BINARY STRUCTURING AND OPTIMIZATION OF MICROELECTROMECHANICAL SYSTEMS WITH "GOLD" PROPORTION

P.Kosobutskyy[1], I.Onishechko[2]

[1] petkosob@gmail.com

[2] onishechko.lviv@gmail.com

In the design of electromechanical systems, discrete physical models in the form of elementary springs and electrical resistances are widely used. By concurrently and sequentially interconnecting them, blocks are formed with subsequent synthesis of structures, which allows to apply mathematical models equivalent to chain fractions. This approach allows to apply the laws of mechanics and electricity in differential form and thus successfully solve both the design problems [1] and the optimization problems [2] of systems, including the methods of gold section (GS) and Fibonacci recurrence relations [3-4]. Both mathematical approaches are based on quadratic irrationalities - properties of the roots

$$x_{\pm} = \frac{-p \pm \sqrt{p^2 - 4q}}{2} = \frac{-p}{2} \pm \sqrt{D} \qquad (1)$$

of the consolidated quadratic equations (QE)

$$x^2 + px + q = 0 . \qquad (2)$$

that fulfills the theorem of Viete`s.

Irrational numbers, like rational numbers, are represented as finite chain fractions [5,6]. Therefore, the question arises whether it is possible to construct an element with a parameter $\frac{p}{q}$. The answer to the question is: why, for any fraction $\frac{p}{q}$ must first connect q elements (springs) in parallel, obtain the characteristic (for example, for the stiffness of the springs) $\frac{1}{q}$, and then multiply this p times, and connect them together successively. The mathematical model of such a compound of $\mathrm{K}(p,q)=\sum_{k=0}^{n} a_k$ elements looks like a chain fraction.

$$\frac{p}{q}=a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\ldots\cfrac{1}{a_n}}} \quad (3)$$

Grade $\mathrm{K}(p,q)=\sum_{k=0}^{n} a_k$ was justified by G. Lame [7], using for this property the Fibonacci numbers

$$F_n=\frac{1}{\sqrt{5}}\left(\Phi^n-(-\Phi)^n\right), \quad \frac{F_{n+1}}{F_n}=1+\cfrac{1}{1+\cfrac{1}{1+\ldots+\cfrac{1}{1}}} \quad (4)$$

with initial conditions $F_0=0, F_1=1$ and golden numbers $\Phi=1.618..$ and $\varphi=0.618..$

$$(-\varphi)^n \langle 1 \Rightarrow F_n \rangle \frac{1}{\sqrt{5}}\Phi^n-1. \quad (5)$$

Lame found that the number of elements does not exceed $\mathrm{K}(p,q)=n$

$$\mathrm{K}(p,q)\langle \frac{\log\left[2.3\frac{\Phi}{\sqrt{5}}\right]}{\log\Phi}. \quad (6)$$

Currently, the regularities of numbers Φ, φ and Fibonacci numbers F_n have been sufficiently studied [3-4, 8-18]. Generalized models of the relation between numbers Φ, φ and Fibonacci numbers F_n [19-20], the model of metallic averages [21-23] and other extensions have been developed [24-26]. The largest contribution to the development of the theory of proportional GS and recurrence numbers was made in 1963 by the Fibonacci Association. From 1963 she began publishing the quarterly mathematical journal The Fibonacci Quarterly, co-founded by V. Hoggatt. R. Knott created the WEB Resourse "Fibonacci Numbers and the Golden Section" [27] and later appeared sites [28-31].

The study of the regularities of the generalization of recurrence sequences $F_n = F_n(a,b,p,q)$ and other forms on the basis of square equations was initiated in [34], the characteristic equation (1.2) had a form with the initial conditions $(F_0, F_1) = (a,b)$. However, in terms of the task of research of our work, the closest in essence are models of generalization of linear functions of the main sequence of Fibonacci in the form of a recurrence ratio $F(n) = aF(n-1) + bF(n-2)$, with $n \geq 2$ [35-36], but without numerical analysis.

On the phase direction

$$|p| = |q| = k\,, \qquad (7)$$

the division model (1) as

$$\frac{x(k)}{L} = k\frac{L - x(k)}{x(k)}\,,\ k \neq 1\,, \qquad (8)$$

which has not investigated recently in the literature, and introduce the coefficient of relative changes [38-39]

$$\varphi(k) = \frac{x(k)}{L} \quad \text{and} \quad \Phi(k) = \frac{k}{\varphi(k)}\ . \qquad (9)$$

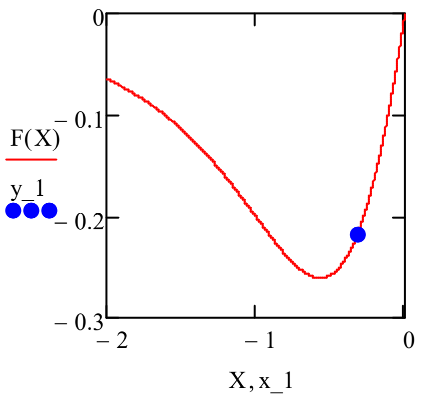
The graphs of dependency (9) for positive solutions for positive solutions of $\varphi(k) = \varphi_+(k)$ and $\Phi(k) = \Phi_+(k)$ are indicate on the fact that the graphs of both solutions converge to a common point indicates the correctness of the proposed model of generalization of the proportional division of the whole into two unequal parts in the phase direction (8). On these phases the point with coordinates $(p,q) = (k_N, k_N)$ is localized as the known point $p = +1, q = -1$ of the "golden" division with quantitative characteristics $\varphi = +0.168,\quad \Phi = -1.618$. It was the idea that allowed us to propose the physical principle of binary structuring of systems [40] and a new approach to the application of the "golden" division and recurrent Fibonacci relations for one-dimensional optimization of target functions. One of the options for implementing this approach is shown in the figure.

Known method

x_1 := optim_1(a, b, E)$_0$ = −0.318

y_1 := F(x_1) = −0.217

k_1 := optim_1(a, b, E)$_1$ = 17

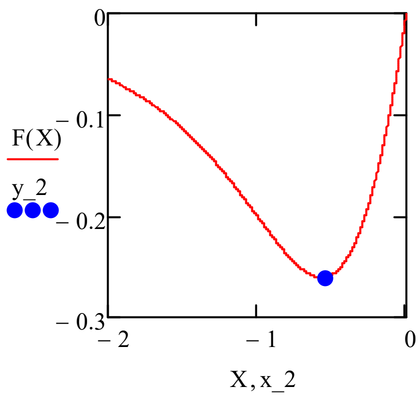


New method

x_2 := optim_2(a, b, E)$_0$ = −0.551

y_2 := F(x_2) = −0.261

k_2 := optim_2(a, b, E)$_1$ = 9



Reference

1.Kosobutskyy P. Modelling of electrodynamic Systems by the Method of Binary Seperation of Additive Parameter in Golden Proportion.. Jour. of Electronic Research and Application, 2019,3(3):8-12,

2. Kosobutskyy P. et.al. Physical principles of Optimization of the Static Regime of a Cantilever-Type Power-effect Sensor with a Constant Rectangular Cross Section. Jour. of Electronic Research and Application, 2018, 2(5) :11-15.

3. Vorobyov N. Fibonacci Numbers. Moscow,1961.

4. Hoggat V., Fibonacci and Lucas Numbers. MA: Houghton Mifflin, Boston, 1969

5. The irrationals are the roots of a quadratic equation with integer coefficients p, q called quadratic irrationalities. The quadratic irrationality is represented by

the form $\alpha = \frac{p+\sqrt{D}}{q}$ where D (D> 1) is an integer non-square number. The second irrational root $\alpha = \frac{p-\sqrt{D}}{q}$ is conjugate to α.

6. Chain fractions were introduced by Bombelli to calculate square roots. The theory of chain fractions was developed by Euler. Important from the practical point of view of the properties of chain fractions discovered Christian Huygens. He already knew that chain fractions were non-redundant and that they represented the best rational approximation, so he used gear wheels to build a planetarium with a gear ratio $\sqrt{2}$.

7. Gelfond A.O. Calculation of finite differences. M.: GI FM, 1959

8. Huntley H. The Divine Proportion: A Study in Mathematical Beauty. Dover Publications,Inc., New York,1970.

9. Vajda S., Fibonacci & Lucas Numbers, and the Golden Section. Theory and Applications. Ellis Horwood limited, 1989.

10. Dunlap R. The golden ratio and Fibonacci numbers. World Scientific Publishing Co. Pte. Ltd. 1997

11. Gazale M. Gnomon. From Pharaohs to Fractals. Princeton, New Jersey: Princeton University Press, 1999

12. Stakhov A., Massingue V., Sluchenkova A . Introduction into Fibonacci coding and cryptography. Kharkiv: Osnova (Ukraine), 1999.

13. Koshy T. Fibonacci and Lucas numbers with application. A Wiley-Interscience Publication: New York, 2001

14. Livio M. The Golden Ratio: The Story of Phi, the World's Most Astonishing Number, Broadway Books, New York, 2002.

15. Smirnov V. The Golden Section – Basic the Mathematics and Physics in Future. The Spiral of the Universe Development, San-Peterb.,2002.

16. Petrunenko V. The Golden Section of quantum states and its astronomical and physical manifestations. – Minsk: Pravo i economika, 2005.

17. Bodnar O. The Golden Section and Non-Euclidean Geometry in Science and Art. Lviv: Publishing House Ukrainian Technologies, 2005

18.Semenute N. Analysis of linear electrical circuits by the method of recurrence numbers. Gomel: Bel.GUT, 2010.

19. Engstrom P. Section, golden and not so golden. Fibonacci Quart.1987,**25**(2) :118-128.

20. Bradley S. A geometric connection between generalized Fibonacci sequences nearly golden section. Fibonacci Quart.,2000, 38(2) : 174-179.

21. V. de Spinadel. The Metallic Means and Design, Nexus II: Architecture and Mathematics. Editor: Kim Williams. Edizioni dell'Erba, 1998

22. V. de Spinadel. The metallic means family and multifractal spectra. Nonlinear Analysis. 1999, 36: 721–745.

23. V. de Spinadel.The Metallic means Family and Art. Aplimat. Journal of Appl. Mathematics.2010, 3(1) :53-64.

24. Shneider R. Fibonacci numbers and the golden ratio. From Web Resource: VarXiv:1611.07384v1 [math.HO] 22 Nov 2016.

25. Falcon S. Generalized (k, r)–Fibonacci Numbers. Gen. Math. Notes.2014, 25(2) :148-158.

26. Agaian S. and J. Gill II. The Extended Golden Section and Time Series Analysis. Frontiers in Signal Processing,2017, 1(2).

27.http://www.mscs.dal.ca/Fibonacci/

28.http://britton.disted.camosun.bc.ca/goldslide/jbgoldslide.htm,

29.http://www.fhfriedberg,

30.de/users/boergens/marken/beispiele/goldenerschnitt.htm

31.http://www.goldenmuseum.com/1801Refer_rus.html.

32. Kharitonov A. Structural exposition of composite systems. Applied Physics, 2007, №1: 5-9

33. The division of the segment in the middle and extreme positions are related to the geometric problem. In its essence, the segment is divided into two parts, of which the greater part, called the "golden section", is the average proportion between the smaller part and the whole segment, and the corresponding proportion is not an identity but an equality.

34. Horadam A. Basic Properties of a Certain Generalized Sequence of Numbers. Fibonacci Quart., 1965, 3(3) :161-176.

35. Kalman D.,Mena R. The Fibonacci Numbers – Exposed. The Mathematical Magazine, 2002, 76(3):167-181.

36. Shneider R. Fibonacci numbers and the golden ratio. VarXiv:1611.07384v1 [math.HO] 22 Nov 2016.

37. Larcombe P. Horadam Sequences: A Survey Update and Extension , Bulletin of the ICA, 2017, 80: 99–118.

38. Kosobutskyy P. S. Phidias numbers as a basis for Fibonacci analogues. Notes on Number Theory and Discrete Mathematics (Bulgaria). **26**(1), 172—178, 2020.

39. P.Kosobutskyy , et.al.. Mathematical methods for cad: the method of proportional division of the whole into two unequal parts 40. Bulletin of the LPNU. Lviv. Collection of scientific works. Scientific publication. Series: Computer Design Systems. Theory and practice. №908, 75-90, 2019

40. P.Kosobutskyy, et.al. Physical principles of optimization of the static regime of a cantilever-type power-effect sensor with a constant rectangular cross-section. Journal of Electronic Research and Application (Australia). **2**(5). – P.11–15, 2018.