

**FOURIER SPECTRUM OF OSCILLATIONS OF THE  
AMPLITUDE FUNCTION OF THE INCOMMENSURATE  
PHASE FROM THE MAGNITUDE OF THE  
ANISOTROPIC INTERACTION, DESCRIBED BY  
DZIALOSZYNSKI INVARIANT**

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The evolution of a incommensurate phase (IP) in the first approximation is determined by two competing influences: long-range and anisotropic interaction. The influence of each role (which is determined by the parameters  $T$  and  $K$ , respectively) at different stages of the dynamics of the superstructure, when the influence of the order parameter is both spontaneous deformation ( $n = 3$ ) and spontaneous polarization ( $n = 4$ ). To this end, the Fourier study of the spectrum of the amplitude modulation function from the magnitude of the anisotropic interaction was performed.

The calculation of the spatial changes in the amplitude of the order parameter was performed for systems described by two second-order differential equations.

$$R'' - R^3 + (1 - \phi'^2 + T\phi')R - R^{n-1}K(\cos n\phi + 1) = 0, \quad (1)$$

$$\phi'' + 2\frac{R'}{R}\left(\phi' - \frac{T}{2}\right) + R^{n-2}K\sin n\phi = 0. \quad (2)$$

where  $T = \frac{\sigma}{(\gamma r)^{\frac{1}{2}}}$ ,  $K = 2^{-\frac{n}{2}}r^{\frac{n}{2}-2}n\omega u^{1-\frac{n}{2}}$  — dimensionless parameters,  $n$  — is an

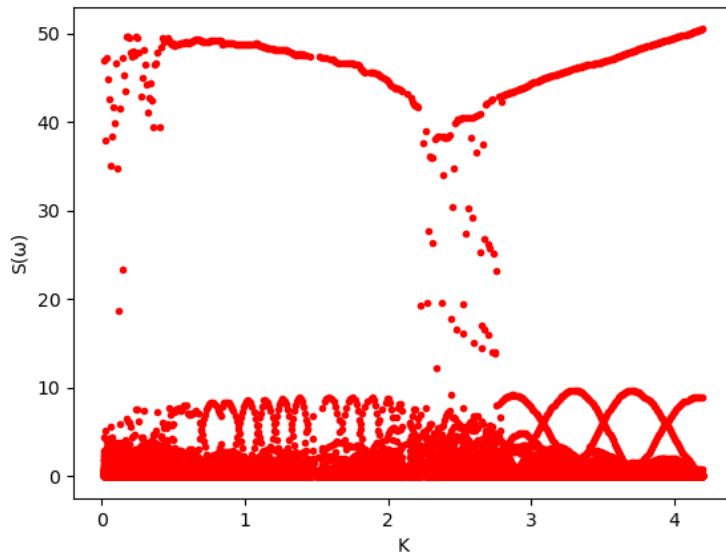
integer characterizing the symmetry of the potential, and dimensionless variables

$$\eta = \left(\frac{r}{2u}\right)^{12} R, \quad z = \left(\frac{\gamma}{r}\right)^{12} \xi.$$

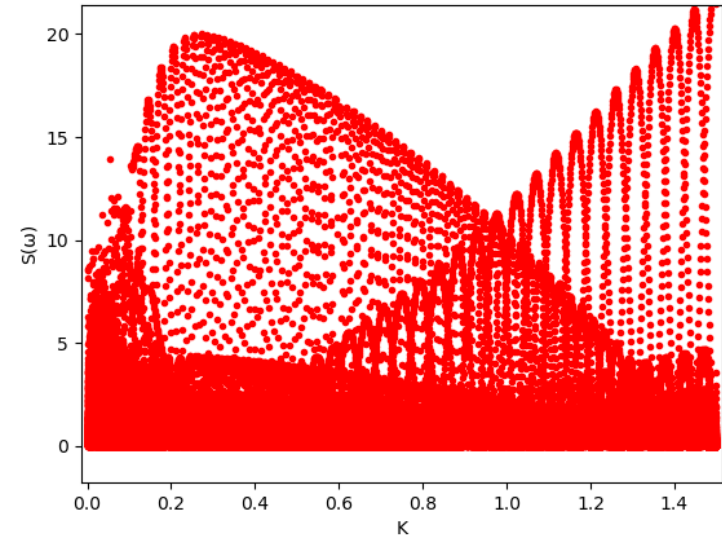
System of (1) and (2), consisting of two second-order differential equations, was solved by the numerical BDF method.

The calculation of the spatial changes of the amplitude and phase of the order parameter was performed in the Python environment using the Skipy and JiTCODE libraries.

According to expressions (1) and (2) the parameter  $T$  describes a long-range interaction and the parameter  $K$  is anisotropic determined by the Dyaloshinsky invariant.



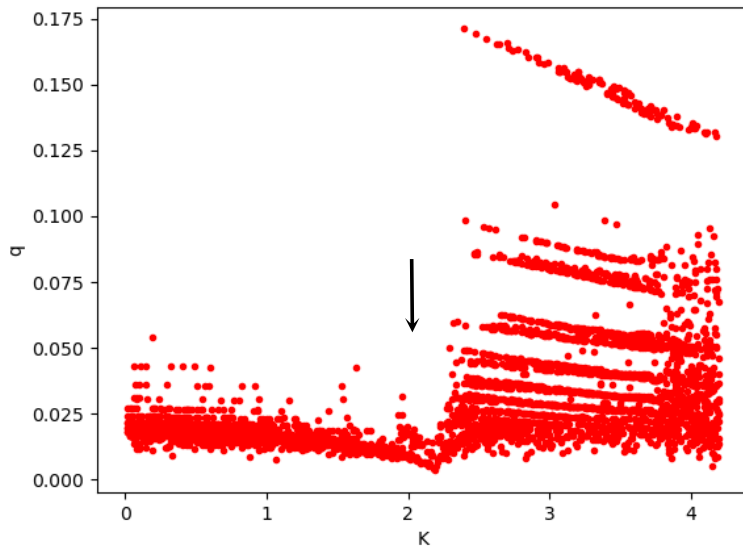
**a**



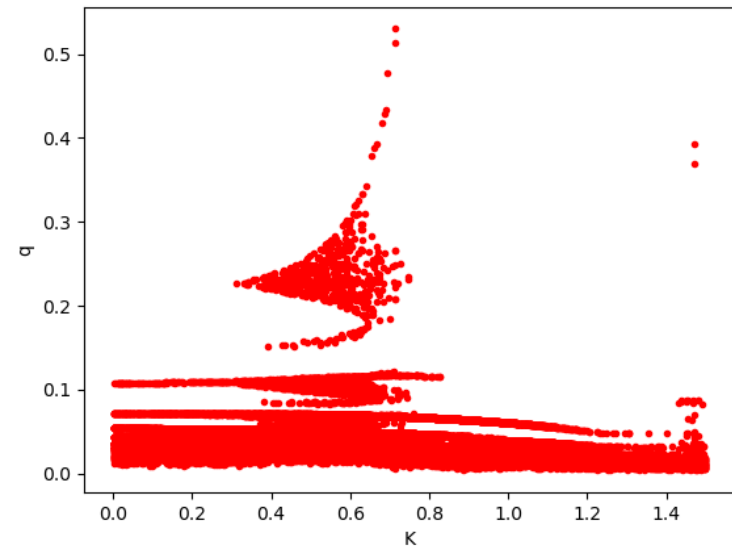
**b**

Fig.1. Fourier spectrum of oscillations of the amplitude function of the superstructure IP from the value of the anisotropic interaction (which is described by the parameter  $K$ ) for: a)  $n = 4$ ; b)  $n = 3$ .

Figure 1 shows the Fourier spectrum of oscillations of the amplitude function of the superstructure of the superstructure from the parameter  $K$ . According to the obtained dependences, this structure is characterized by a chaotic state in the process of its origin ( $K = 0 \div 0.3$  for  $n = 4$  and  $K = 0 \div 0.1$  for  $n = 3$ ). The transition to the soliton regime is accompanied by the appearance of new oscillations ( $K = 2.1$  for  $n = 4$  and  $K = 0.6$  for  $n = 3$ ). Subsequent changes in the parameter  $K$  cause the appearance of chaotic behavior  $q$ , which indicates the transition of the system to the stochastic mode of the superstructure with the emergence of a chaotic phase ( $K > 4.2$  for  $n = 4$  and  $K \Rightarrow 1.5$  for  $n = 3$ ).



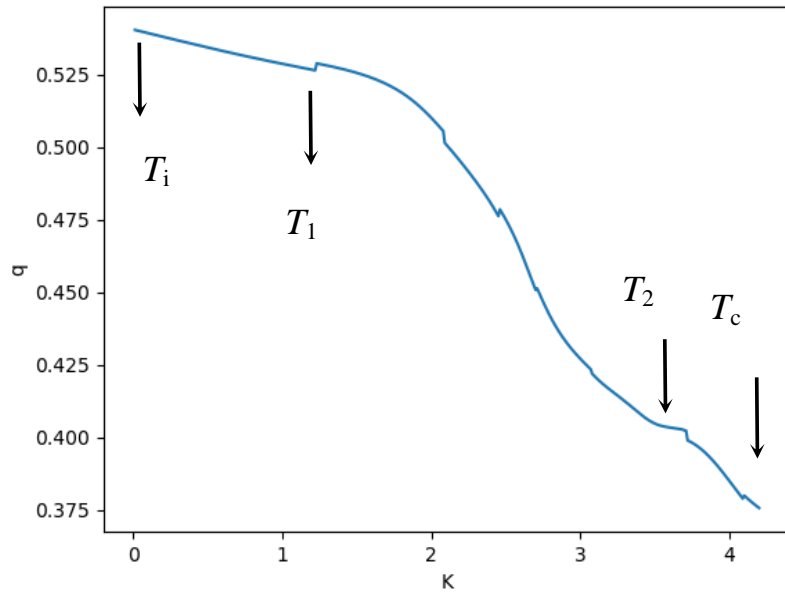
**a**



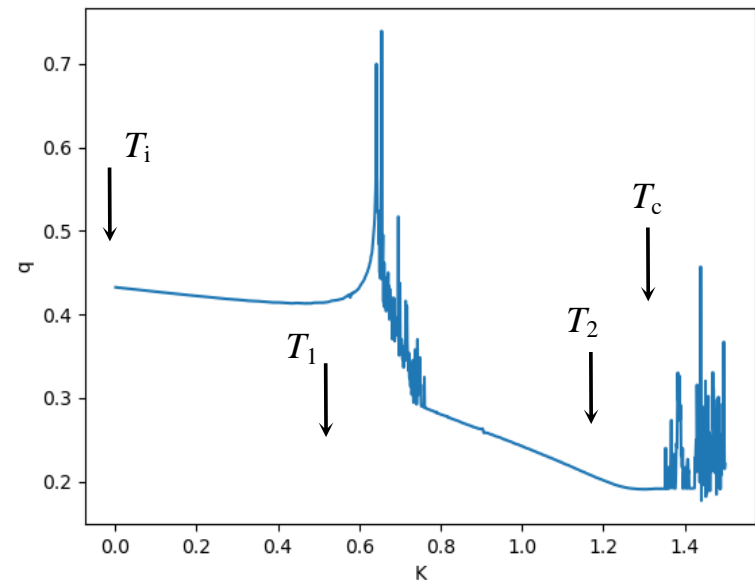
**b**

Fig.2. Dependence of the wave vector ( $q$ ) harmonic of incommensurate modulation on the anisotropic value described by the Dzialoszynski invariant (parameter  $K$ ) under the condition  $n = 4$  a), and  $n = 3$  b).

The behavior of the harmonics of the IP modulation from the parameter  $K$  indicates complex transformations that occur during the transition from sinusoidal to soliton mode (Fig. 2).



**a**



**b**

Fig.3. Evolution of the wave vector of a incommensurate superstructure from the value of the anisotropic interaction  $K$  at a constant value of  $T = 1.0$ . Where  $T_i$  is the temperature of transition to the incommensurate phase;  $T_1$  - transition to the soliton mode of the IP superstructure;  $T_2$  - transition to the stochastic mode of the IP superstructure;  $T_s$ -phase transition to the commensurate phase.

Therefore, based on studies of the harmonics of the IP modulation, it can be confirm that the transition to the soliton mode of the IP superstructure is accompanied by change frequency spectrum of incommensurate modulation.