $H_{osc} = \frac{(p^a)^2}{2m} + \frac{(p^b)^2}{2m} + \frac{m\omega^2 a^2}{2} + \frac{m\omega^2 b^2}{2}$. The parameter of noncommutativity of coordinates is believed to be of the order of the Planck scale. Therefore, we put $\sqrt{\frac{\hbar}{m\omega}} = l_p$, where l_p is the Planck length. We also consider the limit $\omega \to \infty$.

So, the following commutation relations are proposed [5]

$$[X_i, X_j] = i\alpha(a_ib_j - a_jb_i), \qquad [X_i, P_j] = i\hbar\delta_{ij}, \qquad [P_i, P_j] = 0. \tag{2}$$

The coordinates a_i, b_i, and momenta p_i^a, p_i^b satisfy the ordinary commutation relations $[a_i, a_j] = [b_i, b_j] = 0$, $[a_i, p_j^a] = [b_i, p_j^b] = i\hbar\delta_{ij}$, also $[a_i, b_j] = [a_i, p_j^b] = [b_i, p_j^a] = [p_i^a, p_j^b] = 0$, $[a_i, X_j] =$

$[a_i, P_j] = [b_i, X_j] = [b_i, P_j] = 0$. So, X_i, P_i and $\theta_{ij} = \frac{\alpha}{\hbar}(a_i b_j - a_j b_i)$ satisfy the same commutation relations as in the case of the canonical version of noncommutativity (1). Moreover, algebra (2) is rotationally invariant. [5]

Energy levels of the hydrogen atom in noncommutative space

In order to find corrections to the energy levels of the hydrogen atom in rotationally invariant noncommutative space (2) we consider the following Hamiltonian

$$H = H_h + H_{osc}, \tag{3}$$

where $H_{osc} = \frac{(p^a)^2}{2m} + \frac{(p^b)^2}{2m} + \frac{m\omega^2 a^2}{2} + \frac{m\omega^2 b^2}{2}$ and $H_h = \frac{P^2}{2M} - \frac{e^2}{R}$ is the Hamiltonian of the hydrogen atom, here $R = \sqrt{\sum_i X_i^2}$.

It is convenient to use the following representation $X_i = x_i + \frac{1}{2}[\boldsymbol{\theta}\times\mathbf{p}]_i$, $P_i = p_i$, where $\boldsymbol{\theta} = \frac{\alpha}{\hbar}[\mathbf{a}\times\mathbf{b}]$, coordinates x_i and momenta p_i satisfy the ordinary commutation relations. Therefore, we can rewrite Hamiltonian (3) as follows $H = H_0 + V$, here $H_0 = \frac{p^2}{2M} - \frac{e^2}{r} + H_{osc}$, $r = \sqrt{\sum_i x_i^2}$, and V is the perturbation caused by the noncommutativity of coordinates $V = -e^2/R + e^2/r$.

Note that in the case of $\omega \to \infty$ harmonic oscillator put into the ground state remains in it. Therefore, using the perturbation theory, up to the second order in $\boldsymbol{\theta}$ we obtain

$$\Delta E_{n,l} = -\frac{\hbar^2 e^2 \langle\theta^2\rangle}{a_B^5 n^5}\left(\frac{1}{6l(l+1)(2l+1)} - \frac{6n^2 - 2l(l+1)}{3l(l+1)(2l+1)(2l+3)(2l-1)}\right.$$
$$\left. + \frac{5n^2 - 3l(l+1) + 1}{2(l+2)(2l+1)(2l+3)(l-1)(2l-1)} - \frac{5}{6}\frac{5n^2 - 3l(l+1) + 1}{l(l+1)(l+2)(2l+1)(2l+3)(l-1)(2l-1)}\right), \tag{4}$$

where $\langle\theta^2\rangle = \langle\psi^a_{0,0,0}\psi^b_{0,0,0}|\theta^2|\psi^a_{0,0,0}\psi^b_{0,0,0}\rangle = \frac{3\alpha^2 l_P^4}{2\hbar^2}$, here $\psi^a_{0,0,0}$, $\psi^b_{0,0,0}$ are the well known eigenfunctions of the harmonic oscillators, and a_B is the Bohr radius. [5]

It is worth mentioning that in the case of $l = 0$ or $l = 1$ the corrections (4) are divergent. The problem of divergence of the corrections to the ns energy levels was studied in Ref. 5.

Conclusion

We have considered the generalization of the constant antisymmetric matrix θ_{ij} to a tensor that gives the possibility to construct rotationally invariant noncommutative algebra. The hydrogen atom have been studied in noncommutative space (2). We have found the corrections to the energy levels of the hydrogen atom.

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