RESONANT TUNNELING IN A DOUBLE-BARRIER JOSEPHSON JUNCTION

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Abstract

The Josephson supercurrent through double-barrier tunnel junction (SISIS) have been studied analytically in the framework of quasiclassical Nambu – Gorkov equations. We have obtained a non-monotonous dependence of the supercurrent on distance between barriers, with the presence of resonance peaks, that is related to the resonant tunneling of Cooper pairs through the double-barrier structure. We found the values of a thickness of interior superconductor layer that corresponds to the maximum of supercurrent. The critical current was derived and analyzed. The current-phase dependence of a supercurrent deviate from sinusoidal.

1 Introduction

For many important and interesting phenomena discovered in the 20th century the superconductivity has attracted special interest. There are a lot of reasons for this, among them a zero electrical resistance, expulsion of a magnetic field from a superconductor, electric current without dissipation, magnetic flux quantization etc. The superconductors also are the "window into the quantum world", because the quantum effects in them are manifest at a macroscopic scale. Unique to the physics of superconductivity there is Josephson effect or phenomenon of supercurrent — a current that flows without any voltage applied across a Josephson junction, which consists of two superconductors coupled by a weak link.

In recent years, has become possible to create multilayered Josephson structures of the SINIS and SISIS-type [1]. Investigation of the phase coherent transport of the charges through these junctions has an important practical significance to develop SQUID-technologies as well as to realisation quantum qubits for quantum computer.

There are many articles dedicated to the study of a multilayered Josephson junctions. An experimental investigation of the Josephson current in multilayered structures was described in the papers [2]. The theoretical description of a Josephson effect in a SISIS-junction was developed in articles [3], where an implicit expression for the supercurrent has been obtained using Green's function approach. To provide an analysis of current-phase dependence, authors have used numerical methods.

In the present paper, we develop an analytical theory of a supercurrent in the Josephson SISIS-junction. The theoretical description is based on a quasiclassical approximation for the Gorkov equations theory of superconductivity. Quasiclassical equations, which explicitly take account of the fact that the characteristic scale of a spatial variation of the macroscopic quantities substantially exceeds the interatomic distance, has a lower order than Gorkov equations. Using approach of the quasiclassical equations for Green's functions we have obtained an analytical expression for the supercurrent through the double Josephson junction. The value of supercurrent through double-barrier

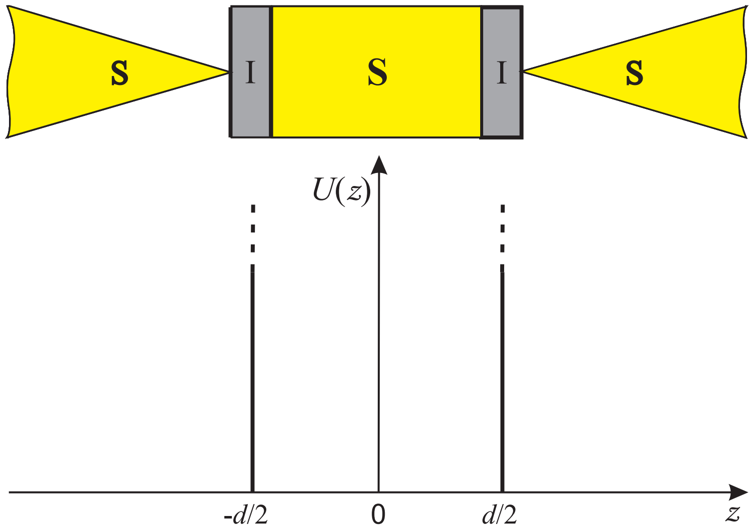

Figure 1: Model of the symmetric SISIS tunnel junction and corresponding potential.

structure has significant difference from the current in the case a SIS-junction. Firstly, we note that the critical value of supercurrent has a non-monotonous dependence on distance between barriers, with the presence of resonance peaks. This is related to the resonant tunneling of Cooper pairs through the double-barrier structure. In this paper we have obtained the values of a thickness when current has a maximum or minimum.

2 Model

Let us consider a double barrier tunnel junction with geometry as follows (S – superconductor, I – isolator)

The insulators are modeled by a delta-Dirac potential barriers

$$U(z) = \alpha \left[\delta\left(z - \frac{d}{2}\right) + \delta\left(z + \frac{d}{2}\right) \right]. \tag{1}$$

There are two mean-field theoretical approaches to description of the inhomogeneous superconductors: the Bogoliubov – de Gennes equations for the quasiparticle amplitudes $(u_{\mathbf{p}}(\mathbf{r}),\ v_{\mathbf{p}}(\mathbf{r}))$ and the Nambu – Gorkov equations for the Green's functions $G_{\omega_n}(\mathbf{r}, \mathbf{r}')$.

The Bogoliubov – de Gennes or equivalent Nambu – Gorkov equations has very complicated mathematical structure, because the unknown functions ($u_{\mathbf{p}}(\mathbf{r})$ and $v_{\mathbf{p}}(\mathbf{r})$ or $\hat{G}_{\omega_n}(\mathbf{r},\mathbf{r}'))$ are nonlinear functionals of $\Delta(\mathbf{r})$. Simplification of these equations is related with quasiclassical motion of the Cooper pairs. Indeed, the center of mass of a Cooper pair moves with velocity v_s much smaller than Fermi velocity v_F. The quasiclassical reduction has a spatial aspect, because $(\mathrm{v}_s)_{crit}/\mathrm{v}_F \sim T_c/T_F \sim a/\xi_0 \ll 1$, where T_c – critical temperature, T_F – Fermi temperature, a – interatomic distance, ξ_0 – coherence length. Therefore, the quasiclassical equations are spatially smoothed over a length scale of order interatomic distance.

Let us now expand the Green's functions with respect to the system eigenstates of the single-particle Hamiltonian with potential (1)

$$\hat{G}_{\omega_n}(\mathbf{r},\mathbf{r}')=\sum_{i,k}\int d\mathbf{p}\int d\mathbf{p}'\hat{G}^{ik}_{\omega_n}(\mathbf{p},\mathbf{p}')\chi^{(i)}_{\mathbf{p}}(\mathbf{r})\chi^{(k)}_{\mathbf{p}'}(\mathbf{r}'), \qquad (2)$$

where

$$\chi^{(1)}_{\mathbf{p}}(\mathbf{r})=\frac{1}{2\pi}e^{i\mathbf{p}_\perp\mathbf{r}}\Psi^{(1)}_{p_z}(z),\ \ \chi^{(2)}_{\mathbf{p}}(\mathbf{r})=\frac{1}{2\pi}e^{i\mathbf{p}_\perp\mathbf{r}}\Psi^{(2)}_{p_z}(z).$$

Here $p_\perp$ and p_z – are the transverse and longitudinal components of a momentum, functions $\Psi^{(1)}_{p_z}$ and $\Psi^{(2)}_{p_z}$ – are the solutions of Schrodinger equation for double delta-Dirac potential barrier (1).

Let us take into account that the values of momentum of the electrons lie close to the Fermi surface. Then $p = p_F + \xi/\mathrm{v}_F$, where $\xi \sim T_c$. The changes of momentum $p_z - p'_z \simeq \frac{\xi-\xi'}{\mathrm{v}_F x}$, $x \equiv \cos\theta$, where θ – is the angle of incidence of the electrons on a barrier. Therefore

$$\delta(p_z - p'_z) \simeq \mathrm{v}_F x\delta(\xi-\xi'),\ \ \int\limits_{-\infty}^{\infty} dp_z \ldots \simeq \frac{1}{\mathrm{v}_F x}\int\limits_{-\infty}^{\infty} d\xi \ldots$$

Next we replace the "energetic" variable ξ by a Fourier-conjugate variable t by using the base functions

$$\langle\xi|t\rangle = \frac{1}{\sqrt{2\pi \mathrm{v}_F x}} e^{-i\xi t}, \ \langle t|\xi\rangle = \frac{1}{\sqrt{2\pi \mathrm{v}_F x}} e^{i\xi t}.$$

The quasiclassical approximation for Nambu-Gorkov equations in t-representation

$$\left(i\omega_n + i\sigma_z \frac{d}{dt}\right) \hat{G}^{ik}_{\omega_n}(t,t') - \sum_j \hat{\Delta}^{ij}(t) \hat{G}^{jk}_{\omega_n}(t,t') = \delta_{ik}\delta(t-t'). \qquad (3)$$

This is first-order differential equation versus a second-order initial Nambu – Gorkov equation. Using equation (3) we should compute a Green's functions and calculate a supercurrent through the barrier.

In the theory of superconductivity widely used a model with a constant order parameter within a superconductor, when we neglect the changes of Δ under the influence of the transparency and current. When the parameter order is changing at the coherence length scale, using this model does not lead to qualitative error.

We consider next model for the order parameter

$$\Delta(z) = \Delta \begin{cases} e^{-i\varphi/2}, & z < -d/2, \\ 1, & |z| < d/2, \\ e^{i\varphi/2}, & z > d/2, \end{cases} \qquad \Delta = \text{const}. \qquad (4)$$

Here φ is the phase difference across the junction.

3 Results

Quasiclassical equations (3) for the first set of Green's functions in the region $z > d/2$ are as follows

$$\begin{cases} \left(i\dfrac{d}{dt} + i\omega_n\sigma_z - \hat{\Delta}_\varphi + 2i\Delta R\sigma_x \sin\varphi\right)\hat{G}^{1,1}_{\omega_n}(t,t') \\ -2\Delta\sqrt{DR}\sigma_x \sin\varphi\, \hat{G}^{2,1}_{\omega_n}(t,t') = \delta(t-t'), \\ \left(i\dfrac{d}{dt} + i\omega_n\sigma_z - \hat{\Delta}_{-\varphi} - 2i\Delta R\sigma_x \sin\varphi\right)\hat{G}^{2,1}_{\omega_n}(t,t') \\ +2\Delta\sqrt{DR}\sigma_x \sin\varphi\, \hat{G}^{1,1}_{\omega_n}(t,t') = 0. \end{cases}$$

Using solution of these equations for Green functions [4] we find a supercurrent through the SISIS Josephson junction at $z = d/2$

$$j = \frac{\pi}{4} e v_F N_F \frac{\Delta D \sin\varphi}{\sqrt{1 - D\sin^2\dfrac{\varphi}{2}}} \mathrm{th} \frac{\Delta\sqrt{1 - D\sin^2\dfrac{\varphi}{2}}}{2T}. \tag{5}$$

Here $N_F = \dfrac{3}{4}\dfrac{n}{E_F}$ is a density of states at the Fermi surface, D is a transmission coefficient for the double delta-function barrier.

The value of supercurrent through SISIS junction has significant difference from the current in the case a SIS-junction. The value of supercurrent has a non-monotonous dependence on distance between barriers, with the presence of resonance peaks (see Fig. 2). For $\kappa = 0.01$ the maximum current (transparency) occurs at $p_F d \simeq 6.30$ (solid line), for $\kappa = 0.02$ at $p_F d \simeq 6.32$ (dotted line), and for $\kappa = 0.03$ we get $p_F d \simeq 6.34$ (dashed line). This is related to the resonant tunneling of Cooper pairs through the double-barrier structure.

When a thickness of interior superconductor layer (in $\hbar = 1$ units)

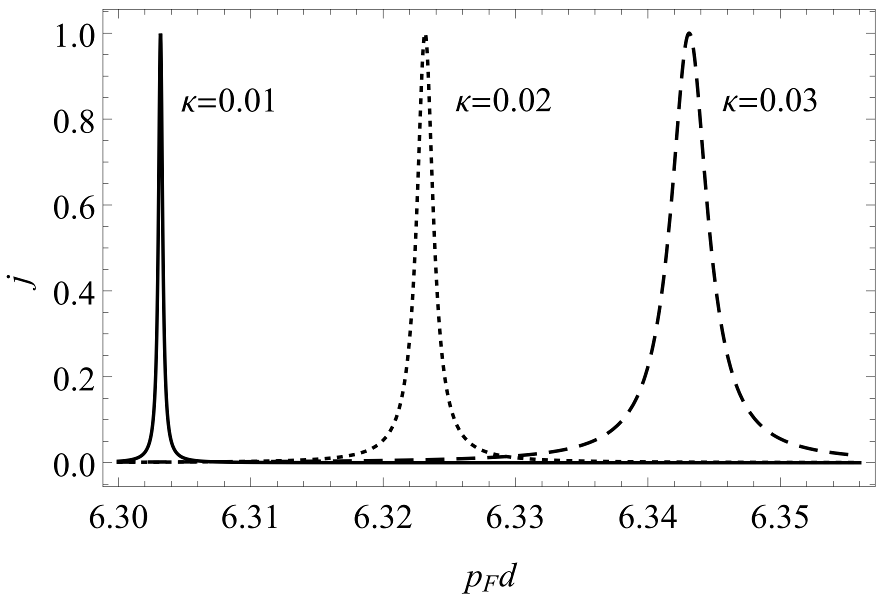

Figure 2: Dependence of a current density on a distance between the barriers (thickness of interior superconductor layer) at the different values $\kappa = \frac{p_F}{2m\alpha}$.

satisfy a condition

$$p_F d_{max} = \frac{1}{2}\left(-\text{arctg}\frac{4\kappa}{4\kappa^2-1} + 2\pi n\right),\ n \in N,$$

then current reaches the maximum values.

Minimum values of the current has been observed when

$$p_F d_{min} = \frac{1}{2}\left(\pi - \text{arctg}\frac{4\kappa}{4\kappa^2-1} + 2\pi n\right),\ n \in N.$$

Typical insulators used for the tunnel junctions (e.g., Al_2O_3) has a thickness of about $10 - 20$ nm. The barrier height is about $1 - 5$ eV. Hence, a constant α for the delta-Dirac potential (1) is about $(1-10)\times 10^{-8}$ eV·m. Since the Fermi energy for metals is about $2-10$ eV, then parameter κ takes on the values $(5-30)\times 10^{-3}$. In the case of $\kappa = 0.01$ the value of supercurrent would be maximum when $p_F d_{max} \simeq 0.019$, $p_F d_{max} \simeq 3.161$, $p_F d_{max} \simeq 6.303$, $p_F d_{max} \simeq 9.444$ etc. For the niobium-based junction with $p_F \simeq 2.192 \times 10^{-13}\ kg \cdot ms^{-1}$, the values of a thickness of interior superconductor layer that corresponds to the

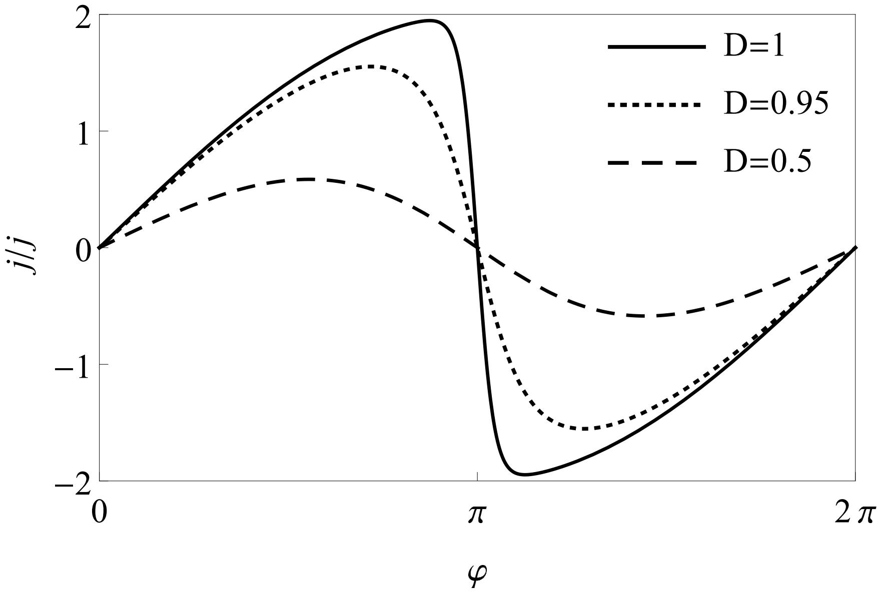

Figure 3: Current-phase relation curves for SISIS-junction at different values of a transparency D. The current density is normalized to the value $j_0 = \frac{\pi}{4} e \mathrm{v}_F N_F \Delta$.

maximum of transparency are $d_{max} \simeq 1.5\mathring{A}$, $d_{max} \simeq 3\mathring{A}$, $d_{max} \simeq 4.5\mathring{A}$ etc.

Another feature of the supercurrent in a double Josephson junction (5) is that it exhibits a non-sinusoidal current-phase relation. Current-phase relation for SISIS-junction at different values of a transparency D is shown in Fig. 3.

The value of critical current j_{max} can be found as an extremum of the supercurrent (5) with respect to phase difference φ. For the critical values of the superconducting phase difference we find

$$\varphi_{max} = \arccos\left[1 - \frac{2}{D}\left(1 - x^2\right)\right],$$

where x is a root of the transcendental equation

$$\mathrm{sh}\left(\frac{\Delta}{T}x\right) = \frac{\Delta}{T}\frac{x^2\left(1-x^2\right)\left(1-D-x^2\right)}{1-D-x^4}.$$

Let us compute the critical current at $T = 2.5$ K for junction based on niobium with energy gap $\Delta \simeq 3$ meV, critical temperature $T_{\mathrm{C}} \simeq$

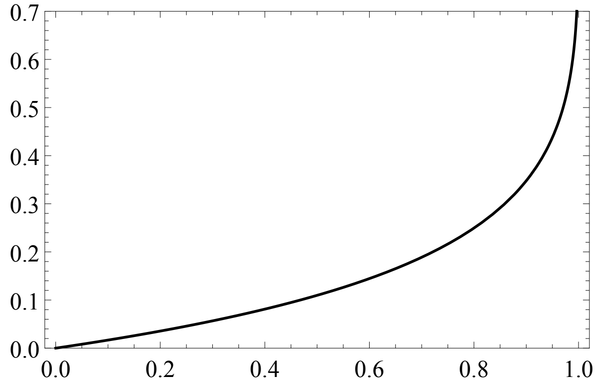

Figure 4: Skewness in the current-phase relation of a SISIS superconducting structure as a function of a transparency D.

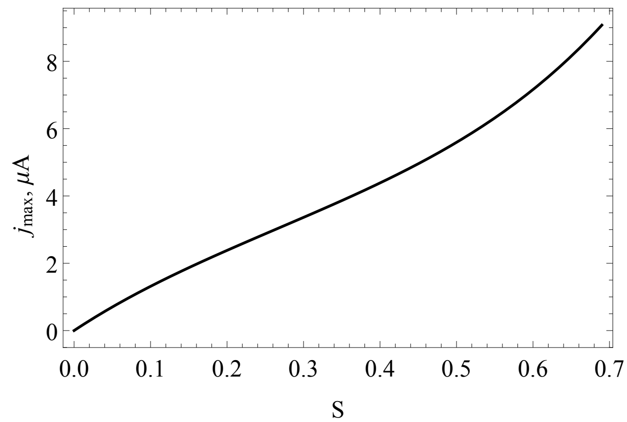

Figure 5: The critical current density j_{max} versus skewness S plot. The skewness linear increases with critical current.

9.5 K, density of states $N_F \simeq 5.56 \times 10^{28}$ m^{-3}, and Fermi velocity $\mathrm{v}_F \simeq 1.37 \times 10^6$ ms^{-1}. For this case $\Delta/T \simeq 14.2$ and

$$j_0 = \frac{\pi}{4} e \mathrm{v}_F N_F \Delta \simeq 4.59\ \mu\mathrm{A/m}^2.$$

For the case of perfect transparency, $D = 1$ the root $x \simeq 0.23$ and $\varphi_{max} = 2.67$, then we have $j_{max} \simeq 1.94 j_0 \simeq 8.9\ \mu\mathrm{A/m}^2$.

For $D = 0.5$ we obtained $x \simeq 0.84$, $\varphi_{max} = 1.75$, $j_{max} \simeq 0.59 j_0 \simeq 2.7\ \mu\mathrm{A/m}^2$.

In the Ref. [5] for identifying the behaviour of SISIS double-barrier Josephson junctions has been defined the skewness S in the current-phase relation, as follows

$$S = \frac{2}{\pi}\varphi_{max} - 1.$$

Skewness in the current-phase relation of a supercurrent is shown in Fig. 4

The dependence of a critical current density j_{max} on a skewness S is approximately a linear (see Fig. 5). The linear increasing of a skewness with critical current was observed experimentally for Josephson junctions having a graphene barrier, obtained by a phase-sensitive SQUID interferometry technique [2].

References

[1] P. Seidel, *Applied Superconductivity: Handbook on Devices and Applications*, Wiley-VCH, Berlin (2015).

[2] English, C. D. and Hamilton, D. R. and Chialvo, C. and Moraru, I. C. and Mason, N. and Van Harlingen, D. J., Phys. Rev. B **94**, 11 (2016).

[3] A. Brinkman, A. A. Golubov, Phys. Rev. B **61**, 11297 (2000).

[4] P. Shygorin, A. Svidzynskyi and I. Materian, Ukr. J. Phys. **62**, 518 (2017).

[5] De Luca R 2011 *Phys. Lett. A* **375** (24) 2441