OPTICAL SPATIAL DISPERSION IN TERM OF JONES MATRICES

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I. Introduction

Jones matrix formalism is a powerful tool for calculating electric field vector of a light wave exiting a system of optical elements. One of the most representative examples of the usefulness and power of the calculus is its application for optical simulation of liquid crystal displays.

If the optical system is a pile of elements each of which is described by its own Jones matrix, then the Jones matrix of the system is a product of the matrices of elements. Jones proposed to calculate the J matrix for a deformed crystal, as an integral of a 2X2 matrix N(z) : $J = e^{\int N(z)dz} J_0$. The matrix J is called ***integral Jones matrix***, the matrix N is called ***differential Jones matrix***.

Traditionally *optical spatial dispersion (OSD)* is defined as the dependence $\varepsilon(\vec{k})$ of the dielectric permittivity tensor ε on the light wave vector $\vec{k}$.

II. The dielectric permittivity tensor with the account of OSD

- The general case:

$$\varepsilon^{eff}=\begin{pmatrix} \varepsilon_{11}^{(3)0}-\frac{1}{\varepsilon_{33}^{0}}\left(\left(\varepsilon_{13}^{(3)0}\right)^2-\left(\lambda\!\!\!^{-} g_{23}^{(3)}\nabla_z\right)^2\right) & \varepsilon_{12}^{(3)0}-\lambda\!\!\!^{-} g_{33}^{(3)}\nabla_z-\left(\varepsilon_{13}^{(3)0}+\lambda\!\!\!^{-} g_{23}^{(3)}\nabla_z\right)\left(\varepsilon_{23}^{(3)0}+\lambda\!\!\!^{-} g_{13}^{(3)}\nabla_z\right) \\ \varepsilon_{12}^{(3)0}-\lambda\!\!\!^{-} g_{33}^{(3)}\nabla_z-\left(\varepsilon_{13}^{(3)0}+\lambda\!\!\!^{-} g_{23}^{(3)}\nabla_z\right)\left(\varepsilon_{23}^{(3)0}+\lambda\!\!\!^{-} g_{13}^{(3)}\nabla_z\right) & \varepsilon_{11}^{(3)0}-\frac{1}{\varepsilon_{33}^{(3)0}}\left(\left(\varepsilon_{23}^{(3)0}\right)^2-\left(\lambda\!\!\!^{-} g_{13}^{(3)}\nabla_z\right)^2\right) \end{pmatrix}$$

- For a medium with the point group symmetry, which is not lower than orthorombic

$$\varepsilon^{eff}=\varepsilon^{0(d)}+\lambda\!\!\!^{-} k g_{33} R\left(\frac{\pi}{2}\right),\quad \varepsilon^{(0)d}=\begin{pmatrix}\varepsilon_{11}^{0(d)} & 0\\ 0 & \varepsilon_{22}^{0(d)}\end{pmatrix},\quad R\left(\frac{\pi}{2}\right)=\begin{pmatrix}\sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right)\\ -\cos\left(\frac{\pi}{2}\right) & \sin\left(\frac{\pi}{2}\right)\end{pmatrix}$$

$\varepsilon_{ij}^{(3)0}$ - components of dielectric permittivity tensor in 3D form $i, j=\overline{1,3}$ without taking into account of OSD,

$\nabla_z=\frac{d}{dz}$ - differential operator

g_{ij} are coefficients of the series expansion of the dielectric tensor

III. OSD in Jones matrix formalism

The contribution of OSD in the integral Jones matrix:

$$J^D = J\left(J^0\right)^{-1} \qquad \varepsilon^{eff} = J^D \varepsilon^0 \left(J^D\right)^{-1}$$

J-the Jones matrix with account of OSD, J^0- the Jones matrix without account of OSD, J^D-matrix that is responsible for OSD

$$\left[\varepsilon_0 + ƛ^2\left(J^D\right)^{-1} + \left(\frac{d^2 J^D}{dz^2} + 2\frac{dJ^D}{dz}\frac{d}{dz}\right)\right]\overrightarrow{E_0} = \frac{d^2\overrightarrow{E_0}}{dz^2}$$

The Maxwell's equation for Jones matrix N^D that is independent on coordinate z :

$$\left[\varepsilon^0 + ƛ^2\left(\left(N^D\right)^2 + 2N^D\frac{d}{dz}\right)\right]\overrightarrow{E_0} = \frac{d^2\overrightarrow{E_0}}{dz^2}$$

The equation for finding the refractive indexes of media, that are the eigen values of differential Jones Matrix taking into account of OSD:

$$\left(\eta^0\right)^4 + a_3\left(\eta^0\right)^3 - a_2\left(\eta^0\right)^2 - a_1\eta^0 + a_0 = 0$$

In a general case, when all components of the OSD DJM N^D are non-zero, the secular equation for the refractive indices of the eigen waves is a quartic equation. The coefficient at the cubic term in the secular equation is non-zero only for non-zero OSD corrections to the average refractive index. For transparent crystals at nonzero OSD correction to the average refractive index and zero all other correction parameters in ND, the secular equation has two distinct real and two complex conjugated roots. We assign the complex conjugated roots to the forward and backward light scattering. Therefore with the account for OSD effect on the refractive index, the Jones calculus becomes capable for description of light scattering.

V. The OSD in the cholesteric

The Maxwell's equation in matrix form for the cholesteric:

$$\left\{\left(N^0\right)^2+2qR\left(\frac{\pi}{2}\right)N^0+\frac{1}{\lambda\!\!\!^{-2}}\hat{\varepsilon}^0-q^2I\right\}\vec{E}^0=0$$

The equation for finding the refractive indexes of cholesteric:

$$\left(\eta^0\right)^4+2a_2\left(\eta^0\right)^2+a_3=0$$

The refractive indexes of the cholesteric:

$$\left(\eta_0^{\pm}\right)^2=\overline{\varepsilon}^0-\frac{1}{4}(\Delta\delta g)^2\pm\sqrt{\frac{\left(\Delta\varepsilon^0\right)^2}{4}-\overline{\varepsilon}^0(\Delta\delta g)^2}$$

VI. The relation between electrical permittivity tensor and differential Jones matrix

The differential Jones matrix is an differential operator:

$$\frac{d\vec{E}}{dz} = N\vec{E}$$

The Maxwell's equation in classical 2D form:

$$\hat{\varepsilon}\vec{E} = -\lambda\!\!\!^{-}{}^{2}\frac{d^{2}\vec{E}}{dz^{2}}$$

The relationship between differential Jones matrix and electrical permittivity tensor :

$$N = -\frac{i}{\lambda\!\!\!^{-}}\sqrt{\varepsilon} = -\frac{i}{\lambda\!\!\!^{-}}\frac{1}{\eta_{+}+\eta_{-}}\begin{pmatrix} \varepsilon_{11}+\eta_{+}\eta_{-} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22}+\eta_{+}\eta_{-} \end{pmatrix}$$

V. Conclusions

- We have developed an approach for description of the OSD phenomena in the framework of Jones calculus. In our approach the integral Jones matrix of the medium with the account of OSD is the product of the integral Jones matrix without the account of OSD by the correction integral Jones matrix which accounts for the OSD effects.
- In a general case, when all components of the OSD differential Jones matrix are non-zero, the secular equation for finding here eigen values is a quartic equation
- With the account of OSD effect on the refractive index, the Jones calculus becomes capable for description of light scattering.
- We employed the introduced approach for OSD in terms of Jones matrixes for cholesteric and got the result that agree with our previous results for the Jones matrix for a cholesteric

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